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ROBUST ADAPTIVE NONLINEAR PARTIAL OBSERVERS FOR TIME-VARYING CHAOTIC SYSTEMS

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Abstract— **Adaptive observers scheme is developed, which provides output tracking of uncertain plant with vector of unknown parameters estimation. These properties are robust with respect to any essentially bounded noise in output measurements channel. Two examples of computer simulation of proposed adaptive observer are included.**

I. INTRODUCTION

FIELD of adaptive observers design for nonlinear systems was intensively researched during last decade. One of the reasons of such interest consists in application of adaptive observers for information encoding and transmission. Particularly, using a chaotic dynamical system as a transmitter it is possible to build a receiver [12, 13, 14], that is a dynamical system, which can track output of transmitter and estimate transmitter parameters under some mild conditions. Potentialities of fast information transmitting in the presence of noise in such systems were demonstrated in [1, 2, 3]. Several techniques were used to design receivers [6, 8, 16, 22], the most of them are based on passifiability property of transmitter under assumption that relative degree of transmitter is equal to zero or one. Other solutions can be found in [10, 11], unfortunately, in those results a state feedback was used for adaptive observer construction and robust properties of proposed schemes were not investigated. Recent paper [9] overcame the relative degree limitation for adaptive observer-based communication systems and extended them to class of nonpassifiable systems. That result was based on new canonical form of nonlinear adaptive observers [7, 18]. In paper [4] result of [9] was developed and applicability conditions are obtained, which allow to enlarge class of transmitter systems. For example, proposed there result permits to use a brussellator model [19] as a transmitter system. In this work robust properties of developed in [4] adaptive observer are proved.

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In Section 2 previous result is discussed. In Section 3 conditions are presented, which ensure robust stability properties for adaptive observer with respect to any essentially bounded noise signals in output measurements channel. Computer simulation results are included in Section 4 with brussellator and Duffing's forced models as transmitter systems. Conclusion finishes the paper in Section 5.

II. PRELIMINARY RESULTS

As in [9] and [4] in this work it is supposed, that model of transmitter can be written as follows:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{y})\mathbf{x} + \varphi(\mathbf{y}) + \mathbf{B}(\mathbf{y})\boldsymbol{\theta}\,,\ \ \mathbf{y} = \mathbf{C}\mathbf{x}\,, \qquad (1)$$

where $\mathbf{x} \in R^n$ is state space vector of transmitter; $\mathbf{y} \in R^m$ is output vector; $\boldsymbol{\theta} \in \Omega_\theta \subset R^p$ is vector of "unknown" parameters of transmitter, or, better to say, it is transmitted vector, which values belong to some known compact set Ω_θ, should be estimated by receiver basing on on-line measurements of transmitter output $\mathbf{y}$. Vector function φ and columns of matrix functions $\mathbf{A}$ and $\mathbf{B}$ are locally Lipschitz continuous, $\mathbf{C}$ is some constant matrix of appropriate dimension. Thus, for any initial conditions $\mathbf{x}_0 \in \Omega_x \subset R^n$ and any $\boldsymbol{\theta} \in \Omega_\theta$ (where Ω_x some known, probably compact, set), solution of (1) $\mathbf{x}(t, \mathbf{x}_0, \boldsymbol{\theta})$ is well defined at the least locally (further we will omit dependence of $\mathbf{x}_0$ and $\boldsymbol{\theta}$ if it is clear from the context and will simply write $\mathbf{x}(t)$; we denote output solution as $\mathbf{y}(t, \mathbf{x}_0, \boldsymbol{\theta})$). In previous paper [4] it was supposed, that output of transmitter is measured without any noises or errors, but on practice such assumption is not satisfied, and in general output of transmitter $\mathbf{y}$ differs from measured by receiver output

$$\mathbf{y}_d(t) = \mathbf{y}(t) + \mathbf{d}(t)\,,\ t \ge 0\,, \qquad (2)$$

where additive noise $\mathbf{d}(t)$ reflects influences of external disturbances and errors in measurement channels, it is Lebesgue measurable essentially bounded function of time. It is usually assumed, that signal $\mathbf{d}$ is not directly available for measurements and its upper bound is not precisely known. For transmitter as in [4] and [9] it is usually to suppose, that its solution is bounded and defined for all $t \ge 0$.

Assumption 1. *For any initial condition* $\mathbf{x}_0 \in \Omega_x$ *and any* $\boldsymbol{\theta} \in \Omega_\theta$, *solution of (1)* $\mathbf{x}(t, \mathbf{x}_0, \boldsymbol{\theta})$ *is an essentially bounded function of time:*

$$|\mathbf{x}(t, \mathbf{x}_0, \boldsymbol{\theta})| \le \sigma_0(|\mathbf{x}_0|),\ \sigma_0 \in \mathcal{K} \text{ for all } t \ge 0. \quad \blacksquare$$

As usually, it is said, that function $\rho : R_{\ge 0} \to R_{\ge 0}$ belongs to class $\mathcal{K}$, if it is strictly increasing and $\rho(0) = 0$; $\rho \in \mathcal{K}_\infty$ if $\rho \in \mathcal{K}$ and $\rho(s) \to \infty$ for $s \to \infty$ (radially unbounded). Function $\mathbf{x} : R_{\ge 0} \to R^n$ is essentially bounded, if

$$\|\mathbf{x}\| = ess\sup\{|\mathbf{x}(t)|, t \ge 0\} < +\infty,$$

where $|\cdot|$ denotes usual Euclidean norm, $R_{\ge 0} = \{\tau \in R : \tau \ge 0\}$. Such assumption is valid for class of system (1) with so-called chaotic dynamics. Also in paper [4] the following two assumptions were introduced.

