Discrete-event implementation of observer-based feedback control of manufacturing system

Antoinette G. N. Kommer, Alexander Y. Pogromsky, Boris Andrievsky and Jacobus E. Rooda

***Abstract*— The control of a manufacturing machine, modeled as an integrator with input saturation, is studied. An unknown demand must be tracked. A proportional feedback controller with reduced-order observer is proposed. This controller guarantees uniformly convergent system behavior in the presence of input fluctuations and external disturbances. The performance of the continuous system is considered in frequency and time domain. It is described how the controller can be implemented in discrete-event setting, and simulation results show that the proposed controller can be used to control a discrete-event model of a machine.**

***Index Terms*— Manufacturing systems, Industry automation, Feedback control, Observers, Discrete-event systems**

I. Introduction

Nowadays, manufacturing lines become more and more complex. In order to stay ahead of the competition, companies must use good control strategies to control their machines.

An efficient method of modeling and controlling a manufacturing system is to use a continuous approximation of a manufacturing machine. Products are represented by a continuous flow. In the so-called *flow models*, the system is modeled using ordinary differential equations (ODEs) [1]. A manufacturing machine is often modeled as an integrator. In this case, the total production output is the integral over the production rate. The production rate is bounded by zero and the maximum machine capacity.

The analysis and control of ODE models is widely studied, and many control strategies are already available. A frequently used control strategy that is based on these ODE models is *model-based predictive control* (MPC) [2]. The advantage of MPC is that it is a robust method resulting in good demand tracking, and that it is able to control a manufacturing system real-time [3]. Moreover, MPC is able to take into account hard constraints, such as the maximum machine capacity. Although MPC is effective in the control of continuous approximations of manufacturing systems, it has two main disadvantages. Firstly, the optimization problem that has to be solved is very detailed, so it requires much computational effort. Secondly, all MPC strategies are based on a forecast of the future demand within a certain control horizon. Since the future demand is difficult to predict, it is often inaccurate.

Another frequently used method is a feedback control strategy that considers only the present and the past of the systems and signals. Like MPC, it uses a continuous approximation of a machine, but the future demand is not estimated and the control problem is less complex. An example of such a control strategy is a feedback control law that includes proportional and integral (PI) components. This is the most frequently used feedback control strategy in industrial processes and manufacturing plants. Due to the bounds in the production capacity, implemented as a saturation of the control signal, the use of a PI-controller can lead to so-called *integrator windup*. The PI-controller assumes the overall system to behave linearly at all time, so integration proceeds even when the actuator cannot provide more power. This results in oscillatory behavior. By adding an *anti-windup strategy* the integrator can be "turned off" whenever the control signal saturates, resulting in good performance of the system [4]–[7]. However, the need of an anti-windup strategy complicates the controller design.

Following [8], a different control scheme is used in the present paper, namely *the proportional feedforward–feedback controller with an observer*. The observer estimates a sum of the external disturbance and the averaged desired production rate. Because no integral action is used in this controller, integrator windup cannot occur, so an anti-windup strategy is not required. In the present paper, these results are extended to a discrete-event model of a single manufacturing machine. In future research the observer-based feedback controller will be applied to lines of manufacturing machines to analyze the performance of more complicated systems.

This paper is outlined as follows. The output feedback control law is presented in Section II. In Section III the closed-loop system with the continuous flow manufacturing machine model is analyzed, including frequency-domain analysis via the modified harmonic linearization method and the simulation results for typical production demands. In Section IV an implementation of the proposed controller in a discrete-event setting is given. The simulation results for a discrete-event system with and without external disturbance are presented in Section V. Section VI concludes this paper.

II. Control of a manufacturing machine

A. Problem statement: A continuous flow model

Following [7], [9], let us use a continuous approximation of the single discrete-event manufacturing machine. Namely,

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consider a manufacturing machine that produces products with a *production rate* $u_\mathrm{p}(t) \in \mathbb{R}$, $t \in \mathbb{R}$. Assume that there is always sufficient raw material to feed the machine. The total amount of products produced by the machine is denoted by $y(t) \in \mathbb{R}$ and is related to production rate $u_\mathrm{p}(t)$ by the following equation:

$$\dot{y}(t) = u_\mathrm{p}(t) + f(t), \tag{1}$$

where $f(t) \in \mathbb{R}$ stands for an unknown *external disturbance*. This term may describe manufacturing losses, or variations of the machine capacity, for example. The production rate u_p can not be negative and has a certain upper bound $u_{\mathrm{p,max}}$ caused by the machine capacity limitation. Therefore, the following bounds are valid for the production rate:

$$0 \leq u_\mathrm{p} \leq u_{\mathrm{p,max}}. \tag{2}$$

Inequalities (2) introduce a *saturation* in the control loop. The saturation effect complicates design of the control law and the system performance analysis.

