COMBINED ADAPTIVE FLIGHT CONTROL SYSTEM

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Abstract. Combined adaptive control scheme is presented and used for adaptive control for attitude of the aircraft with uncertain parameters. The proposed controller ensures finite time convergence of augmented error omitting to relay term in control law and exponential convergence of the parameter error under the condition of persistent excitation. This allows achieving the desired dynamics to the true plant output. The simulation results demonstrate high adaptability properties of the proposed controller.

1. Introduction

In recent years an interest raised to the development of adaptive schemes for plants with unknown relative degree by using output measurements only. The most interesting approach is in introduction the *parallel feedforward compensator.* The main idea of the method is to ensure the hyper minimum phase property (HMP) of the augmented plant (plant and compensator), see [11, 13]. This procedure simplifies the design of the adaptive controller. In [3, 5, 6, 8] a simplified adaptive control scheme has been presented, which performs the regulation of uncertain plants via pole assignment without requiring the perfect knowledge of the relative degree of the controlled plant and independently of the magnitude of the unmodelled dynamics. In this paper the method of [3] is used for aircraft adaptive flight control.

2. Combined Adaptive Controller

2.1. Control aim and the controller structure

Consider linear time-invariant SISO plant presented in the following form

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t), \quad y_p(t) = C_p x_p(t),$$

where $x_p(t) \in \Re^n$, $u(t) \in \Re$, $y_p(t) \in \Re$. The plant transfer function is

$$W_p(s) = C_p\left(sI_n - A_p\right)^{-1} B_p = \frac{B_p(s)}{A_p(s)} \qquad (1)$$

where $s \in \mathcal{C}$ denotes Laplas transform variable, $\deg A_p(s) = n$, $\deg B_p(s) = m$, $k = n - m$ is a plant *relative degree*. It is assumed that $W_p(0) > 0$, $k > 1$. Let plant be time-invariant has a priori undefined parameters and only output signal $y(t)$ is measurable. The control aim is to achieve desired closed-loop system performance described by following *reference model equation* (see also [5])

$$A_m(p) y_p(t) = K \cdot B(p) r(t), \qquad (2)$$

where $r(t)$ is a reference input signal, p denotes the time derivative operator $p \equiv d/dt$, $A_m(p)$ is an arbitrary chosen Hurwitz polynomial degree n, $K = A_m(0)/B(0)$. This equation corresponds to so-called "*implicit reference model*" [1, 10] and imposes less restrictions on system performance than for explicit reference model. Gain K is introduced to achieve astatism of the system. To achieve the aim (2) let us provide an accurate tracking to transformed reference signal $y_f(t)$ which is generated by adjustable prefilter described below. This tracking problem can be solved by means of organizing a *sliding motion* [16]. It is well known [10, 12], that the *hyper minimum phase* (HMP) condition is sufficient for existence of the stable sliding-modes as well as for direct adaptive control problem solution. For SISO plants the HMP condition means that plant transfer function has all zeros in the left half-plane and $k = 1$. These conditions are not assumed to be valid in the considered problem. It seems to be perspective to use parallel feedforward compensator (or "*shunt*") [4, 14, 15]. It makes possible to ensure requirement mentioned above for *augmented plant* (AP), consisting of controlled plant and shunt and allows to design adaptive control schemes that do not require plant output derivatives. Denote the shunt transfer function as $W_c(s) = \frac{B'(s)}{A'(s)}$, $\deg A'(s) = n'$. The AP output is $y = y_p + y_c$ and the AP transfer function has a form:

$$W(s) = W_p(s) + W_c(s) = \frac{F(s)}{A_p(s) A'(s)}, \qquad (3)$$

where $F(s) = A_p(s)B'(s) + A'(s)B_p(s)$. Let us notice that the AP output $y(t)$ is not identical to the plant output $y_p(t)$ and the ideal tracking of $y(t)$ to $y_f(t)$ does not involve those one for $y_p(t)$. Hence prefilter equations must be chosen properly. For this purpose let us find a transfer function $W_r(s)$ from r to y_p, assuming that $y(t) \equiv y_f(t)$. Taking into account (3) and the shunt equation one obtains that

