Estimation and control under information constraints for LAAS Helicopter Benchmark

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***Abstract*— A scheme for state estimation and control under information constraints is implemented for "LAAS Helicopter benchmark". Parameter estimates of the "Helicopter" pitch dynamics model are obtained by means of the real-time identification algorithm. The hybrid continuous–discrete observation procedure for transmission of the measured data over the limited-band communication channel with adaptive tuning of the coder range parameter is proposed and used in the experiments for pitch motion control of the "Helicopter". Experimental results for pitch motion control of the "Helicopter" are presented, showing efficiency of the proposed method.**

***Index Terms*— helicopter control, parameter identification, state estimation, information capacity**

I. Introduction

During the last decades various computer-controlled equipment units have been described in the literature: Schmid pendulum [1], Furuta pendulum [2], pendubot [3], etc. The usage of such equipment is threefold. First, it may be used for demonstration, attracting newcomers to the control systems area. Secondly, it is used for education, allowing students to enhance their skills in control systems design. Finally, such units are useful for research since they may serve as testbeds for testing new control algorithms under real world conditions. In this connection the control problems for different kinds of laboratory helicopters attracts significant interest [4]–[12]. One of impressive laboratory setups is a laboratory-scale bench-top three degrees of freedom (3DOF) helicopter produced by Quanser Consulting Inc. [5]. This setup was modified under demand of LAAS–CNRS [13] to form the "LAAS Helicopter Benchmark", allowing testing 3DOF flight control algorithms under time-varying conditions.

Manuscript received December 16, 2007; revised April 23, 2009, September 17, 2009. This work was supported by the Russian Foundation for Basic Research, projects RFBR # 08-01-00775, # 09-08-00803, PICS cooperation project between CNRS and RFBR # 4281, # 07-01-92166, the Council for grants of the RF President to support young Russian researchers and leading scientific schools (project NSh-2387.2008.1), the Program of basic research of OEMPPU RAS #2 "Control and safety in energy and technical systems" and by Russian Federal Program "Research and Teaching Cadres", contract # 02.740.11.5056. The authors are grateful to Professor Robin J. Evans for his fruitful and stimulating comments and to the members of MAC group of LAAS–CNRS for the help in the work with the "Helicopter".

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In the present paper the "LAAS Helicopter Benchmark" (hereafter, the "Helicopter") is used as a testing setup for examination of the methods for estimation and control in presence of constraints imposed by finite information capacity of communication channels. A fast growth of interest to the problem of estimation and control under information constraints has been observed within the control community during the last decade. Recently the limitations of control under constraints imposed by a finite capacity information channel have been investigated in detail in the control theoretic literature, see [14]–[20] and references therein. For unstable linear systems it was shown that stabilization of the system at the equilibrium under information constraints is possible if and only if the capacity of the information channel exceeds the entropy production of the system at the equilibrium (so called *Data-Rate Theorem* [15], [16]. The above results have clear physical interpretation: information must be transported as fast as the system generates it, or else instability occurs. However, they are mathematically justified only for linear systems or near equilibrium. For discrete-time nonlinear systems, the concept of feedback topological entropy was introduced and the minimum data-rate for local stabilization was given in [16].

Continuous-time nonlinear systems were also considered in [21]–[27], where several sufficient conditions for different stabilization problems were obtained. For nonlinear stabilization problems it was mainly assumed that the full states of the systems were measurable [16], [21]–[24]. An output feedback stabilization problem is attacked in [25]–[27]. In [25], the uniformly observable systems were considered and an "embedded-observer" decoder and a controller were designed, which semi-globally stabilizes this class of systems under data-rate constraints. In [27], an output feedback stabilization problem of a class of nonlinear systems with nonlinearities satisfying the non-decreasing property was considered. The communication channel was assumed to be free from noise and time delay. An encoding/decoding scheme was introduced and sufficient conditions for the stabilization problem were obtained.

The known results are close to complete for the equilibrium stabilization problem. However, they leave open similar questions for many other important problems. If the desired state vector of the system is time-varying (e.g. for tracking problems) no tight bounds for transmission rate ensuring convergence of the error to zero are known.

In a number of engineering applications (e.g. in distributed sensor networks, or remote surveillance systems), there is no possibility to mount advanced measurement/estimation devices on the transmitter (target) side [14], [15], [21], [28]–[31]. In these cases only measurements of some scalar output variable of the transmitter system are available. Such a problem was studied in [32]–[36], where results on synchronization of nonlinear systems, represented in the Lurie form are given. It is shown that an upper bound on the limit synchronization error is proportional to a certain upper bound on the transmission error. Under the assumption that a sampling time may be properly chosen, optimality of binary coding in the sense of demanded transmission rate is established, and the relationship between synchronization accuracy and an optimal sampling time is found. On the basis of these results, the present paper deals with a binary coding procedure.

The results of [32]–[34], [36], [37] are applied to the problem of state estimation via the limited-band communication channel for specific flight control system – the "Helicopter" benchmark. Adaptation law for tuning the coder range parameter is proposed and the experimental results for pitch motion of the "Helicopter" are presented. Preliminary results of the work are given in [38].

The paper is organized as follows. Modeling the "Helicopter" pitch dynamics is considered in Sec. II. Section III is devoted to the problem of state estimation over the limited-band communication channel. The hybrid continuous–discrete observation procedure is presented. The adaptation law for tuning the coder range parameter is proposed. The experimental results for feedback control of the "Helicopter" pitch angle are presented in Sec. IV.

II. Modeling the "Helicopter" motion

A. LAAS Helicopter benchmark

The 3DOF helicopter setup is manufactured by *Quanser Consulting Inc.*, [4], [10]. The MAC Group of LAAS-CNRS uses its improved version as a benchmark for implementation and testing robust control laws. The "Helicopter" consists of a base on which a long arm is mounted. The arm carries the helicopter body on one end and a counterweight on the other end. The arm can tilt on an elevation axis as well as swivel on a vertical (travel) axis. Quadrature optical encoders mounted on these axes measure the elevation and travel of the arm. The *helicopter body*, which is mounted at the end of the arm, is free to pitch about the *pitch axis*. The pitch angle is measured via a third encoder. Two motors with propellers mounted on the helicopter body can generate a force proportional to the voltage applied to them. The force, generated by the propellers, causes the helicopter body to lift off the ground and/or to rotate about the pitch axis. All electrical signals to and from the arm are transmitted via a slipping with eight contacts. The system is also equipped with a motorized *lead screw* that can drive a mass along the main arm in order to impose known controllable disturbances.