The control aim is to track the non-decreasing *reference production signal* $y_\mathrm{d}(t)$. In what follows we assume that $y_\mathrm{d}(t)$ may be modeled as:

$$y_\mathrm{d}(t) = y_{\mathrm{d},0} + v_\mathrm{d} t + \varphi(t), \tag{3}$$

where $y_{\mathrm{d},0}$ denotes the average desired production at $t = 0$, v_d is a constant that represents the average desired production rate, and $\varphi(t)$ is a bounded function, describing a fluctuation of the desired production from the linearly increasing time-varying demand, caused by market (e.g. seasonal) fluctuation. It is natural to suppose that $0 \leq v_\mathrm{d} \leq u_{\mathrm{p,max}}$. It may be also assumed that $\varphi(t)$ has a "zero mean" in a some sense because its averaged value may be considered as a part of $y_{\mathrm{d},0}$.

B. Feedforward-feedback controller

Assuming that the variables $y(t)$, $f(t)$ and the value of v_d may be measured, let us take the control law in the following feedforward–feedback form:

$$u_\mathrm{p}(t) = \mathrm{sat}_{[0,u_{\mathrm{p,max}}]}\left(k_\mathrm{p} e(t) + v_\mathrm{d} - f(t)\right), \tag{4}$$

where $e(t) = y_\mathrm{d}(t) - y(t)$ denotes the *tracking error*, $k_\mathrm{p} > 0$ is the controller parameter (a *proportional gain*), and $\mathrm{sat}(\cdot)$ denotes the *saturation function*:

$$\mathrm{sat}_{[a,b]}(z) = \min\left(b, \max(a,z)\right) = \begin{cases} b & \text{if } z \geq b, \\ a & \text{if } z \leq a, \\ z & \text{otherwise}, \end{cases} \qquad (b > a). \tag{5}$$

Equations (1) and (4) describe the closed-loop manufacturing system model for time-varying demand $y_\mathrm{d}(t)$ given by (3).

C. Observer-based feedback controller

The assumption that the average desired production rate v_d and the external disturbance $f(t)$ can be measured is rather impractical. To design a controller that uses only the measurements of the tracking error $e(t)$, replace v_d and $f(t)$ in law (4) by their estimates $\hat{v}_d(t)$ and $\hat{f}(t)$, which are produced by an observer (state estimator). An observer reconstructs the state of a system by estimating external signals using the signals measured and a certain dynamical system [10]. To keep the observer dynamics simple, it is assumed that v_d and $f(t)$ are constants and that $\varphi(t) \equiv 0$. Differentiating $e(t) = y_\mathrm{d}(t) - y(t)$ with respect to t obtains $\dot{e}(t) = v_\mathrm{d} - u_\mathrm{p}(t) - f(t)$. It can be seen that v_d and $f(t)$ cannot be estimated separately if only $e(t)$ is measured. However, it is possible to estimate the combined signal $r(t) = v_\mathrm{d} - f(t)$. This allows us to rewrite the system as:

$$\begin{cases} \dot{e}(t) = -u_\mathrm{p}(t) + r(t), \\ \dot{r}(t) = 0. \end{cases} \tag{6}$$

Following [11], [12], a new variable $\sigma(t)$ is introduced and defined as a linear combination of the measured variable $e(t)$ and the estimated variable $\hat{r}(t)$: $\sigma(t) = \hat{r}(t) - Le(t)$, with $L > 0$ the observer gain. The reduced-order observer that computes the estimate $\hat{r}(t)$ can now be described as follows:

$$\begin{cases} \dot{\sigma}(t) = -L\sigma(t) - L^2 e(t) + L u_\mathrm{p}(t), \\ \hat{r}(t) = \sigma(t) + Le(t). \end{cases} \tag{7}$$

In control action (4) the estimate $\hat{r}(t)$ can be used instead of $v_\mathrm{d} - f(t)$ $\left(= r(t)\right)$:

$$u_\mathrm{p}(t) = \mathrm{sat}_{[0,u_{\mathrm{p,max}}]}\left(k_\mathrm{p} e(t) + \hat{r}(t)\right), \tag{8}$$

Equations (7) and (8) describe the first-order controller that uses only the tracking error $e(t)$ to compute the control signal $u_\mathrm{p}(t)$. Gains k_p and L are the controller design parameters.

