NUMERICAL AND LABORATORY EXPERIMENTS WITH CONTROLLED COUPLED PENDULUMS

B.R. Andrievsky[1], K.B. Boykov[2]

[1] *Institute for Problems of Mechanical Engineering of RAS, 61 Bolshoy ave.,V.O., Saint-Petersburg, 199178, Russia,*
e-mail: andrh@ipme.ru
[2] *Baltic State Technical University, 1st Krasnoarmejskaja, 1, Saint-Petersburg, 199005, Russia.*

Abstract: A problem of oscillation excitation and synchronization is considered on the example of two coupled pendulum-like mechanical systems. Electrical equipment, data exchange interface and software for on-line laboratory experiments and control are described. Pulse-modulated control law for oscillation excitation/synchronization is obtained via the speed-gradient method. Laboratory experiments are used for verification and parameter estimation of the adopted mathematical model. Comparison results of the simulations and laboratory experiments for oscillation control is given.

Keywords: nonlinear dynamics, nonlinear control, sinchronization, computer-aided system analysis.

INTRODUCTION

The problem of oscillatory mechanical systems control has the significant theoretical interest and growing value in practice. For the purposes of the scientific research and control engineering education it would be necessary to build up appropriate laboratory equipment and software to work out approaches for investigation of this kind of systems. There are many papers where this problem was considered and the significant results have been achieved (Andrievsky *et al.,* 1999; Åström and Furuta, 2000; Bakker *et al.*, 1996; Blackburn *et al.*, 1992; Christini *et al.,* 1996; Dunnigan, 1998; Furuta and Yamakita, 1998; Furuta *et al.,* 1999; Gäfvert it et al, 1999; Huang S. and Huang C. 2000; Lenci and Rega, 2000; Miroshnik and Bobtsov, 2000; Shiriaev *et al.,* 2000; Wu *et al.,* 2000; Yagasai and Yamashita, 1999; Yi and Yubazaki, 2000). In this paper the results of (Andrievsky *et al.,* 1998; Andrievsky *et al.,* 1999) are further developed.

This article deals with two similar mechanical toys. Each of them includes two coupled pendulums, one into another. The toys are connected by means of the resilient tie. In Sec. 1 the brief description of the construction and its mathematical model is given. For making laboratory experiments and on-line control, the electrical design, data exchange interface and software tools were created. Their description is given in paragraphs 1.3, 1.4. Section 2 demonstrates the usage of laboratory experiments for getting more precise values of the plant model parameters. In the considered system only period of oscillations is measurable and only pulsed control torque can be applied to the plant. Under this condition the problem of feedback control on the plant behavior turns into the difficult one. The problem of the real-time state estimation, based the accessible flow data

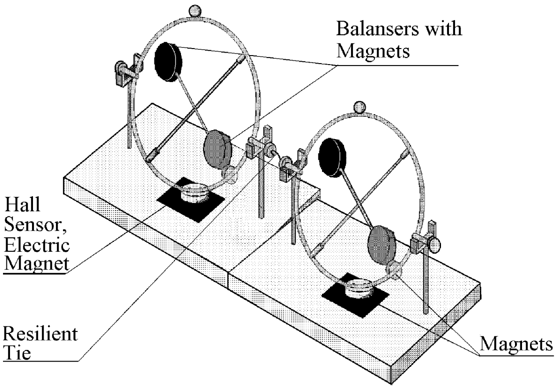

Fig. 1. Coupled pendulums with the resilient tie.

and the system model is considered in Sec. 3. The control law, ensuring prescribed energy level of whole mechanical system and synchronization of the pendulums is obtained by speed-gradient technique (Andrievsky *et al.,* 1996; Fradkov *et al.,* 1998; Fradkov and Andrievsky, 1999) and described in Sec. 4.

1. LABORATORY DEVICE

Laboratory device includes: *mechanical oscillating system*; *electrical equipment* (with *computer interface facilities*), and *the personal computer* for experimental data processing, representation of the results and the real-time control. For data exchange via standard In-Out ports of the computer, the special *exchange routine* is written.