B. Nomenclature

Following notation is used through the paper (see Fig. 2):

- $\theta(t)$–pitch angle, rad;
- $\dot{\theta}(t)$–pitch angular rate, rad/s;
- $\theta_*(t)$–pitch reference signal, rad;

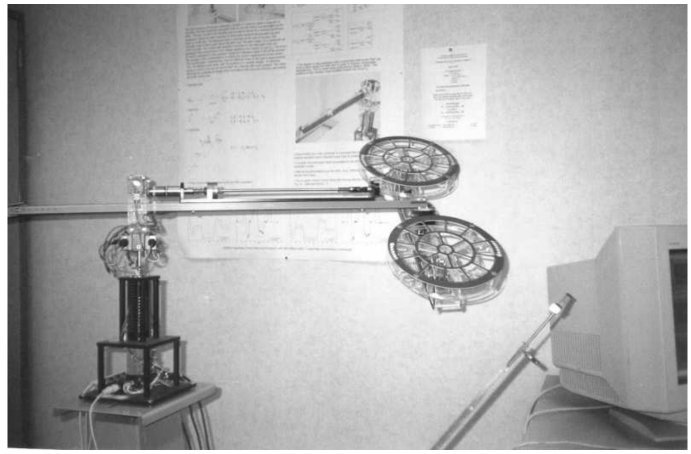

Fig. 1. Photo of the "Helicopter".

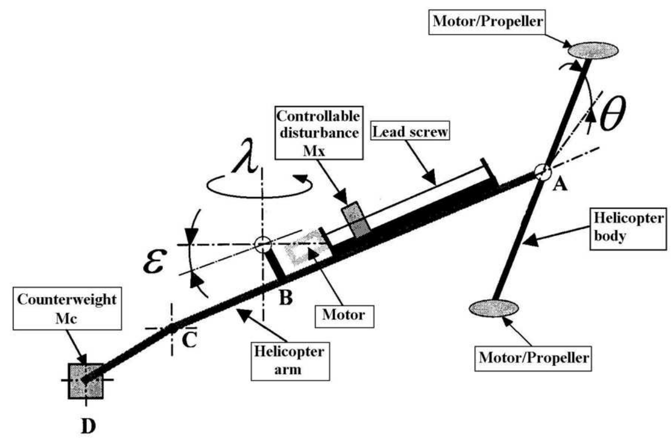

Fig. 2. Lay-out of the "Helicopter".

- $v_f(t)$–control voltage of the (conditionally) "front" motor, V;
- $v_r(t)$–control voltage of the "rear" motor, V;
- $u(t)$–pitch torque command signal, V;
- $w(t)$– normal force command signal (used for elevation/travel control), V,
- $f_f(t)$, $f_r(t)$– tractive forces of front and rear propeller, N.

The control voltages $v_f(t)$ and $v_r(t)$, applied to the front and rear motors are computed from the command signals $u(t)$, $w(t)$ as follows:

$$v_f = 0.5(w+u), \quad v_r = 0.5(w-u). \tag{1}$$

The motor control voltages have saturation level 5 V on magnitude.

C. Simplifying assumptions

Following simplifications are made to build the model of the "Helicopter" motion:

- The "Helicopter" is considered as a rigid body, i.e. it is assumed that the long arm of the "Helicopter" rotates about the long axis together with the crossbar, and bending of structural components is neglected.

- The gyroscopic torques, developed by motor/propeller pairs are neglected.
- Dependence of motor/propeller force gain on the "Helicopter" airspeed is neglected.
- Influence of aerodynamical pressure forces on the "Helicopter" body is neglected.
- Dry friction in the pivots is neglected.

Taking into account that dynamics of the pitch motion are faster than the elevation and travel dynamics, and assuming that the rotation rates about the travel and elevation axes are relatively small, consider the pitch motion independently of the other ones. It makes it possible to use the following simplified model of the pitch dynamics:

$$\ddot{\theta}(t)+a_m^{\omega_x}\dot{\theta}(t)+a_m^{\theta}\sin\big(\theta(t)-\theta_0\big)=k_m^m m_x(t), \qquad (2)$$

where $m_x(t)$ denotes the scaled controlling torque about the pitch axis; θ_0 stands for the equilibrium pitch angle; $a_m^{\omega_x}$, a_m^{θ}, k_m are pitch model parameters. The torque $m_x(t)$ is produced by the propeller pair. This torque is proportional to the difference between the propeller tractive forces $f_f(t)$ and $f_r(t)$:

$$m_x(t)=k_m^f\big(f_f(t)-f_r(t)\big). \qquad (3)$$

Each force is assumed to be proportional to the rotation speed of the corresponding motor.[1] Neglecting the small time lags, resides in the air-blast speed of the funs and the motor electric circuits, one may write down the following equation for the tractive forces $f_i(t)$:

$$T_m\dot{f}_i(t)+f_i(t)=k_f^v v_i(t), \quad i\in\{f,r\}, \qquad (4)$$

where $v_i(t)$ is the control voltage, applied to the corresponding motor; T_m is an electric motor time constant; k_f^v is the gain parameter.

Parameters $a_m^{\omega_x}$, a_m^{θ}, k_m^m, k_f^v, k_m^f of the system (2), (4) depend on the design features of the setup, including characteristics of interaction between the propeller blades with ambient air, and assumed to be subjected the identification procedure. The approximate value of the electric motor parameter $T_m\approx 0.08$ s is taken from the "Helicopter" User's Guide, presented by the manufacturer.

D. Identification of the "Helicopter" pitch model parameters

The least-squares-like plant estimator is used for on-line identification of the "Helicopter" parameters. The identification algorithm uses the measurement of the input/output signals only. To avoid measuring of the pitch angular rate and time derivatives of higher order, the state filters are introduced.

1) General form of the identification algorithm: The identification algorithm in a general form is described in [39], [40]. The essentials of the algorithm are briefly presented below.

Consider the following LTI SISO plant model:

$$\begin{aligned}&y^{(n)}(t)+a_1y^{(n-1)}(t)+\ldots+a_ny(t)\\&=b_0u^{(m)}(t)+b_1u^{(m-1)}(t)+\ldots+b_mu(t),\end{aligned} \qquad (5)$$

[1] More accurate model should represent asymmetry of the propeller tractive force from sign of rotation [10].