In [8] it is shown that the controller (7), (8) ensures vanishing tracking error if $\varphi(t)$ and $f(t)$ are constants, and that for differentiable, time-varying signals $\varphi(t)$ and $f(t)$ the controller ensures convergence to the same trajectory for different initial conditions, i.e. the control system possesses the *uniform convergence property* (see [9], [13]–[16] for details). The bounded tracking error is as desired and cannot be improved by using integral action in the controller. Therefore, PI-control with an anti-windup strategy is not required.

III. Performance analysis for continuous flow manufacturing machine model

A. Frequency domain analysis of a manufacturing system

In this section the system response to a harmonic excitation $\varphi(t)$ is studied. The complementary sensitivity function is used as a measure of how well the system can track different frequencies of $\varphi(t)$. In order to use this function, the system is first linearized using the modified harmonic linearization method. See e.g. [7] for details about well-posedness and accuracy of harmonic linearization of Lur'e type systems, which is the type of system considered here.

1) Coordinate transformation and state space description: To perform a coordinate transformation of system (6)–(8), new variables are introduced:

$$\begin{aligned} m &= u_{\mathrm{p,max}}/2, \\ z(t) &= y(t) - y_{\mathrm{d},0} - v_\mathrm{d} t, \\ \psi(t) &= \sigma(t) - m, \\ \rho(t) &= Le(t) + \sigma(t) - m, \\ \eta(t) &= m + f(t) - v_\mathrm{d} = m - r(t). \end{aligned}$$

Additionally, the symmetric saturation function is used:

$$\mathrm{sat}_a(z) = \min\big(a, \max(-a,z)\big) \tag{9}$$

With the "unsaturated" control signal $u(t) = k_\mathrm{p}e(t) + \hat{r}(t)$ and the shifted control signal $u_\mathrm{m}(t) = u(t) - m$, the control signal $u_\mathrm{p}(t)$ (8) can be described as:

$$u_\mathrm{p}(t) = \mathrm{sat}_{[0,u_{\mathrm{p,max}}]}\big(u(t)\big) = \mathrm{sat}_m\big(u_\mathrm{m}(t)\big) + m. \tag{10}$$

With these new variables the tracking error becomes $e(t) = \varphi(t) - z(t)$ and the observer (7) becomes:

$$\dot{\sigma}(t) = -L\sigma(t) - L^2(\varphi(t) - z(t)) + L\big(\mathrm{sat}_m(u_\mathrm{m}(t)) + m\big). \tag{11}$$

Now, the system can be described as follows:

$$\begin{cases} \dot{z}(t) = \dot{y}(t) - v_\mathrm{d} = \mathrm{sat}_m(u_\mathrm{m}(t)) + \eta(t), \\ \dot{\psi}(t) = -L\psi(t) - L^2(\varphi(t) - z(t)) + L\,\mathrm{sat}_m(u_\mathrm{m}(t)), \\ u_\mathrm{m}(t) = (k_\mathrm{p} + L)(\varphi(t) - z(t)) + \psi(t). \end{cases} \tag{12}$$

This system can be written in the Lur'e form:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\phi(u_\mathrm{m}) + Fw(t), \\ u_\mathrm{m}(t) = Cx(t) + Dw(t), \\ z(t) = Hx(t). \end{cases} \tag{13}$$

with the nonlinearity $\phi(u_\mathrm{m}) = \mathrm{sat}_m(u_\mathrm{m}(t))$, the state vector $x(t) \in \mathbb{R}^2$ defined as $x(t) = [z(t),\ \psi(t)]^\mathrm{T}$, and the external input vector $w(t) \in \mathbb{R}^2$ as $w(t) = [\varphi(t),\ \eta(t)]^\mathrm{T}$. The following matrices are obtained:

$$A = \begin{bmatrix} 0 & 0 \\ L^2 & -L \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ L \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 1 \\ -L^2 & 0 \end{bmatrix}, \tag{14}$$
$$C = \big[-(k_\mathrm{p} + L),\ 1\big], \quad D = \big[k_\mathrm{p} + L,\ 0\big], \quad H = [1, 0].$$