1.1. *Mechanical system*

The controlled plant consists of the *mechanical* and *electromagnetic devices.* The mechanical part of the system, in its turn, consists of two similar subsystems (mechanical toys). Each of them includes two coupled pendulums. These subsystems are connected one with another by the resilient tie, see Fig. 1.

Pendulums have displaced centers of parts weight and sliding support of an external link. An external part of each subsystem is a metal ring with located on it a massive ball and two cylindrical magnets. The ball and magnets displace the center of weight of the ring. Magnets, in addition, provide transfer of control efforts to the both links. On an outside surface of the ring two opposite directed half-axes, ensuring its support on two flat platforms with terminators of a course, are located. System is fixed on the massive basis, in center of which the Holl sensor of the first link zero state and electromagnet; transmitting control efforts on cylindrical magnet are located.

The second part of coupled pendulum is presented by two cylindrical loads with magnets that established on axis symmetric concerning its center. This part rotates inside an external ring with an axis of rotation fixed on it with the help of cylindrical hinges. These hinges are on an axis that is turned by 45 degrees with respect to the axis of an external ring rotation.

Each mechanical subsystem has three degrees of freedom:

1. Parallel carry of an external ring on flat support of the basis;

2. Rotations of an external ring with the above-stated half-axes;

3. Rotation of the second link of a rather external ring.

On the specified mechanical system the following control efforts are applied:

1. Influence of an electromagnet of the basis on one of magnets, established on an external ring;

2. Influence of the second magnet, established on an external ring on magnets of second (internal) link.

Measuring value is an interval of time between transitions of an external ring magnet above an electromagnet of the basis. These events are determined by means of the Hall sensors.

1.2. *Simplified mathematical model of the mechanical system*

Let us consider, at first, a single subsystem. Its detailed model in the *aggregative form* is given in Andrievsky *et al.* (1999). For the purposes of the present research the simplified model is used. Taking into account that in the considered plant the inner pendulum exerts weak influence on the motion of the external ring, one can use given below reduced plant model.

By means of transforms and simplified suggestions, plant model can be written as:

$$\begin{aligned} &J_e\ddot{\varphi} = -m_1 g(R\sin\varphi - (\rho + r)\cos\varphi) - \\ &m_2 g(f\sin\varphi - (\rho + r)\cos\varphi) + \\ &m_p g(\rho + r)\cos\varphi + \\ &m_3 g(R_m\sin\varphi + (\rho + r)\cos\varphi) + M(\varphi, u), \end{aligned} \quad (1)$$

where $J_e = m\rho^2 + J_0$ is *the equivalent moment of inertia* of the plant. The coefficients m_i, r, R_m, ρ denote the mass-geometric parameters of the system. The last term $M(\cdot)$ describes the torque of *the electromagnetic forces.* The electromagnetic attraction excites by *the residual magnetization* of the core. Repulsive electromagnetic force caused by *the controlling signal* $u(t)$, applied to the clips of the electromagnet. Assuming that the electromagnetic force changes inversely as the square of the distance between the magnets, one can get the following formulas for $M(\varphi, u)$:

$$\begin{aligned} f_m(\varphi) &= \big((R(1-\cos\varphi)+\delta)^2 + \\ &\quad R^2\sin^2\varphi\big)^{-1}, \\ \gamma(\varphi) &= \pi - \arctan\frac{R\,(1-\cos\varphi)+\delta}{R\sin\varphi}, \\ \mu(\varphi) &= -f_m(\varphi)\cos\gamma, \\ M(\varphi,u) &= (A_0 - A_u u)\mu(\varphi), \end{aligned} \tag{2}$$

where δ is the minimal value of the gap between the magnets, R denotes the radius of the external ring. A_0, A_m are assumed to be constant. Their values depend on the residual magnetization and the properties of the magnetoelecrtric circut. Let us rewrite model (1) in the state-space form, where the viscous friction is taken into account. One obtains the following equations:

$$\begin{cases} \dot\varphi(t) = \omega(t), \\ \dot\omega(t) = -a_1\sin(\varphi(t)-\psi) - a_2\omega(t) + \\ \quad (a_0 + a_u u(t))\mu(\varphi). \end{cases} \tag{3}$$