where a_1, $\ldots a_n$, b_0, $\ldots,b_m$ are unknown plant parameters (index n means the nth time derivative of the signal). Equation (5) may be rewritten as:

$$y^{(n)}(t)=\varphi(t)^{\mathrm{T}}\Omega^*, \qquad (6)$$

where the *regressor vector* $\varphi=[y^{(n-1)},\ \ldots,\dot{y},y,\ u^{(m)},\ldots,u]^{\mathrm{T}}$, $\Omega^*=[-a_1,\,-a_2,\,\ldots,\,-a_n,\,b_0,\,b_1,\ldots,b_m]^{\mathrm{T}}$, $\varphi(t),\Omega^*\in\mathbb{R}^{n+m+1}$. Introducing the filtered signals $\tilde{y}(t)$, $\tilde{\varphi}(t)$ satisfying equations $D(p)\tilde{y}^{(n)}(t)=y^{(n)}(t)$, $D(p)\tilde{\varphi}(t)=\varphi(t)$, where $D(p)=p^n+d_1p^{n-1}+\cdots+d_n$ is an arbitrary Hurwitz polynomial of $p\equiv d/dt$, one gets from (6) the relation: $\tilde{y}^{(n)}=\tilde{\varphi}(t)^{\mathrm{T}}\Omega^*$. The signals $\tilde{y}(t)$, $\tilde{\varphi}(t)$ are outputs of the following *state filters*

$$\dot{\xi}=A_d\xi+b_dy(t),\ \ \dot{\psi}=A_d\psi+b_du(t), \qquad (7)$$

where $\xi(t)$, $\psi(t)\in\mathbb{R}^n$; pair (A_d,b_d) has a regular canonical form, $\det(sI_n-A_d)=D(s)$. Notice, that both signals $\xi(t)$ and $\psi(t)$ can be implemented without measurement of time derivatives. It is easy to see that $\tilde{\varphi}=\big[\xi_n,\,\ldots,\xi_2,\,\xi_1,\psi_{m+1},\,\ldots,\,\psi_1\big]^{\mathrm{T}}$, $\tilde{y}^{(n)}=y(t)\ -\sum_{i=1}^n d_{n-i+1}\xi_i$. In the sequel, the variable $\tilde{y}^{(n)}$ is denoted as $\tilde{\xi}$.

Introduce the vector of *estimates* $\Omega(t)\in\mathbb{R}^{n+m+1}$ as $\Omega(t)=[-\hat{a}_1(t),\ -\hat{a}_2(t),\ \ldots,\ -\hat{a}_n(t),\ \hat{b}_0(t),\ldots,\hat{b}_m(t)]^{\mathrm{T}}$, where $\hat{a}_i(t)$, $\hat{b}_j(t)$ are estimates of the corresponding plant model parameters a_i, b_j. Consider Ω^* as an unknown initial state of the system

$$\dot{\Omega}(t)=0, \quad \tilde{\xi}(t)=\tilde{\varphi}(t)^{\mathrm{T}}\Omega, \quad \Omega(0)=\Omega^*. \qquad (8)$$

Application of the Kalman filtering technique to the system (8) leads to the following identification algorithm:

$$\dot{\Omega}(t)=-\Gamma(t)\tilde{\varphi}(t)\tilde{\varphi}(t)^{\mathrm{T}}\Omega(t)+\Gamma(t)\tilde{\varphi}(t)\tilde{\xi}(t), \qquad (9)$$

$$\dot{\Gamma}(t)=-\Gamma(t)\tilde{\varphi}(t)\tilde{\varphi}(t)^{\mathrm{T}}\Gamma(t)+\alpha\Gamma(t), \qquad (10)$$

where $\Gamma(t)$ is the gain matrix. Initial value $\Gamma(0)=\Gamma(0)^{\mathrm{T}}>0$ is the algorithm parameter.

It is well known that the so-called *persistent excitation* (*PE*) *condition* is important ensuring convergence of the parameter estimates to their true values [41]–[43]. Fulfillment of this condition is an open issue for the closed-loop systems, because the input signal is produced by the controller as a function on current state of the plant. In the present work the parameter identification procedure is carried out at the stage of controller design. The open-loop system may be used at that stage and the input signal may be chosen to assure fulfillment of the PE condition [43].

2) Algorithm for identification of the pitch model parameters: Following [44], [45], let us apply the described above method to designing the parameter identification algorithm for the plant model (2)–(4). Let us use knowledge of the parameter T_m and introduce the overall gain k_m^v as $k_m^v=k_f^vk_m^mk_m^f$. Then the plant equations may be rewritten in the form

$$\ddot{\theta}+a_m^{\omega_x}\dot{\theta}+a_m^{\theta}\sin\big(\theta-\theta_0\big)=k_m^v\mu(t), \qquad (11)$$

$$T_m\dot{\nu}_i(t)+\nu_i(t)=v_i(t), \quad i\in\{f,r\}. \qquad (12)$$

where $\mu(t)=\nu_f(t)-\nu_r(t)$. The signal $\mu(t)$ is considered as a new control action, applied to the plant (11). This signal may be reproduced as difference between outputs of the low-pass filters (12).

The "bias" parameter θ_0 was found as the equilibrium angle of pitch for $v_f(t) = v_r(t) \equiv 0$. This parameter is estimated outside the identification algorithm.

To apply the aforementioned procedure, let us define the variable $s(t)$ as $s(t) = \sin(\theta - \theta_0)$ and consider $s(t)$ as an additional input signal. Then introduce the vectors $\Omega^* = [-a_m^{\omega_x}, -a_m^{\theta}, k_m^v(t)]^{\mathrm{T}} \in \mathbb{R}^3$ and $\Omega(t) = [-\hat{a}_m^{\omega_x}(t), -\hat{a}_m^{\theta}(t), \hat{k}_m^v(t)]^{\mathrm{T}} \in \mathbb{R}^3$, where $\hat{a}_m^{\omega_x}(t)$, $\hat{a}_m^{\theta}(t)$, $\hat{k}_m^v(t)$ are estimates of the corresponding parameters of the plant model (11).

The state filters (7) for the considered case are described by the following equations:

$$\begin{cases} \dot{\vartheta}_1(t) = \xi_2(t), \\ \dot{\vartheta}_2(t) = \omega_f^2(\theta(t) - \vartheta_1(t)) - 2\omega_f\rho\vartheta_2(t), \end{cases}$$
$$\dot{\theta}_f(t) = \vartheta_2(t),$$
$$\tilde{\xi}(t) = \omega_f^2(\theta(t) - \vartheta_1(t)) - 2\omega_f\rho\vartheta_2(t), \tag{13}$$
$$\begin{cases} \dot{\eta}_1(t) = \eta_2(t), \\ \dot{\eta}_2(t) = \omega_f^2(s(t) - \eta_1(t)) - 2\omega_f\rho\eta_2(t), \end{cases}$$
$$s_f(t) = \eta_1(t), \tag{14}$$
$$\begin{cases} \dot{\psi}_1(t) = \eta_2(t), \\ \dot{\psi}_2(t) = \omega_f^2(\mu(t) - \psi_1(t)) - 2\omega_f\rho\psi_2(t), \end{cases}$$
$$\mu_f(t) = \psi_1(t), \tag{15}$$

where parameters ω_f and ρ are the pass band and the damping coefficient of the filters. The filter (13)–(15) outputs are used to form the regressor vector $\tilde{\varphi}(t) = [\dot{\theta}(t), s(t), \mu(t)]^{\mathrm{T}} \in \mathbb{R}^3$. The signals $\tilde{\varphi}(t)$, $\tilde{\xi}(t)$ are used in the identification algorithm (9), (10). For the considered case, the gain $\Gamma(t)$ is (3×3)-matrix.