2) Harmonic linearization: Due to the convergency property of system (13), (14) the steady-state solution $\bar{x}(t)$ is unique for given input signals. Consequently, system (13) can be approximated by the following linear system, which has steady-state solution $\bar{\xi}(t)$:

$$\begin{cases} \dot{\xi}(t) = A\xi(t) + BK\zeta(t) + Fw(t), \\ \zeta(t) = C\xi(t) + Dw(t), \\ z(t) = H\xi(t). \end{cases} \tag{15}$$

The nonlinear function $\phi(u_\mathrm{m})$ is replaced by a linear gain K, which is to be determined. Because the response of the system to only the input $w(t) = \varphi(t)$ is analyzed here, the second columns of matrices F and D are omitted; the other matrices are defined by (14). If system (15) is forced by a harmonic input $w(t) = b\sin(\omega t)$ with $b > 0$ and $\omega > 0$ and if $(A + BKC)$ does not resonate at ω, then the steady-state output is also harmonic: $\bar{\zeta}(t) = a\sin(\omega t + \alpha)$ with phase shift α, amplitude a and the same frequency ω as the input. The gain K varies with the amplitude a. For a saturation nonlinearity the relation between K and a is given by the following *describing function*:

$$K(a) = \begin{cases} 1, & a \le m, \\ \dfrac{2}{\pi}\left(\arcsin\left(\frac{m}{a}\right) + \frac{m}{a}\sqrt{1 - \frac{m^2}{a^2}}\right), & a > m. \end{cases} \tag{16}$$

The relation between the amplitudes b and a is given by the following *harmonic balance equation*:

$$\big|1 - K(a)C(i\omega I - A)^{-1}B\big|^2 a^2 = |C(i\omega I - A)^{-1}F + D|^2 b^2, \tag{17}$$

where I is the appropriate identity matrix and $i^2 = -1$.

As is shown in [7], if a Lur'e system meets the following requirements:

- The matrix A is Hurwitz,
- The nonlinearity $\phi(y)$ is odd and satisfies the incremental sector condition for some $\mu > 0$: $0 \le \frac{\phi(y_1) - \phi(y_2)}{y_1 - y_2} \le \mu$, for all y_1, y_2, $y_1 \ne y_2$,
- $\mathrm{Re}\big(C(i\omega I - A)^{-1}B\big) > -\frac{1}{\mu}$, for given $\omega \in \mathbb{R}$,

then for any $b > 0$ there is a unique positive real solution $a(b,\omega)$ to the harmonic balance equation (17). This is the case for system (13). Because one input leads to one uniquely defined output, the system can be analyzed using the sensitivity functions, as is described hereafter.

3) Sensitivity functions of the linearized system: Two performance measures for analysis in frequency domain are the sensitivity and complementary sensitivity functions, which can be used for linear ODE systems. In [17] these functions are generalized for convergent nonlinear Lur'e systems, such as the system described in this paper. Because these functions are nonlinear and depend on both the input frequency ω as well as the amplitude b, the computational costs are high. Therefore, instead of the generalized sensitivity functions the linear sensitivity functions are used here for the linearized system (15). As this system depends on both ω and b, these sensitivity functions do as well.

In [7] the accuracy of the harmonic linearization method can be found. It is expressed as the bounds of the $\mathscr{L}_2$-norm of the error between outputs of nonlinear system and its harmonically linearized model with harmonic input. Recall that $\mathscr{L}_2$-norm of the T-periodic function $z(t)$ is understood as $\|z\|_2 = \left(T^{-1}\int_0^T z(\tau)^2\,d\tau\right)^{1/2}$.