Preliminary mass–geometrical examination gives the foolowing rough values of the model (3) parameters: $a_1 \approx 44\ s^{-2}$; $\psi \approx 0.13\ rad$; parameter a_2 belongs to the interval $[0.1 \div 0.5]\ s^{-1}$. Parameters a_0, a_m are obtained from relations $a_0 = A_0/J_e, a_m = A_m/J_e$. Their values should be found experimentally. Let us turn now to the composite mechanicat construction with a resilient tie between subsytems. Assuming resilience of the tie to be linear, one obtains mechanical system model in the following form:

$$\begin{cases} \dot\varphi_1(t) = \omega_1(t), \\ \dot\omega_1(t) = -a_1\sin(\varphi_1(t)-\psi) - a_2\omega_1(t) + \\ \quad k(\varphi_2-\varphi_1) + (a_0 + a_u u_1(t))\mu(\varphi_1). \\ \dot\varphi_2(t) = \omega_2(t), \\ \dot\omega_2(t) = -a_1\sin(\varphi_2(t)-\psi) - a_2\omega_2(t) - \\ \quad k(\varphi_2-\varphi_1) + (a_0 + a_u u_2(t))\mu(\varphi_2). \end{cases} \tag{4}$$

Parameter k in (4) is depends on the rigidity of the spring. Variables φ_i stand for deviation angles of the i-th ring.

1.3. *Hardware environment*

Oscillation control is provided on the basis of combined hard/soft-ware method. The energy for excitation is transmitted by the pulse-width modulated (PWM) signal with the constant level and variable duty cycle. From the programming point of view, hardware represents by the *write-only registers* (WO) for putting in the prescribed duty cycle of control signal from the computer, and the *read-only registers* (RO) for transfer oscillation half-period duration values to the computer. The PWM based method provides more precise control than numeric-pulse one, because of integration the high frequency pulses by the mechanical subsystem (the weight of one discrete of a control have more less value, than on numeric-pulse control method).

Hardware has two identical channels for each mechanical subsystem and the common *peripheral controller*. The measuring unit is built using the *quartz oscillator* to calibrate the main clock pulses. This pulses after the frequency divider to 1000 Hz determine sample time of 1 ms. Maximum measurable time interval, provided by 12-bit counter is from 0.001 to 4.095 s. The beginning and the end of count interval is determined by the *Hall sensors* and *zero-crossing detector*. For data transfer the peripheral controller uses the *Standard Parallel Port* (SPP). in byte bi-directional mode.

The *control unit* generates the exciting action applied to the pendulums via the opposite magnetic fields. It includes bi-channel *asinchronous pulse-width modulator* (APWM), logical command interpreting automat and the *power amplifiers* to drive the electromagnets.

1.4. *Software facilities*

As a basic development software for plant control and measured data processing is used programming package MATLAB^R, supplied with the *Parallel In/Out Toolbox* (PIO-Toolbox) (Boykov, 1999). The communications protocol based on the *sub-addressing method*, arrays (vectors in MATLAB) are considered as data units for transfer.

Main software routine is programming function *pio - Parallel In/Out.* This function makes available vector elements output through the parallel 8-bit channel (commonly used for a printer). There are some advantages of *pio*-function over the similar one from the Realtime Toolbox^R (for details, see Boykov (1999)). Function *pio* is written in assembly language using the means of *Borland C* compiler to connect with MATLAB and also may be re-compiled for SCO UNIX, LINUX as ELF- or DEE-modules.