3) Parameter identification results: The identification algorithm (9), (10), (13)–(15) was implemented on-line in the MATLAB/Simulink and WinCon software environment to obtain the estimates of the "Helicopter" pitch model parameters. The special exciting signal $\mu(t)$ ("square waveform") satisfying the PE condition, was applied to the "Helicopter" control input. To have a better idea of eventual accuracy of the estimated parameters and the repeatability of the identification trials, two sets of the exciting signal parameters (frequency, bias and magnitude) were used for the experiments. Obtained data of the pitch angle $\theta(t)$ and the input signal $\mu_f(t)$ for both experiments are plotted in Fig. 3.

The following parameters of the identification algorithm (9), (10), (13)–(15) were taken: $\Gamma(0) = 10^3 \cdot \mathbf{I}_{3\times3}$, $\alpha = 0.01$, $\omega_f = 1$ s^{-1}, $\rho = 0.7$. The bias angle θ_0 was measured at the quiescent state, $\theta_0 = -0.115$ rad. In (12) the motor time constant $T_m = 0.08$ s. Results of the on-line parameter identification procedures for both shapes of the exciting signal (Fig. 3, plots *a* and *b* respectively) are shown in Fig 4, where the time histories of the estimates $\hat{a}_m^{\theta}(t)$, $\hat{k}_m^v(t)$ and $\hat{a}_m^{\omega_x}(t)$ are plotted.

Based on the identification results, the following parameters of the pitch motion model (11) were obtained and used for state estimation design: $k_m^v(t) = 0.35$ V^{-1}s^{-2}, $a_m^{\theta} = 0.5$ s^{-2}, $a_m^{\omega_x} = 0.1$ s^{-1}.

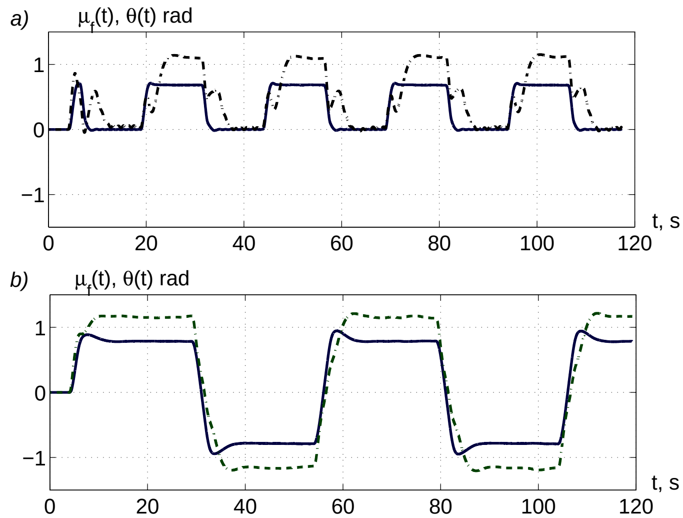

Fig. 3. Experimental results for two sets of the exciting signal $\mu(t)$ parameters. Time histories: pitch angle $\theta(t)$ (solid line), input signal $\mu_f(t)$ (dash-dotted line).

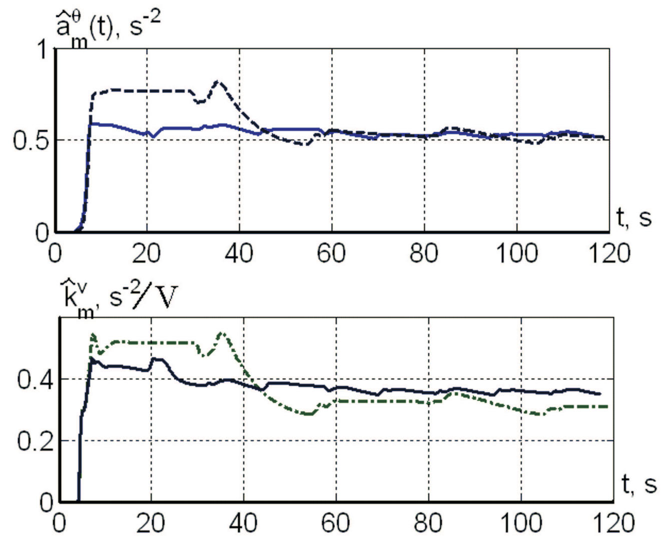

Fig. 4. Parameter identification. Time histories of the parameter estimates $\hat{a}_m^{\theta}(t)$, $\hat{k}_m^v(t)$ for different exciting signals: *a* – solid line, *b* – dash-dotted line.

III. STATE ESTIMATION AND CONTROL OF THE "HELICOPTER" OVER THE LIMITED-BAND COMMUNICATION CHANNEL

The problem of state estimation in presence of information constraints has a growing interest in the recent years due to various engineering applications, such as distributed sensor networks remote surveillance systems, targets tracking, etc., [14], [15], [19], [21], [28]–[31]. In the present work the "Helicopter" setup is used as a benchmark for testing procedures of coding and transmission the sensor measurements over the communication channel with finite information capacity and plant control based on the decoded data. Thereby, let us reproduce a procedure of coding the observation signal (the pitch angle $\theta(t)$) with symbols from a finite alphabet at discrete sampling time instants and its transmission over a digital communication channel with a limited data rate. Although the transmissions delay and transmission channel distortions usually appear in practice, in the present study it is assumed that the coded symbols are available at the receiver side at the same sampling instant, as they are generated by the coder, and transmission channel distortions are absent.

The present study is based on the results of [37], [38], were a full-order observer-based coding procedure was proposed. Those results are used in the present work to design the coder/observer algorithm for the "Helicopter". An improved coding procedure based on the adaptive tuning of the coder range parameter, is proposed and experimentally studied.

A. *Coding procedure*

In the paper [32] the properties of observer-based synchronization for Lurie systems over a limited data rate communication channel with a one-step memory time-varying coder are studied. It is shown that an upper bound on the limit synchronization error is proportional to a certain upper bound on the transmission error. Under the assumption that a sampling time may be properly chosen, optimality of binary coding in the sense of demanded transmission rate is established, and the relationship between synchronization accuracy and an optimal sampling time is found. It is worth to mention that the tight data-rate bound for stabilizability of a scalar system was given in [46]. In this article is shown that even for the *binary control* the bound is achievable. The linear plant stabilization problem was further considered in [47], where it was shown that for the first-order linear plant, binary control is the most robust control strategy under varying data-rate constraint and asynchronism of sampling and control actuation. The paper [32] deals with synchronization problem of nonlinear n-th order systems. On the basis of the mentioned results, the present paper deals with a *binary* coding procedure.