In Fig. 1 the complementary sensitivity function $T(b,\omega)$ for the harmonically linearized system (15) is plotted for different values of the maximum amplitude b of the fluctuation on the input $\varphi(t)$. In addition, the complementary sensitivity function of the linear system, system (13) without the saturation nonlinearity, is plotted. An $*$ in the figure indicates the maximum values of ω (for each different value of b if $u_{\mathrm{p,max}} = 1$ and $v_\mathrm{d} = 0.5$) for which the derivative of the demand y_d defined by (3) is non-negative, as is one of the requirements stated in Section II.

B. Time-domain analysis for continuous flow model

In this section the simulation results for typical demand signals for the continuous system are presented. The following parameters are used for simulations:

– Machine: $u_{\mathrm{p,max}} = 1.0$ and $y(0) = 1$,
– Reference: $y_{\mathrm{d},0} = 0$, $v_\mathrm{d} = 0.5$ and harmonic fluctuation $\varphi(t) = b\sin(\omega\ t)$ with $b = 1$ and $\omega = 0.5$,
– Controller: $k_\mathrm{p} = 5$ and $L = 25$.

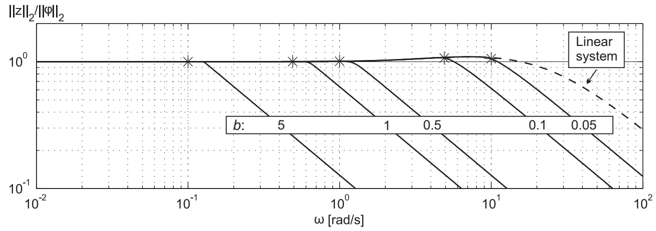
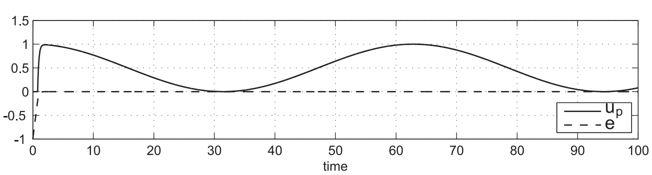


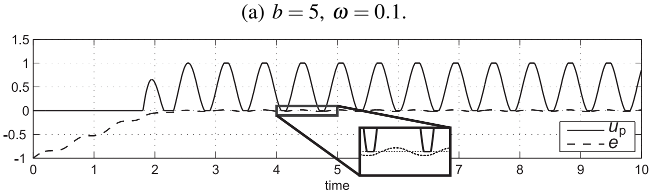
Fig. 1: $T(b,\omega)$ of system (15) for various values of the amplitude b of the fluctuation $\varphi(t)$, and $T(\omega)$ of the linear system, all for $u_{\mathrm{p,max}} = 1$, $k_{\mathrm{p}} = 5$ and $L = 25$. For each b the maximum value of ω is indicated with an $*$.

The observer gain L is chosen five times as large as the control gain k_{p} to ensure that the observer dynamics are five times faster than the other system dynamics.

The time responses of the continuous system (13) for two different amplitudes b of the demand fluctuations $\varphi(t)$, each with their own maximum frequency ω (see Fig. 1) are shown in Fig. 2. Note the difference in time scales. As can be seen, for $b = 5$ and $\omega = 0.1$ the control signal u_{p} varies just within its bounds (2), and the controller is able to track the demand with the output; the tracking error $e(t)$ becomes zero after a transient response. This response is as expected, as the value of the complementary sensitivity function equals 1 for this input signal, see Fig. 1. For $b = 0.05$ and $\omega = 10$, Fig. 2b shows that the input causes the control signal u_{p} to saturate, resulting in a steady-state error, see the part of Fig. 2b that is zoomed in. This also corresponds with Fig. 1, as for this combination of frequency and amplitude the system is more sensitive.



(a) $b = 5$, $\omega = 0.1$.



(b) $b = 0.05$, $\omega = 10$.

Fig. 2: Time responses of the continuous system (13) for different harmonic demand fluctuations $\varphi(t)$.

Fig. 2 shows that the results of the continuous flow model of a manufacturing machine controlled by an observer-based feedback controller are satisfactory. In the following sections it is described how this controller can be used to control a discrete-event model of a manufacturing machine and examples of time-domain analysis are given.

IV. Discrete-event domain implementation

In the previous sections control law (8) with observer (7) was derived for the continuous approximation of the manufacturing machine (1) with reference output (3). The performance was analyzed in frequency and time domain. In this section it is described which steps have to be taken to implement the controller in a discrete-event setting.