2. PARAMETER IDENTIFICATION BASED ON THE LABORATORY EXPERIMENT

Let us find model (3) parameters more exactly on the basis of laboratory experiment. Complexity of this problem increases because only time intervals between zero-cross instants are measurable. For

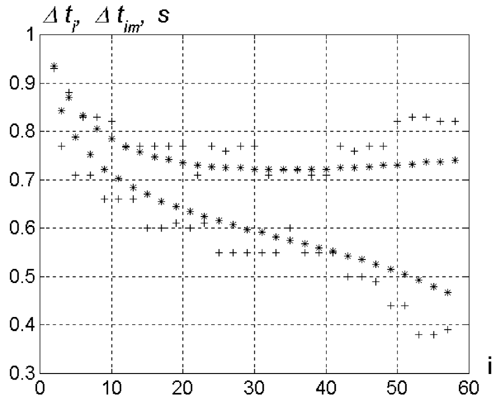

Fig. 2. The sequences of measured (+) and modelling (*) intervals Δt_i, Δt_{i_m} between zero-crossing.

making experiment the field magnet was switched off ($u(t) \equiv 0$), the conjunctive spring was removed and initial ring deflection at the angle of 150 degrees was given. As a result, the sequence of N intervals $\{\Delta t_i\}$ (where $i = 1, 2, \ldots, N$), was obtained. This sequence is shown in Fig. 2.

This experimental sequense is compared with the simulation results, taken by means of the model (3) with choosen parameter values. After numerical optimization procedure the revised parameter are found. For usage the searching optimization technique two lost functionals are used: $Q_1 \triangleq \frac{1}{N} \sum_{i=1}^{N} (t_{k_i} - t_{m_i})^2$ is the mean square value between real and modelling zero-cross instants. This lost functional is considered as a main one. An additional functional $Q_2 \triangleq \frac{|N_m - N|}{N}$, where N_m is a number of zero-crossing in the modelling realization during the same period of time. Functional Q_2 is considered as a restriction. Finally, the lost functional Q is defined as:

$$Q = \begin{cases} 10^4 \cdot Q_2, & \text{when } Q_2 > 0.05, \\ Q_1, & \text{else.} \end{cases}$$

Using the standard optimization routine the following parameter estimates were found: $a_0^* = 0.89\ s^{-2}$, $a_1^* = 27\ s^{-2}$; $a_2^* = 0.06\ s^{-1}$; $\psi^* = 0.08\ rad$. Comparative results are presented in Fig. 2.

3. REAL-TIME STATE ESTIMATION

The identification of the system state vector based on the measurements of the time intervals $\Delta t_k = t_k - t_{k-1}$ (where $\varphi(t_k) = 0, k = 0, 1, 2, \ldots$) is a challenging task. In order to investigate its feasibility the state observer design was performed.

It is known (Migulin *et al.*, 1978), that for a simple pendulum can be written the following equation:

$$T = \sqrt{2} \int_{A_1}^{A_2} \frac{dx}{\sqrt{h - P(x)}}, \tag{5}$$

where A_1 and A_2 - deviation of a pendulum in moment, when its speed is equal to zero; h - total energy of system; $P(x)$ - potential function, i.e. a function, proportional to potential energy of the system. This relationship can be used for the state estimation of the considered system (Andrievsky *et al.*, 1999).

Again neglecting the motion of the second link and the friction the simplified pendulum-like system model (3) is used to obtain the state estimation algorithm.

The observer equation in the interior of the interval $[t_{k-1}, t_k]$ coincides with (3). By means of relation (5) one can calculate the state variables of the observer at time t_k as follows: $\tilde{\varphi}(t_k) = 0$, $\tilde{\omega}(t_k) = \Omega(\varphi_0)$. The initial (amplitude) value φ_0, in its turn, can be found as $\varphi_0 = \Phi(T)$, where $\Phi(T)$ is the inverse function (5) resolved for A_2 ($A_1 = 0$). It can be determined numerically *a priori*. Observer correction algorithm takes the following form:

$$\begin{aligned} &\hat{\varphi}(t_k) = 0, \quad \tilde{\omega}_k = \Omega(\varphi_0(T_k)), \\ &\hat{\omega}(t_k) = d\hat{\omega}(t_{k-1}) + (1-d)\tilde{\omega}_k. \end{aligned} \tag{6}$$

Renewal coefficient d, $|d| < 1$, is used to provide the algorithm with the disturbance filtering properties. If $d = 0$, only current measurements are taken into account.