Consider the memoryless (static) binary quantizer to be a discretized map $q : \mathbb{R} \to \mathbb{R}$ as

$$q(y, M) = M \operatorname{sign}(y), \tag{16}$$

where $\operatorname{sign}(\cdot)$ is the *signum* function: $\operatorname{sign}(y) = 1$, if $y \geq 0$, $\operatorname{sign}(y) = -1$, if $y < 0$. Parameter M may be referred to as the *quantizer range*. Notice that for a binary coder each codeword symbol contains one bit of information. Therefore the transmission rate is $R = 1/T_s$, where T_s is a *sampling time*. The discretized output of the considered quantizer is given as $\bar{y} = M \operatorname{sign}(y)$. We assume that the coder and decoder make decisions based on the same information. The output signal of the quantizer is represented as a one-bit information symbol from the coding alphabet $\mathscr{S}$ and transmitted over the communication channel to the decoder.

In *time-varying quantizers* [18], [21], [32], [48], [49] the range M is updated with time and different values of M are used at each step, $M = M_k$. Using such a "zooming" strategy it is possible to increase coder accuracy in the steady-state mode and at the same time, to prevent coder saturation at the beginning of the process [48]. Zooming is effective for coders with *memory* [21]. To describe this kind of coders, introduce the sequence of *central numbers* c_k, $k = 0, 1, 2, \ldots$ with initial condition $c_0 = 0$. At step k the coder compares the current measured output y_k with the number c_k, forming the deviation signal $\partial y_k = y_k - c_k$. Then this signal is discretized with a current $M = M_k$ according to (16). The output signal

$$\bar{\partial} y_k = M_k \operatorname{sign}(\partial y_k) \tag{17}$$

is transmitted over the communication channel to the receiver. Then the central number c_{k+1} and the range parameter M_k are renewed based on the available information about the driving system dynamics. The following update algorithm is used in one-step memory coders:

$$c_{k+1} = c_k + \bar{\partial} y_k, \quad c_0 = 0, \quad k = 0, 1, \ldots. \tag{18}$$

The values of M_k may be precomputed (the *time-based* zooming), or, alternatively, current quantized measurements may be used at each step to update M_k (the *event-based* zooming).

In the present work the *adaptive tuning* of a coder, proposed in [37], [38], is used: if the coder is not saturated, the quantizer range M is exponentially decreased; in the case of saturation the quantizer range M is increased. Note that the binary coder saturation leads to a chain of identical bits at the coder output. Therefore the moving average of the output signal may be used as an indicator of saturation. Such an adaptive tuning makes it possible to maintain the minimal range M and, at the same time, to prevent fail in tracking the signal y_k due to saturation. Formerly the adaptive quantizer with a memoryless (static) coder was proposed in [50]. The adaptive coding for a special case of first-order system is analyzed in [51].

The proposed method for adaptive tuning the quantizer range M_k realizes the following recurrent algorithm:

$$\begin{aligned} &\lambda_k = (\bar{\partial} y_k + \bar{\partial} y_{k-1})/2, \\ &M_{k+1} = m + \begin{cases} \rho M_k, & \text{if } \ |\lambda_k| \leq 0.5 \\ M_k/\rho, & \text{otherwise,} \end{cases} \end{aligned} \tag{19}$$

where $0 < \rho \leq 1$ is a decay parameter; $m = (1-\rho) M_{\min}$, $M_{\min}$ assigns the possible minimal value for M_k, $k = 1, 2, \ldots$. Unlike the case of time-based zooming [21], the initial value $M_0 > 0$ can be chosen arbitrarily, since it is adjusted during the zoom-out stage. In practice the value M_0 should correspond to the uncertainty region for y_0. The procedure (19) leads to time-based decreasing of M_k while signs of the successive values of ∂y_k alternate. The sort of discrete-time sliding-mode tracking the plant output appears in that case, and M_k recursively tends to the limiting value $M_{\min}$. When the moving average of the transmission error exceeds the threshold, the second alternative of algorithm (19) is realized, and the quantizer range M_k increases.

Equations (17), (19) describe the coder algorithm. The same algorithm is realized by the decoder. Namely, the decoder calculates the variables $\tilde{c}_k$, $\tilde{M}_k$ based on received codeword flow similarly to c_k, M_k.

B. *Hybrid observer for estimation of the "Helicopter" pitch motion*

The hybrid continuous-discrete observation procedure may be effectively used for transmission of the observations over the limited-band communication channel. Firstly, observer generates estimates of the unmeasured state variables. Secondly, implementation of the state observation algorithm in the data transmission procedure, leads to the full-order memory coder/decoder pair, which makes possible, in the ideal case, to achieve vanishing the transmission error. It should be noticed that because the coder and decoder use the same information, the observer should be realized both at the coder and decoder nodes. Despite the measurements are assumed to be accurate at the transmitter end, the observer, implemented at this end should use the quantized signal in the feedback, making possible to follow the state estimation procedure at the decoder node.

Following [38], let us describe the state estimation algorithm, implemented for the "Helicopter" pitch channel. Consider that the controlling signals $v_f(t)$, $v_r(t)$ are generated based on the state estimates, available at the coder and decoder ends. These signals are applied to the filters (12) to form the signal $\mu(t)$, used as the "control" input of the observer. The second ("error") observer input is produced on the base of the coding/decoding procedure, described above. Namely, at the sampling time instants $t_k = kT_s$ $(k = 0, 1, \dots)$ the *state estimation error* $e_k = \theta(t_k) - \hat{\theta}(t_k)$ is calculated at the sensor node and coded in accordance with the binary quantization algorithm as

$$\sigma_k = \operatorname{sign} e_k, \tag{20}$$

The coded representation of σ_k as the sequence of symbols from a binary alphabet is transmitted over the channel and recovered by the decoder. In compliance with the zooming strategy, the signal σ_k is considered as the quantized value of e_k, obtained by means of the quantization algorithm (16), (19). Then the decoder output $\bar{e}_k$ is found as

$$\bar{e}_k = M_k \sigma_k, \tag{21}$$

where the weights M_k satisfy (19). The discrete-time signal $\bar{e}_k$ is expanded over the sampling interval $[t_k, t_{k+1})$ to form the piecewise constant continuous-time signal

$$\bar{e}(t) = e_k \quad \text{as} \quad t \in [t_k, t_{k+1}), \tag{22}$$

where $t_k = kT_s$, $k = 0, 1, \dots$. Finally, the observer is described by the following equations:

$$\begin{cases} \dot{\hat{\theta}}(t) = \hat{\omega}(t) + l_1 \bar{e}(t), \\ \dot{\hat{\omega}}_x(t) = -a_m^{\omega_x} \hat{\omega}_x(t) - a_m^{\theta} \sin\left(\hat{\theta}(t) - \theta_0\right) \\ \qquad\qquad + k_m^{v} \mu(t) + l_2 \bar{e}(t), \end{cases} \tag{23}$$

where $\hat{\theta}(t)$, $\hat{\omega}_x(t)$ are the estimates of the pitch angle $\theta(t)$ and pitch angular rate $\omega_x(t)$; parameters l_1, l_2 are the observer gains.