In the discrete-event domain the manufacturing machine produces discrete products $y(t) \in \mathbb{N}$. In this paper it is assumed that the machine produces products only at a fixed rate, so the machine is either busy or idle. Because of that, the production rate of the discrete-event manufacturing machine can have only two values: 0 or $u_{\mathrm{p,max}}$. The processing time, which is the time it takes to produce one product, is $t_0 = 1/u_{\mathrm{p,max}}$.

Although in discrete-event setting the output $y(t)$ is not continuous anymore, the continuous controller can still be used to control the machine, as it uses only the tracking error $e(t)$ as its input. However, the control signal $u_{\mathrm{p}}(t)$ is not sent directly to the machine any longer; two intermediate steps are taken between the computation of $u_{\mathrm{p}}(t)$ and the control of the machine. First, $u_{\mathrm{p}}(t)$ is modulated into an "on-off' signal $u_{\mathrm{mod}}(t)$, so that it matches the discrete idle-busy capabilities of the machine. This is done with pulse-width modulation (PWM), see Section IV-A. If the production rate of the machine is set equal to $u_{\mathrm{mod}}(t)$, the machine could have to stop several times during the production of one particular product because $u_{\mathrm{mod}}(t)$ is not necessarily "on" for an uninterrupted period of t_0 time units. For this reason, it is calculated how long $u_{\mathrm{mod}}(t) = u_{\mathrm{p,max}}$, to determine when the controller must send a command signal to the machine to make it start producing one complete product. A possible procedure for this is described in Section IV-B.

A. Pulse-width modulation of control signal

Naturally-sampled pulse-width modulation (PWM) can be used to modulate the production rate $u_{\mathrm{p}}(t)$ into an "on-off" signal $u_{\mathrm{mod}}(t)$ [18]. As a modulating waveform a sawtooth is chosen with period Δt_{saw} and peak-to-peak amplitude $u_{\mathrm{p,max}}$. Whenever $u_{\mathrm{p}}(t)$ is larger than the sawtooth, $u_{\mathrm{mod}}(t)$ becomes $u_{\mathrm{p,max}}$, and whenever $u_{\mathrm{p}}(t)$ is smaller than the sawtooth, u_{mod} is switched back to 0, see Fig. 3.

The sampling period Δt_{saw} can be determined using the sampling theorem of Nyquist and Shannon [10]: the sampling frequency must be chosen at least twice the bandwidth of the signal to be sampled.

B. Starting time of production

The pulse signal $u_{\mathrm{mod}}(t)$ can be used to determine the instances in time at which the machine must start producing an entire product. To this end, a variable $timer(t)$, with $timer(0) = 0$, is introduced:

$$timer(t) = \frac{u_{\mathrm{mod}}(t)}{u_{\mathrm{p,max}}}\, t. \qquad (18)$$

By the time $timer(t) = t_0$, so when $u_{\mathrm{mod}}(t)$ has been equal to $u_{\mathrm{p,max}}$ for t_0 time units, one product should have

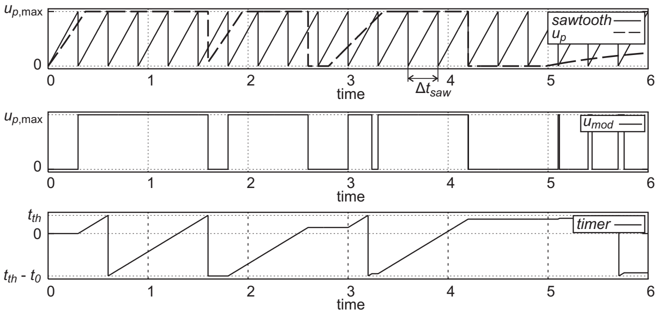


Fig. 3: $u_\mathrm{p}(t)$ is modulated into $u_\mathrm{mod}(t)$ using PWM with a sawtooth with slope $\frac{u_\mathrm{p,max}}{\Delta t_\mathrm{saw}}$. $timer(t)$ is switched on and off with $u_\mathrm{mod}(t)$ and is decreased by t_0 after reaching t_th.

been finished. If the machine had been able to produce products instantaneously, then sending a command signal to the machine when $timer(t) = t_0$ to let it produce a product would have resulted in $e(t) = 0$ at that time instant. However, producing one product takes t_0 time units, so sending a command signal when $timer(t) = t_0$ results in the finish of a product t_0 time units later. If one is satisfied with this behavior, then $timer(t) = t_0$ can be the trigger for the controller to send the command signal to the machine.