Another *gradient energy based correction* algorithm is proposed to reduce the influence of the disturbances. This algorithm has the form

$$\begin{aligned} &\hat{\varphi}_{k+1} = \Big(\hat{\varphi}_k + \\ &\alpha \frac{\Delta H_k mgl \sin(\hat{\varphi}_k - \varphi_0)}{\big(mgl_e \sin(\hat{\varphi}_k - \varphi_0)\big)^2 + (J_e \hat{\omega}_k)^2}\Big) \bmod 2\pi, \\ &\hat{\omega}_{k+1} = \hat{\omega}_k + \alpha \frac{\Delta H_k J_e \hat{\omega}_k}{\big(mgl_e \sin(\hat{\varphi}_k - \varphi_0)\big)^2 + (J_e \hat{\omega}_k)^2}, \end{aligned}$$

where $\alpha > 0$ is a gain coefficient, $\Delta H_k = \hat{H}_k - \tilde{H}_k$, $\tilde{H}_k = J_e \tilde{\omega}(t_k)^2/2$, $\hat{H}_k = J\hat{\omega}_k^2/2 + mgl_e\big(1 - \cos(\hat{\varphi}_k - \varphi_0)\big)$.

4. EXCITING AND SYNCHRONIZATION OF TWO COUPLED PENDULUMS

Consider two pendulums model (3). For lossless case it has the form

$$\begin{cases} \ddot{\varphi}_1(t) + \omega^2 \sin\varphi_1(t) = \\ \qquad k\big(\varphi_2(t) - \varphi_1(t)\big) + u(t), \\ \ddot{\varphi}_2(t) + \omega^2 \sin\varphi_2(t) = \\ \qquad k\big(\varphi_1(t) - \varphi_2(t)\big), \end{cases} \tag{7}$$

where $\varphi_i(t)$, $(i = 1, 2)$ are the rotation angles of pendulums, $u(t)$ is the external torque, (control action), applied to the first pendulum, ω, k are the system parameters: ω is the natural frequency of small oscillations, k is the coupling strength (e.g. stiffness of the string).

Introduce the state vector $x(t) \in \mathcal{R}^4$ as $x(t) \triangleq \mathrm{col}\{\varphi_1, \varphi_2, \dot\varphi_1, \dot\varphi_2\}$. The total energy of the system (7) $H(x)$ can be written as follows

$$\begin{array}{l} H(x) = \frac{1}{2}\dot\varphi_1 + \omega^2(1 - \cos\varphi_1) + \\ \frac{1}{2}\dot\varphi_2^2 + \omega^2(1 - \cos\varphi_2) + \frac{k}{2}\left(\varphi_1 - \varphi_2\right)^2 \end{array} \qquad (8)$$

Consider the problem of excitation a “wave” with the desired amplitude by means of small feedback. The problem can be understood as achieving the given energy level of the system with additional requirement that pendulums should have the opposite phases of oscillation.

In order to apply the Speed-gradient procedure (Fradkov *et al.* 1998; Fradkov and Andrievsky (1999)) introduce *the objective functions* as

$$\begin{array}{l} Q_\varphi(\dot\varphi_1, \dot\varphi_2) \triangleq \frac{1}{2}\left(\delta_\varphi\right)^2 \\ Q_H(x) \triangleq \frac{1}{2}(H(x) - H_*)^2. \end{array} \qquad (9)$$

where $\delta_\varphi = \dot\varphi_1 + \dot\varphi_2$ and H_* is the prescribed value of the total energy.