$$W_r(s) = W_f(s) \frac{B(s) A'(s)}{F(s)}, \qquad (4)$$

where $W_f(s)$ is the *prefilter* transfer function. From (2), (4) it is follows that the control aim is achieved if $y(t) \equiv y_f(t)$ and $W_f(s)$ is taken as

$$W_f(s) = \frac{K \cdot F(s)}{A_m(s)A'(s)}, \quad (5)$$

where $K = A_m(0)B(0)^{-1}$. Notice that (4) describes a time-invariant filter for nonadaptive case. In presence of the plant parameters uncertainty the following *tunable prefilter* should be used instead of (4):

$$\dot{x}_f = A_f x_f + B_f r(t), \; y_f = \Omega(t)^T x_f, \quad (6)$$

Nominal value of $\Omega(t) \equiv \Omega_*(t)$ depends on the plant parameters. The latter ones are estimated by means of on-line identification algorithm described in Sec. 3. The shunt transfer function is taken as

$$W_c(s) = \frac{\kappa\varepsilon(\varepsilon s + \lambda)^{k-2}}{(s+\lambda)^{k-1}}, \quad \lambda > 0. \quad (7)$$

The following Theorems 1, 2 [3, 11, 12] give the necessary property of AP (3) with shunt (7).

Theorem 1. Let $W_p(s)$ (1) be minimum-phase ($B(s)$ be a Hurwitz polynomial) with the relative degree $k > 1$ and $W_p(0) > 0$. Then there exist $\kappa_0 > 0$ and function $\varepsilon_0(\kappa) > 0$ such that transfer function $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa > \kappa_0$ and $0 < \varepsilon < \varepsilon_0(\kappa_0)$.

Theorem 2. Let $W_p(s)$ be stable ($A(s)$ be a Hurwitz polynomial) with the relative degree $k > 1$ and $W_p(0) > 0$. Then for every $\varepsilon > 0$ there exists sufficiently large κ_0, such that $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa \geq \kappa_0$.

Theorem 1 shows, that one can introduce shunt (7) with order $\deg A_s(s) = k - 1 = n - m - 1$ providing for sufficiently large κ and small ε augmented plant (3) satisfying HMP condition for arbitrary given minimum-phase plant parameters domain. As it is follows from the Theorem 2, another way of shunt (7) parameters choosing provides HMP condition for stable (and, possible, nonminimum-phase) plants. For this case, the shunt equation can be simplified; namely $W_c(s) = \dfrac{\kappa}{s+\lambda}$ may be taken instead of (7).

2.2. The adjustment law

The continuous-time least-squares-like estimator is used for unknown plant parameter identification. That estimator does not need time derivatives of input/output signals. The first step is designing filters to avoid the measurements of the derivations of plant output. The plant equations (1) are rewritten in the following form:

$$\begin{aligned} &y^{(n)}(t) + a_1 y^{(n-1)}(t) + \cdots + a_n y(t) \\ &\quad = b_0 u^{(m)}(t) + b_1 u^{(m-1)}(t) + \cdots + b_m u(t), \end{aligned} \quad (8)$$

where $a_1,\ldots,a_n,b_0,\ldots,b_m$ are unknown plant parameters (index n means the $n\,th$ time derivative of the signal). It gives the following plant equation:

$$y^{(n)}(t) = \varphi(t)^T \theta_*, \quad (9)$$

where the *data vector* φ and the *parameter vector* θ_* are given by:

$$\varphi = \left[-y^{(n-1)},\ldots,-\dot{y},y,u^{(m)},\ldots,u\right]^T,$$

$$\theta_* = \left[a_1,a_2,\ldots,a_n,b_0,b_1,\ldots,b_m\right]^T,$$

$\varphi(t),\theta(t) \in \Re^{n+m+1}$. Introducing filtered signals $\tilde{y}(t)$, $\tilde{\varphi}(t)$, as solutions of the equations: $D(p)\tilde{y}(t) = y(t)$, $D(p)\tilde{\varphi}(t) = \varphi(t)$, where an *estimator filter polynomial* $D(p)$ is an arbitrary chosen Hurwitz polynomial of $p \equiv d/dt$, $\deg D(p) \geq n$, one obtains from (9) that it is valid $\tilde{y}^{(n)}(t) = \tilde{\varphi}(t)^T \theta_*$. Signals $\tilde{y}$, $\tilde{\varphi}$ could be generated by means of the following filters:

$$\dot{\xi} = A_d \xi + b_d y(t), \; \dot{\psi} = A_d \psi + b_d u(t), \quad (10)$$

where $\xi(t),\psi(t) \in \Re^n$; A_d, b_d have regular canonical form, $\det(sI - A_d) = D(s)$. Notice, that both ξ and ϕ can be implemented by using input/output measurements only. It is straightforward to see that

$$\tilde{\varphi}(t) = \left[\xi_n(t),\ldots,\xi_1(t),\psi_{m+1}(t),\ldots,\psi_1(t)\right]^T,$$

$$\tilde{y}^{(n)}(t) = y(t) - \textstyle\sum_{i=1}^{n} d_{n-i+1}\xi_i(t).$$

The identification algorithm has the following form:

$$\dot{\theta}(t) = \Gamma(t)\varphi(t)\sigma(t), \quad (11)$$

where $\sigma(t) = \tilde{y}^{(n)}(t) - \varphi(t)^T\theta(t)$ is an *identification error*, the matrix gain $\Gamma(t)$ is governed by the equation:

$$\dot{\Gamma}(t) = -\Gamma(t)\varphi(t)\varphi(t)^T\Gamma(t) + \alpha\Gamma(t), \quad (12)$$

where $\alpha > 0$ is referred to as the *forgetting factor*. The *covariance matrix* can also be found via algorithm

$$\dot{\Gamma}(t) = -\Gamma(t)\varphi(t)\varphi(t)^T\Gamma(t) + \left(\Gamma(t) - \Gamma(t)^2/\beta_0\right), \quad (13)$$

where $\beta_0 I > \Gamma(0) = \Gamma(0)^T > 0$, see [3] for details.

2.3. The variable structure control law

The *variable structure controller* (VSC) is designed to provide the sliding mode on the surface $s \equiv y - y_f = 0$ under the assumption that the system is HMP. This implies that A_* is a Hurwitz matrix and $c_2 b > 0$ [16]. The control action is taken as follows: $u(t) = -k_s s(t) - \gamma\,\mathrm{sign}\,s(t)$, where the positive parameters k_s and γ are specified below. The skeleton diagram of the combined adaptive control system is shown in Fig. 1.

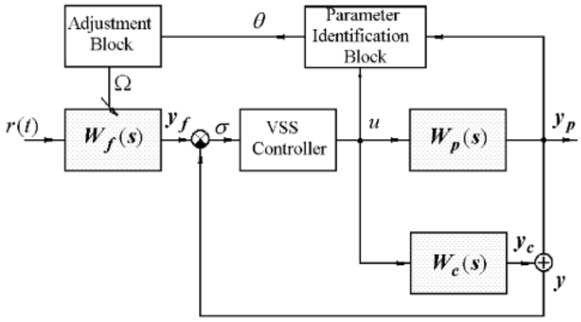

Fig. 1. Skeleton diagram of the combined adaptive control system

3. Aircraft Attitude Control

Let us apply the proposed scheme to the aircraft attitude control problem. The aircraft is modeled as a linear time invariant plant with uncertain parameters. The following aircraft lateral dynamics model is used:

$$\begin{cases} \dot\beta(t) = \omega_y(t) + a_z^\beta \beta(t) + a_z^\delta \delta_r(t), \\ \dot\omega_y(t) = a_m^\beta \beta(t) + a_m^\omega \omega_y(t) + a_m^\delta \delta_r(t), \\ \dot\psi(t) = \omega_y(t), \end{cases} \quad (14)$$

where $\psi(t), \omega(t)$ are the yaw angle and its time derivative correspondingly, $\beta(t)$ is the glancing angle; $\delta_r(t)$ is the rudder angle, a_i^j are aircraft model parameters. Their true values depend on flight conditions (such as altitude, speed and load) and are changing in the wide range during the flight. They assumed to be *a priory* unknown. It is also assumed that $\delta_r(t)$ is a control action and $\psi(t)$ is a measurable output. Equations (14) correspond plant transfer function (1) with $\deg A(s) = 3$, $\deg B(s) = 1$, $k = 2$. The problem is to achieve desired closed-loop flight control system dynamics in accordance with the reference-model (2), where $A_m(p) = p^3 + \alpha_1 p^2 + \alpha_2 p + \alpha_3$. Coefficients of the plant equation (8) can be found through parameters of the aircraft model (14). It is clean that it does not need to estimate $a_3 \equiv 0$, therefore in the considered example the number of estimated parameters can be decreased and the order of the estimation algorithm can be reduced. In the considered case plant relative degree $k = 2$ and shunt (7) has a form $W_c(s) = \dfrac{\kappa}{s + \lambda}$. The estimator filters (10) in this case have degree 3. Vector $\theta(t) \in \Re^4$ represents the estimates of aircraft transfer function coefficients, $\theta_{*=} = [a_1, a_2, b_0, b_1]^T$. The gain matrix (covariance matrix) $\Gamma \in \Re^{4\times 4}$, $\Gamma(0) = k_0 I$.

Let us consider now a variable-structure controller. Introduce the augmented output $y_a(t) = \psi(t) + y_c(t)$, where $y_c(t)$ is the shunt output and choose the control action as $\delta_r(t) = -k_s s(t) - \gamma \mathrm{sign} s(t)$, where $s(t) = y(t) - y_f(t)$; $y_f(t)$ is an output signal of the prefilter (6). To achieve the control aim (2) one has found the prefilter parameters so that under the assumption of convergence of the parameter estimates to their true values, the prefilter equation (6) satisfies (4). In the considered case it gives the following expression:

$$\begin{aligned} F(s) = \kappa s^3 + (\kappa a_1 + b_0)s^2 \\ + (\kappa a_2 + b_0\lambda + b_1)s + b_1\lambda \end{aligned} \quad (15)$$

The denominator of $W_f(s)$ (4) is the fourth-order polynomial $A_m(s)A'(s) = s^4 + (\lambda + d_1)s^3 + (\lambda d_1 + d_2)s^2 + (\lambda d_2 + d_3)s + \lambda d_3$. Finally one gets the equations of the adjustable prefilter (6). Let us find an area in the aircraft model parameter space, in which the HMP condition is satisfied. For the considered plant model the numerator of AP (3) transfer function $F(s)$ is given as (15) and has to be Hurwitz polynomial degree 3. It gives the following inequalities for the aircraft and shunt parameters

$$\begin{aligned} &\kappa > 0, \quad b_0 + \kappa a_1 > 0, \\ &\lambda b_0 + \kappa a_2 + b_1 > 0, \\ &\lambda b_0^2 + \kappa\lambda a_1 b_0 + \kappa^2 a_1 a_2 + b_0 b_1 - \kappa\lambda b_1 > 0, \end{aligned}$$

The hyper-minimum phase (HMP) domain is shown in Fig. 2. Evidently, the HMP property takes place for the wide range of the aircraft model parameters.