If it is not acceptable that $y(t)$ lags behind on $y_\mathrm{d}(t)$ with t_0, the production must be started earlier. Starting the production when $timer(t) = 0$ solves this problem, but it could also lead to the production of unnecessary products. This occurs, for instance, if the initial number of products is sufficient for the demand and no further production is required ($y_0 \geq y_{\mathrm{d},0}$, $v_\mathrm{d} = 0$ and $\varphi(t) = 0$). In conclusion, the controller must send a command signal to the machine when $timer(t)$ exceeds some threshold value $t_\mathrm{th} > 0$.

After sending the command signal, the value of $timer(t)$ must be reset to $t_\mathrm{th} - t_0$ in order to determine when the next product must be produced, see Fig. 3. In resetting $timer(t)$, a decrease of t_0 is necessary to anticipate the finish of the last product commanded for.

When $timer(t)$ reaches t_th, it could be the case that the machine is unable to start with the production of a new product, for instance because it is in repair. In that case, the controller waits before sending the command signal. As soon as the machine is ready to produce a new product, it receives the command signal from the controller and it starts producing a product. At that time, the controller resets $timer(t)$ to determine when the production of the next product must be started.

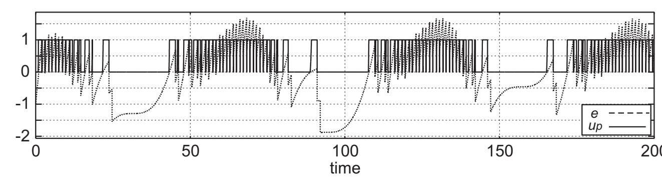
V. Performance analysis for discrete-event manufacturing system model

In the previous section the practical implementation of the observer-based controller in discrete-event domain was discussed. In this section it is shown that the continuous controller can be used to control a discrete-event manufacturing machine. This is done by means of a simulation in the *Hybrid* χ (Chi) [19], [20] language, developed at the Eindhoven University of Technology, which can be used for modeling, simulation and verification of hybrid systems. The combination of the continuous controller and discrete-event manufacturing machine is such a hybrid system.

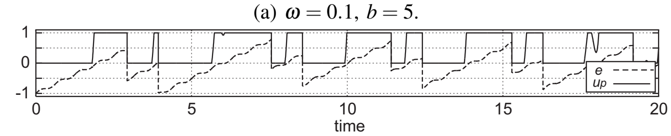
For the simulation the same parameters as in Section III-B are used with the following addition for the implementation in discrete-event domain: $\Delta t_\mathrm{saw} = 0.1$ and $t_\mathrm{th} = 0.1$.

A. Disturbance-free case

In Fig. 4 time responses of the system with the discrete-event machine and the continuous controller are shown for two different values of the amplitude b of the demand fluctuation $\varphi(t) = b\sin(\omega t)$. Equal to Section III-B, the maximum value of ω is chosen for which the derivative of reference $y_\mathrm{d}(t)$ is non-negative. Note the differences in time scale. One can see that when the machine finishes a product the tracking error $e(t)$ decreases with one product, indicating that $y(t)$ has increased by 1. In the discrete-event setting an error smaller than 1 can be neglected, as this is less than one product.



(a) $\omega = 0.1$, $b = 5$.



(b) $\omega = 10$, $b = 0.05$.

Fig. 4: Time response of the tracking error $e(t) = y_\mathrm{d}(t) - y(t)$ with $y(t) \in \mathbb{N}$ and the continuous (saturated) control signal $u_\mathrm{p}(t)$ for different fluctuations $\varphi(t) = b\sin(\omega t)$.