The minimum value of the function Q_φ meets the "opposite phases" requirement (at least for small initial phases $\varphi_1(0), \varphi_2(0)) : Q_\varphi(\dot\varphi_1, \dot\varphi_2) \equiv 0$ iff $\dot\varphi_1 \equiv -\dot\varphi_2$. The minimization of Q_H means achievement of the desired amplitude of the oscillations.

In order to design the control algorithm the weighted objective function $Q(x)$ is introduced as the weighted sum of Q_φ and Q_H, namely

$$Q(x) \triangleq \alpha Q_\varphi(\dot\varphi_1, \dot\varphi_2) + (1 - \alpha) Q_H(x), \qquad (10)$$

where α, $0 \le \alpha \le 1$ is a given weighting coefficient.

The speed-gradient (SG) *method* with the local objective functional $Q(x)$ gives the control law in the *finite form* as

$$u(t) = -\gamma \nabla_u \phi(x, u, t), \qquad (11)$$

where $\gamma > 0$ is a gain coefficient, x is the plant state vector, $\nabla_u(\cdot)$ stands for the gradient in u, $\phi(x, u, t)$ is the time derivative (speed) of $Q(x)$ with respect to the equations (7). Performing calculations according to the Speed-gradient procedure (Fradkov *et al.,* 1998) one arrives to the following control law

$$\begin{array}{l} u(t) = -\gamma\left(\alpha\delta_\varphi(t) + \left(1 - \alpha\right)\delta_H(t)\dot\varphi_1(t)\right), \\ \delta_\varphi(t) = \dot\varphi_1(t) + \dot\varphi_2(t), \\ \delta_H(x(t)) = H(t) - H_*, \end{array}$$

where $\gamma > 0$ is a gain coefficient.

Applying the results of Fradkov *et al.* (1998), Fradkov and Andrievsky (1999) yields that sufficient conditions for the achievement of the control goal $Q(x(t)) \to 0$ are valid if the desired level of energy does not exceed the value $H_* = 2\omega^2$, corresponding to the upper equilibrium of one pendulum and lower equilibrium of another one.

Computer simulations were performed in order to complement the theoretical conclusions. After some transient time both pendulums oscillate with the opposite phases while both goal functions approach the prescribed values. The transient time for both H and for Q_φ is about 100 time units. The relation between transient times for H and for Q_φ can be changed by means of changing the weight coefficient α. The control amplitude can be arbitrarily decreased by means of decreasing the gain γ (when damping is not taken into account).

CONCLUSION

The laboratory equipment for experiments with the controlled double links pendulums is described. The simple pulsing control law is used to provide the given total energy (or, in other words, given amplitude of oscillations) of the pendulum. Results of the numerical experiments are close to the laboratory ones. The special form of the observer is developted to obtain the state estimates via conditions when only period of oscillations can be measured. The algorithms for oscillation excitation and synchronization are described. More detailed results may be found in Blekhman and Fradkov (2001).

ACKNOWLEDGMENTS

The work was supported in part by the Federal Program “Integration” (project *N A0151*) and also by grant of the St.Petersburg Scientific Center of RAS.

REFERENCES

Andrievsky, B.R., A.L. Fradkov and P.Yu. Guzenko (1996). Control of nonlinear oscillations of mechanical systems by speed-gradient method. *Autom. Remote Control.* **57** (4). 456–467.

Andrievsky, B.R., A.L. Fradkov, V.A. Konoplev and A.P. Konjukhov (1998). Control, State Estimation and Laboratory Experiments with Oscillatory Mechanical System. In: *Prepr. Nonlin. Contr. Design Symp. NOLOCOS'98,* Twente, Holland. 761 – 764.

Andrievsky, B.R., V.A. Konoplev, A.P. Konjukhov and A.L. Fradkov (1999). Modeling, Simulation and Experiment with Double Pendulum Chaotic Toy. In: *Proc. 5th European Contr.Conf. Karlsruhe, Aug. 31–Sep. 3.*

Åström, K.J. and K. Furuta (2000). Swinging up a pendulum by energy control. *Automatica*, **36**: (2), 287–295.