Summarizing, the data coding/decoding and state estimation procedure for the "Helicopter" pitch motion is described by Eqs. (12), (19), (21)–(23). This procedure should be realized both at the transmitter (sensor) and at the receiver (controller) nodes of the system.

IV. Experimental results

In the experiments, the following PID control law was implemented in the MATLAB/Simulink and WinCon software environment:

$$u(t) = k_P \varepsilon(t) + k_I \int_0^t \varepsilon(t) \mathrm{d}t - k_D \hat{\omega}_x(t), \tag{24}$$

where $\varepsilon(t)$ denotes the reference error, $\varepsilon(t) = \theta_*(t) - \hat{\theta}(t)$; $\theta_*(t)$ is the reference signal; k_P, k_I, k_D stand for the controller gains. The control law uses the transmitted/estimated values $\hat{\theta}(t)$, $\hat{\omega}_x(t)$ of the "Helicopter" pitch angle and angular rate $\theta(t)$, $\omega_x(t)$. The estimates are produced based on the discrete signal $\bar{e}_k$, transmitted over the communication channel by the algorithm (12), (19), (21)–(23).

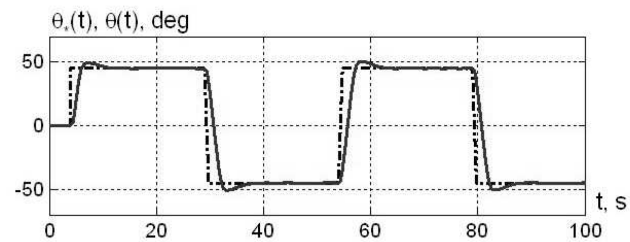

Fig. 5. Time histories of the pitch angle $\theta(t)$ (solid line) and reference signal θ_* (dash-dotted line) for the case of the "ideal" controller.

The square waveform reference signal $\theta_*(t)$ of $\pi/2$ rad in amplitude and 25 s in period was used. Initial pitch angle and pitch angular rate were zeros. PID controller parameters $k_I = 0.053$ s^{-1}, $k_D = 0.12$ s^{-1}, $k_P = 0.1$ were obtained applying the standard Zieger–Nichols design method to the model (2), (4). Time responses for the case of the controller with "ideal" measurements $\hat{\theta}(t) \equiv \theta(t)$, $\hat{\omega}_x(t) \equiv \omega_x(t)$ are shown in Fig. 5.

The observer (23) gains $l_1 = 4.1$, $l_2 = 8.6$ were found based on the poles placement technique, applied to the plant model, linearized at the equilibrium point. Parameters of the range tuning rule (19) were picked up as: $M_0 = 0.5$, $\rho = 0.95$, $M_{\min} = 0.05$. The sampling rate of transmission the control action (24) from the computer to the "Helicopter" was 100 Hz.

Some experimental results for $T_s = 0.05, 0.10$ s are depicted in Figs. 6–8. It is shown that the proposed method makes it possible to use the communication channel with the data rate $R = 10 \div 20$ baud ensuring the pitch transmission error not greater than 5% of the pitch amplitude. Time histories of the range parameter M_k demonstrate effect of the adaptive tuning: M_k decreases at the intervals near the rapid changes of $\theta(t)$, preventing losses in signal transmission due to the coder saturation. Fig. 8 makes it possible to compare results of the pitch rate estimation by means of numerical differentiation of the pitch measurements (typically used for the "Helicopter" benchmark) with the proposed observer-based scheme. Usually the "Helicopter" pitch angle is measured with ten binary digits and transmitted to the control computer with the sampling time $T_s = 10^{-2}$ s, which leads to the bit-per-second rate $R = 10^3$ baud (Fig. 8, curve $\dot{\theta}_{\mathrm{d}}$). For the proposed estimation algorithm (curve $\dot{\theta}_{\mathrm{est}}$), the bit-per-second rate $R = 10$ baud.

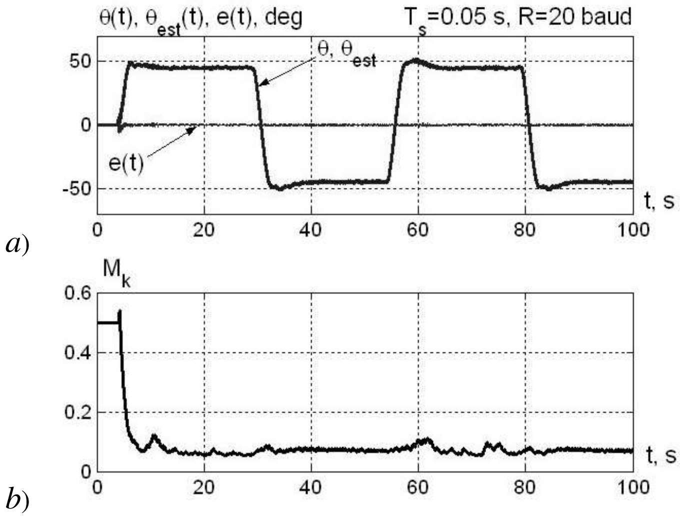

Fig. 6. The case $T_s = 0.05$ s ($R = 20$ baud). Time histories: (*a*) pitch angle $\theta(t)$, pitch estimate $\hat{\theta}(t)$, error $e(t) = \theta(t) - \hat{\theta}(t)$; (*b*) range parameter M_k.