Comparing the time responses of the continuous machine in Fig. 2 with those of the discrete-event machine in Fig. 4, one can observe the following. Where a harmonic fluctuation with $b = 5$ and $\omega = 0.1$ results in a vanishing error with the continuous machine, the same fluctuation results in an error of ± 2 products with the discrete-event machine. For a fluctuation with $b = 0.05$ and $\omega = 10$ the error in both the continuous and discrete-event domain are negligible (smaller than one product). Obviously, different system parameters (such as $u_\mathrm{p,max}$, k_p and L) lead to different results with both the continuous and discrete-event models of a manufacturing machine.

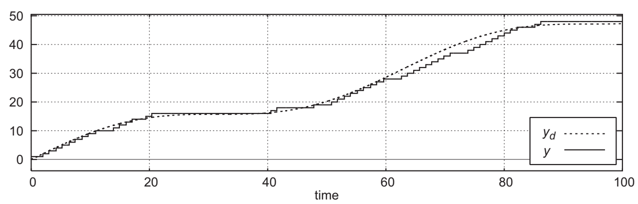
B. Case of the disturbed system

Previously, it was assumed that the external disturbance $f(t)$ was zero. To show that the observer-based controller is capable of dealing with demand fluctuations $\varphi(t)$ as well as external disturbances $f(t)$, both are taken into account in the simulations presented in this section.

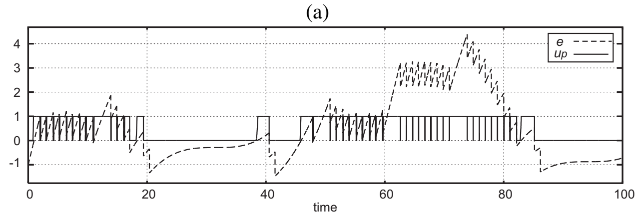
The type of disturbance taken here is an operation dependent machine failure (the machine fails while processing a product) because this is the most common type of machine failure [1]. Here, the disturbance affects 10% of the products and a repair time of $t_{\text{repair}} = 2t_0$ is required. As an example, one could think of a manufacturing machine that uses a tool that gets worn out. The tolerance of every tenth product falls outside the acceptable tolerance range and is discarded. In addition, the tool needs to be changed, which takes $2t_0$ time units in this example.

The results of the simulation with disturbance are shown in Fig. 5. In the top figure the demand $y_{\text{d}}(t)$ and the real output $y(t)$ are plotted. In Fig. 5b the tracking error $e(t)$ and the control signal $u_{\text{p}}(t)$ are shown and Fig. 5c shows $timer(t)$. The high peaks in this plot can be explained by looking at Fig. 5d, in which the state of the machine (idle, busy or in repair) is plotted. When the machine is in repair, for example around $t = 60$, the controller cannot force the machine to start with the production of a new product, so $timer(t)$ keeps increasing. Because the machine starts producing a product later then desired, the error increases. Between $t = 60$ and $t = 80$ the error causes the machine to produce constantly (when it is not in repair), as can be seen in Fig. 5d.

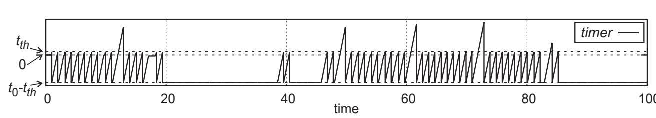
Although the tracking error does not become zero if an external disturbance or fluctuation in the demand is present, the observer-based controller is able to track the reference output reasonably well.



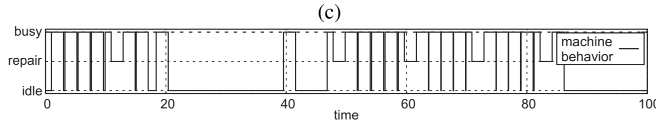
(a)



(b)



(c)



(d)

Fig. 5: Time response of the discrete-event machine controlled by the continuous controller, with operation dependent disturbance, $b = 5$ and $\omega = 0.1$.

VI. Conclusion

This paper shows the possibility of controlling a single manufacturing machine which has a certain minimum and maximum capacity by an observer-based feedback controller. This controller uses only the tracking error to calculate the control signal. In the continuous domain, convergence to one trajectory for different initial conditions is guaranteed if the reference demand contains some fluctuations or if some external disturbance is present. Simulation results show that the same controller can be used to control a machine in discrete-event domain. In future research these results will be extended to a line of manufacturing machines. The performance of a manufacturing line controlled by observer-based feedback controllers will be analyzed.

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