Bakker, R., J.C. Schouten, F. Takens, *et al.* (1996). Neural network model to control an experimental chaotic pendulum. *Phys. Rev.* E **54**: (4), 3545–3552.

Blackburn, J.A., *et al.* (1992). Stability and Hopf bifurcations in an inverted pendulum. *Amer. Journ. Phys.* **60**, 903–908.

Blekhman, I.I. and Fradkov, A.L., Eds. (2001) *Control of Mechatronic Vibrational Units.* St.Petersburg, Nauka (in Russian).

Boykov, K.B. (1999). Usage of MATLAB^R software for compilation of external devices data. In: *Proc. the XI Conf. "Sensors and Data Processors for Measurement and Control (DATCHIK-99)", Gurzuf.* Moscow Inst. Elect. Mashin. (in Russian).

Christini D.J., J.J Collins and P.S. Linsay (1996). Experimental control of high-dimensional chaos: The driven double pendulum. *Phys. Rev.* E **54**: (5) 4824–4827 Nov.

Dunnigan, M.W. (1998). Computer based control assignment digital control of an inverted pendulum. *Int. Journ. Elec. Eng. Educ.* **35**: (2) 162–174.

Fradkov, A. and B. Andrievsky (1999). Feedback Resonance in 1-DOF and 2-DOF Nonlinear Oscillators. In: *Proc. 7th Meditterranean Conference on Control and Automation (MED'99). June 28 – 30* Haifa, Israel. No TA4–2. 1244 – 1253.

Fradkov, A.L., I.V. Miroshnik and V.O. Nikiforov (1998). *Nonlinear and Adaptive Control of Complex Systems.* Kluwer, Dordrecht.

Furuta, K. and M. Yamakita (1991). Swing up control of inverted pendulum. In: *IECON'91.* 2193–2198.

Furuta, K., R. Gabasov and F. Kirillova (1999). Bounded Stabilizing Feedbacks for Nonlinear Inverted Pendulum Models. In: *Proc. of the "6th St.Petersburg Symp. on Adaptive Systems Theory (SPAS'99)", St.Petersburg, Sept. 7 – 9.* **1**. 86 – 87.

Gäfvert M., J. Svensson and K.J. Åström (1999). Friction and Friction Compensation in the Furuta Pendulum. In: *Proc. "European Control Conference (ECC'99)", Karlushe, Germany, Aug. 31 – Sept. 3.* No F1007–5.

Huang S.J. and C.L. Huang (2000). Control of an inverted pendulum using grey prediction model. *IEEE Trans. Ind. Appl.* **36**: (2) 452–458.

Lenci, S and G. Rega (2000). Numerical control of impact dynamics of inverted pendulum through optimal feedback strategies *Journ. Sound Vib.* **236**: (3) 505–527.

Migulin, V.V., V.I. Medvedev, E.R. Mustel, and V.N. Parygin (1978). *Basics of Oscillation Theory* Moscow, Nauka (in Russian).

Miroshnik, I. and A. Bobtsov (2000). Stabilization of Motions of Multi-pendulum Systems In: *Proc. 2nd Intern. Conf. "Control of Oscillations and Chaos (COC2000)", St.Petersburg, July 5 – 7.* **1**. 22 – 25.

Shiriaev, A., A. Pogromsky, H. Ludvigsen *et al.* (2000). On global properties of passivity-based control of an inverted pendulum. *Int. Journ. Robust. Nonlin.* **10**: (4) 283–300.

Wu, Q., N. Sepehri and S. He (2000). On control of a base-excited inverted pendulum using neural networks. *Journ. Franklin Inst.* **337**: (2–3) 267–286.

Yagasaki, K. and S. Yamashita (1999). Controlling chaos using nonlinear approximations for a pendulum with feedforward and feedback control. *Int. Journ. Bifurcat. Chaos.* **9**: (1) 233–241.

Yi, J. and N. Yubazaki N. (2000). Stabilization fuzzy control of inverted pendulum systems. *Artif. Intell. Eng.* **14**: (2) 153–163.