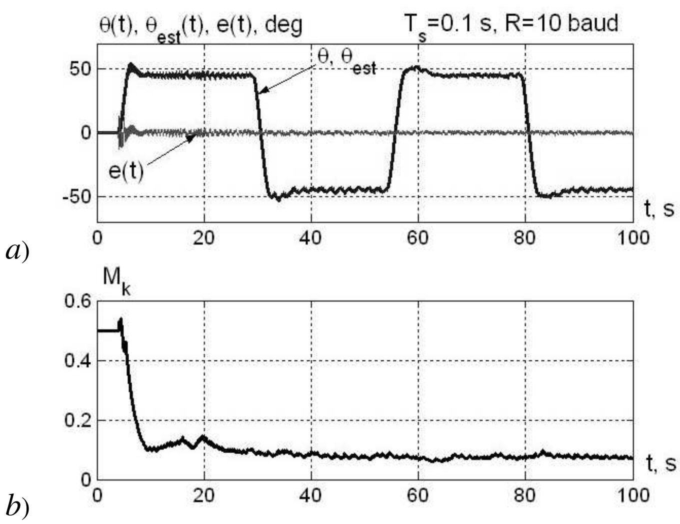

Fig. 7. The case $T_s = 0.10$ s ($R = 10$ baud). Time histories: (*a*) pitch angle $\theta(t)$, pitch estimate $\hat{\theta}(t)$, error $e(t) = \theta(t) - \hat{\theta}(t)$; (*b*) range parameter M_k.

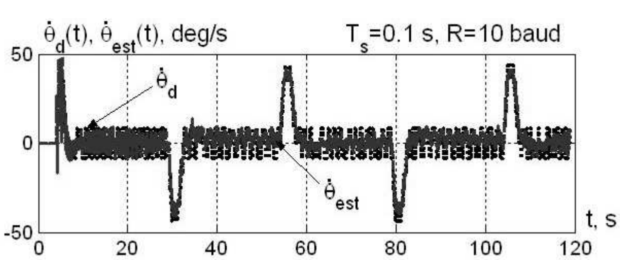

Fig. 8. Time histories of the pitch rate estimates; $\dot{\theta}_d$ – differentiator output (dotted line); $\hat{\dot{\theta}}$ observer output (solid line) for $T_s = 0.10$ s ($R = 10$ baud).

V. Conclusions

In the present paper the “LAAS Helicopter benchmark” is used for testing the algorithms of state estimation and control under constraints, imposed by finite information capacity of communication channel in the control loop. The real-time identification algorithm of the “Helicopter” pitch model parameters is described and the experimental results are given. The hybrid continuous–discrete observation procedure for transmission of the measured data over the limited-band communication channel with adaptive tuning of the coder range parameter is proposed and used in the experiments for pitch motion control of the “Helicopter”. Experimental results demonstrate effectiveness of the proposed procedure for control under the information constraints.

Future research is aimed at taking into account transmission errors for other signals involved and considering the 3D control of the “Helicopter”.

References

[1] C. Schmid, “An autonomous self-rising pendulum. Invited paper,” in *Proc. European Control Conference ECC’99*, Karlsruhe, 1999.
[2] K. Furuta, M. Yamakita, S. Kobayasji, and M. Nishimura, “A new inverted pendulum apparatus for education,” in *Proc. IFAC Symp. on Advances Contr. Education*, Tokyo, 1994, pp. 191–194.
[3] M. W. Spong and D. Block, “The Pendubot: A mechatronic systems for control research and education,” in *Proc. 35th IEEE Conf. Dec. Control (CDC’96)*, New Orleans, USA, 1996, pp. 555–556.
[4] J. Apkarian. (1999) Internet control. Circuit Cellar. [Online]. Available: http://www.circuitcellar.com
[5] Quanser Co. [Online]. Available: http://www.quanser.com/choice.asp
[6] D. R. Chen, H. S. Chen, and J. D. Wang, “Comparison between the system identification and the neural network methods in identifying a model helicopter’s yaw movement,” *Int. J. Nonlinear Sciences and Numerical Simulation*, vol. 3-4, pp. 391–394, 2002.
[7] A. Dzul, R. Lozano, and P. Castillo, “Adaptive control for a radio-controlled helicopter in a vertical flying stand,” *Int. J. Adapt. Control Signal Process.*, vol. 18, pp. 473–485, 2004.
[8] A. T. Kutay, A. J. Calise, M. Idan, and N. Hovakimyan, “Experimental results on adaptive output feedback control using a laboratory model helicopter,” *IEEE Trans. Contr. Syst. Technol.*, vol. 13, no. 2, pp. 196–202, 2005.
[9] K. Tanaka, H. Othake, and O. Wang, “A practical design approach to stabilization of a 3-DOF RC Helicopter,” *IEEE Trans. Contr. Syst. Technol.*, vol. 12 (2), pp. 315–325, 2004.
[10] T. Kiefer, A. Kugi, K. Graichen, and M. Zeitz, “Feedforward and feedback tracking control of a 3 DOF helicopter experiment under input and state/output constraints,” in *Proc. 45rd IEEE Conf. Dec. Control (CDC’06)*, San Diego, CA, Dec. 13–15, 2006, pp. 1586–1593.
[11] M. Nishi, M. Ishitobi, and K. Nakasaki, “Nonlinear adaptive control system design and experiment for a 3-DOF model helicopter,” *Artif. Life Robotics*, vol. 13, no. 1, pp. 50–53, Dec. 2008.
[12] A. Fradkov, B. Andrievsky, and D. Peaucelle, “Adaptive control design and experiments for LAAS “Helicopter” benchmark,” *Europ. J. Contr.*, vol. 14, no. 4, pp. 329–339, 2008.
[13] LAAS-CNRS. [Online]. Available: http://www.laas.fr
[14] W. S. Wong and R. W. Brockett, “Systems with finite communication bandwidth constraints – Part I: State estimation problems,” *IEEE Trans. Automat. Contr.*, vol. 42, no. 9, pp. 1294–1299, 1997.
[15] G. N. Nair and R. J. Evans, “Exponential stabilisability of finite-dimensional linear systems with limited data rates,” *Automatica*, vol. 39, pp. 585–593, 2003.
[16] G. N. Nair, R. J. Evans, I. Mareels, and W. Moran, “Topological feedback entropy and nonlinear stabilization,” *IEEE Trans. Automat. Contr.*, vol. 49, no. 9, pp. 1585–1597, Sept. 2004.
[17] L. Bazzi and S. Mitter, “Endcoding complexity versus minimum distance,” *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 2103–2112, 2005.
[18] G. N. Nair, F. Fagnani, S. Zampieri, and R. Evans, “Feedback control under data rate constraints: an overview,” *Proc. IEEE*, vol. 95, no. 1, pp. 108–137, Apr. 2007.

[19] J. Baillieul and P. J. Antsaklis, “Control and communication challenges in networked real-time systems,” *Proc. IEEE*, vol. 95, no. 1, pp. 9–28, 2007, Special Issue on Technology of Networked Control Systems.

[20] A. S. Matveev and A. V. Savkin, *State Estimation and Control over Communication Networks*. Boston: Birkhäuser, 2008.

[21] D. Liberzon, “Hybrid feedback stabilization of systems with quantized signals,” *Automatica*, vol. 39, pp. 1543–1554, 2003.