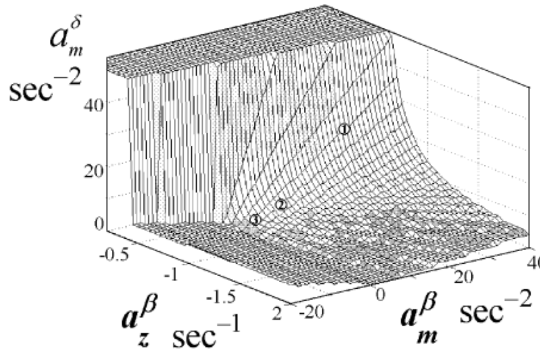

Fig. 2. The HMP domain

Simulation results are described below. The values of aircraft parameters are given in Tabl. 1. Notice that parameters are changed in the wide range and the regime No 1 corresponds to unstable aircraft dynamics. Transient processes of the yawing angle for the reference signal

$r(t) = r_0 \mathrm{sign}(\sin \omega_0 t)$, $r_0 = 5$ deg, $\omega_0 = \pi / 5\, s^{-1}$

and different flight conditions (see Tabl. 1) are given in Fig. 3. Figure 4 demonstrates convergence of the parameter estimates to their true values. Simulation results confirm the theoretical statements and demonstrate high adaptability of the proposed control system.

Table 1. Aircraft model parameters

	a_z^β	a_m^β	a_m^ω	a_z^δ	a_m^δ
N	s^{-1}	s^{-2}	s^{-1}	s^{-1}	s^{-2}
1	−0.4	15.0	−0.6	0.08	25
2	−0.4	−1.5	−0.6	0.06	2.5
3	−0.24	−1.5	−0.06	0.06	5.0

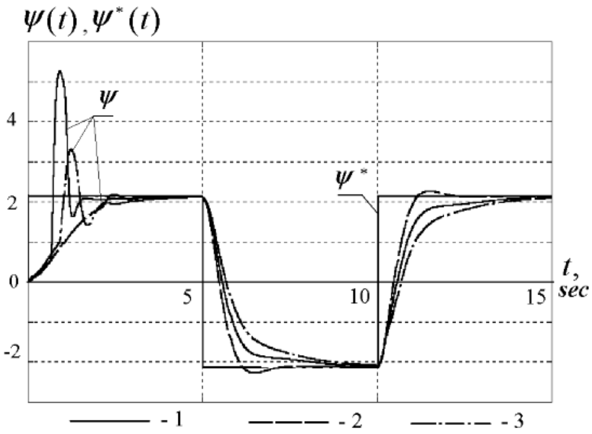

Fig. 3. The yawing angle $\psi(t)$ time histories.

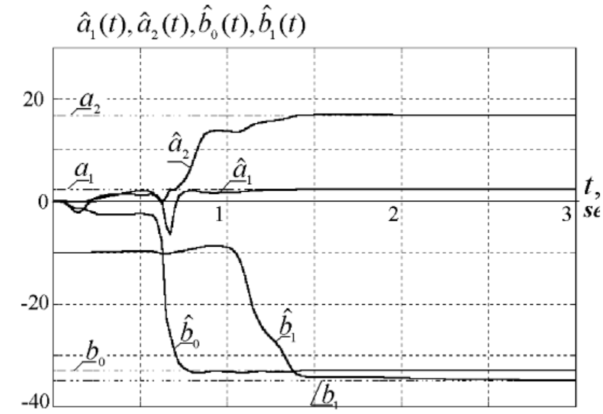

Fig. 4. Estimation process.

Conclusion

In this paper a combined adaptive control scheme is presented for control of unstable and nonminimum-phase uncertain plants by means of parallel feedforward compensator (shunt) which ensures hyper-minimum phase property of augmented plant. The proposed controller ensures finite time convergence of augmented error omitting to relay term in control law and exponential convergence of the parameter error under the condition of persistent excitation. This, in turn allows achieving the desired dynamics to the true plant output. The proposed method was applied and numerically examined to the problem of aircraft attitude adaptive control.

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