[22] D. Liberzon and J. P. Hespanha, “Stabilization of nonlinear systems with limited information feedback,” *IEEE Trans. Automat. Contr.*, vol. 50, no. 6, pp. 910–915, June 2005.

[23] C. De Persis, “n-Bit stabilization of n-dimensional nonlinear systems in feedforward form,” *IEEE Trans. Automat. Contr.*, vol. 50, no. 3, pp. 299–311, 2005.

[24] C. De Persis and A. Isidori, “Stabilizability by state feedback implies stabilizability by encoded state feedback,” *Systems & Control Letters*, vol. 53, pp. 249–258, 2004.

[25] C. De Persis, “On stabilization of nonlinear systems under data rate constraints using output measurements,” *Int. J. Robust Nonlinear Control*, vol. 16, pp. 315–332, Feb. 2006.

[26] C. De Persis and D. Nešić, “Practical encoders for controlling nonlinear systems under communication constraints,” in *Proc. 44th IEEE Conference on Decision & Control, and European Control Conference*, vol. MoA13.3, Seville, Spain, Dec. 2005, pp. 434–439.

[27] T. M. Cheng and A. V. Savkin, “Output feedback stabilisation of nonlinear networked control systems with non-decreasing nonlinearities: A matrix inequalities approach,” *Int. J. Robust Nonlinear Control*, vol. 17, pp. 387–404, Oct. 2007.

[28] A. S. Matveev and A. V. Savkin, “An analogue of Shannon information theory for networked control systems: State estimation via a noisy discrete channel,” in *Proc. 43rd IEEE Conf. Dec. Control (CDC’04)*, vol. 4, Atlantis, Bahamas, 2004, pp. 4485–4490.

[29] B. La Scala and R. J. Evans, “Minimum necessary data rates for accurate track fusion,” in *Proc. 44th Conf. Dec. Contr., Europ. Contr. Conf.*, Seville, Spain, Dec. 12–15, 2005, pp. 6966–6971.

[30] R. Evans, V. Krishnamurthy, G. Nair, and L. Sciacca, “Networked sensor management and data rate control for tracking maneuvering targets,” *IEEE Trans. Signal Processing*, vol. 53, no. 6, pp. 1979–1991, June 2005.

[31] V. Malyavej, I. R. Manchester, and A. V. Savkin, “Precision missile guidance using radar/multiple-video sensor fusion via communication channels with bit-rate constraints,” *Automatica*, vol. 42, pp. 763–769, 2006.

[32] A. L. Fradkov, B. Andrievsky, and R. J. Evans, “Chaotic observer-based synchronization under information constraints,” *Physical Review E*, vol. 73, p. 066209, 2006.

[33] ——, “Controlled synchronization under information constraints,” *Physical Review E*, vol. 78, pp. 036 210 1–6, Sept. 2008.

[34] ——, “Synhronization of nonlinear systems under information constraints,” *Chaos*, vol. 18, no. 4, pp. 037 109, 1–11, 2008.

[35] ——, “Adaptive observer-based synchronization of chaotic systems with first-order coder in presence of information constraints,” *IEEE Trans. Circuits Syst. I*, vol. 55, no. 6, pp. 1685–1694, July 2008.

[36] ——, “Synchronization of passifiable Lurie systems via limited-capacity communication channel,” *IEEE Trans. Circuits Syst. I*, vol. 56, no. 2, pp. 430–439, Feb. 2009.

[37] B. Andrievsky, “Adaptive coding for transmission of position information over the limited-band communication channel,” in *Proc. 9th IFAC Workshop “Adaptation and Learning in Control and Signal Processing” (ALCOSP’2007)*, St. Petersburg, Russia, Aug. 29-31, 2007.

[38] B. Andrievsky, A. L. Fradkov, and D. Peaucelle, “State estimation over the limited-band communication channel for pitch motion control of LAAS Helicopter benchmark,” in *Proc. 17th IFAC Symposium on Automatic Control in Aerospace (ACA’2007)*, Toulouse, France, June 2007.

[39] P. J. Gawthrop, *Continuous-Time Self-Tuning Control*. Letchworth, U.K.: Research Studies Press, 1987.

[40] A. L. Fradkov and B. Andrievsky, “Combined adaptive controller for UAV guidance,” *Europ. J. Contr.*, vol. 11, no. 1, pp. 71–79, 2005.

[41] K. S. Narendra and A. M. Annaswamy, *Stable adaptive systems*. New Jersey: Englewood Cliffs, Prentice-Hall, 1989.

[42] A. L. Fradkov, *Adaptive control in large-scale systems*. Moscow: Nauka, 1990, (in Russian).

[43] A. Loría, E. Panteley, D. Popović, and A. Teel, “A nested Matrosov theorem and persistency of excitation for uniform convergence in stable non-autonomous systems,” *IEEE Trans. Automat. Contr.*, vol. 50, no. 2, pp. 183–198, 2005.

[44] D. Peaucelle, A. L. Fradkov, and B. Andrievsky, “Adaptive identification of angular motion model parameters for LAAS Helicopter Benchmark,” in *Proc. 16th IEEE Int. Conf. Control Applications*, Singapore, Oct. 1-3, 2007, pp. 825–830.

[45] S. Le Gac, D. Peaucelle, and B. Andrievsky, “Adaptive parameter identification for simplified 3D-motion model of ‘LAAS Helicopter Benchmark’,” in *Proc. 9th IFAC Workshop “Adaptation and Learning in Control and Signal Processing” (ALCOSP’2007)*, St. Petersburg, Russia, Aug. 29-31, 2007.

[46] J. Baillieul, “Feedback designs for controlling device arrays with communication channel bandwidth constraints,” in *ARO Workshop on Smart Structures*, University Park, PA, Aug. 16–18, 1999.

[47] K. Li and J. Baillieu, “Robust quantization for digital finite communication bandwidth (DFCB) control,” *IEEE Trans. Automat. Contr.*, vol. 49, no. 9, pp. 1573–1584, Oct. 2004.

[48] R. W. Brockett and D. Liberzon, “Quantized feedback stabilization of linear systems,” *IEEE Trans. Automat. Contr.*, vol. 45, pp. 1279–1289, 2000.

[49] S. Tatikonda and S. Mitter, “Control under communication constraints,” *IEEE Trans. Automat. Contr.*, vol. 49, no. 7, pp. 1056–1068, 2004.

[50] D. J. Goodman and A. Gersho, “Theory of an adaptive quantizer,” *IEEE Trans. Commun.*, vol. COM-22, no. 8, pp. 1037–1045, Aug. 1974.

[51] F. Gomez-Estern, C. Canudas de Wit, F. Rubio, and J. Fornés, “Adaptive delta-modulation coding for networked controlled systems,” in *Proc. Amer. Contr. Conf.*, N.Y., USA, July 11–13, 2007, frA20.6.