Assumption 2. *There exists continuous matrix function* $\mathbf{K} : R^m \to R^{n \times m}$, *such, that there exists function* $V : R^n \to R_{\ge 0}$, $\alpha_1(|\mathbf{x}|) \le V(\mathbf{x}) \le \alpha_2(|\mathbf{x}|)$

$$\partial V(\mathbf{x})/\partial \mathbf{x}\, \mathbf{G}(\mathbf{y})\mathbf{x} \le -\alpha_3 |\mathbf{C}\mathbf{x}|^2$$

for any bounded values of $\mathbf{y} \in R^m$ *and* $\mathbf{x} \in R^n$, *where* α_1, α_2 *are some functions from class* $\mathcal{K}_\infty$ *and* α_3 *is a positive constant, matrix* $\mathbf{G}(\mathbf{y}) = \mathbf{A}(\mathbf{y}) - \mathbf{K}(\mathbf{y})\mathbf{C}$. ■

Assumption 3. *For any initial conditions* $\mathbf{s}_0 \in R^n$ *solution of system*

$$\dot{\mathbf{s}} = \mathbf{G}(\mathbf{y})\mathbf{s} + \mathbf{r} \tag{3}$$

is bounded for any essentially bounded Lebesgue measurable inputs $\mathbf{r}$ *and* $\mathbf{y}$:

$$|\mathbf{s}(t, \mathbf{s}_0, \mathbf{r})| \le \sigma_1(|\mathbf{s}_0|) + \sigma_1(\|\mathbf{r}\|),\ \sigma_1 \in \mathcal{K} \text{ for all } t \ge 0. \quad \blacksquare$$

According to classical results [21] Assumptions 1 and 2 mean, that auxiliary system (3) combined with system (1) is partially stable with respect to variable $\mathbf{s} \in R^n$ and partially asymptotically stable with respect to variable $\mathbf{Cs}$ for $\mathbf{r} = 0$. Assumptions 2 and 3 allow to design for system (1) an adaptive observer as in papers [4] and [9] ($\hat{\mathbf{y}} = \mathbf{C}\mathbf{z}$):

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{y}_d)\mathbf{z} + \varphi(\mathbf{y}_d) + \mathbf{B}(\mathbf{y}_d)\boldsymbol{\theta} + \mathbf{K}(\mathbf{y}_d)(\mathbf{y}_d - \hat{\mathbf{y}}); \tag{4}$$

$$\dot{\boldsymbol{\eta}} = \mathbf{G}(\mathbf{y}_d)\boldsymbol{\eta} - \boldsymbol{\Omega}\dot{\hat{\boldsymbol{\theta}}}; \tag{5}$$

$$\dot{\boldsymbol{\Omega}} = \mathbf{G}(\mathbf{y}_d)\boldsymbol{\Omega} + \mathbf{B}(\mathbf{y}_d); \tag{6}$$

$$\dot{\hat{\boldsymbol{\theta}}} = \gamma \boldsymbol{\Omega}^T \mathbf{C}^T (\mathbf{y}_d - \hat{\mathbf{y}} + \mathbf{C}\boldsymbol{\eta}), \tag{7}$$

where $\mathbf{z} \in R^n$ is vector of estimates of unmeasurable state space vector of system (1); $\hat{\mathbf{y}} \in R^m$ is vector of output $\mathbf{y}$ estimates; vector $\boldsymbol{\eta} \in R^n$ and matrix $\boldsymbol{\Omega} \in R^{n \times p}$ are auxiliary variables, which help to overcome high relative degree obstruction; $\hat{\boldsymbol{\theta}} \in R^p$ is vector of estimates of "transmitted" vector $\boldsymbol{\theta}$; $\gamma > 0$ is a design parameter. Also the following useful property was introduced in [4].

Definition 1. *Function* $a : R_{\ge 0} \to R$ *is called* (μ, Δ)*–positive in average (PA), if for any* $t \ge 0$ *and any* $\delta \ge \Delta$, $\mu > 0$,

$$\int_t^{t+\delta} a(\tau) d\tau \ge \mu\delta. \quad \blacksquare$$

In other words, time function $a(t)$ is (μ, Δ)–PA, if its average value a_{av} on any large enough time interval $[t, t+\delta]$, $\delta \ge \Delta$,

$$a_{av} = \frac{1}{\delta} \int_t^{t+\delta} a(\tau) d\tau$$

is not smaller than some positive constant μ. This property is equivalent [4] to persistent excitation (PE) property from [5, 15, 17, 20]. Such PE condition was frequently used in adaptive control and identification theories [5]. The following Lemma with minor differences was proved in [4], it explains advances of PA property.

Lemma 1. *Let us consider time-varying linear dynamical system*

$$\dot{p} = -a(t)p + b(t),\ t_0 \ge 0, \tag{8}$$

where $p \in R$, $p(t_0) \in R$ *and functions* $a : R_{\ge 0} \to R$, $b : R_{\ge 0} \to R$ *are Lebesgue measurable,* b *is essentially bounded, function* a *is* (μ, Δ)*–PA for some* $\mu > 0$, $\Delta > 0$ *and essentially bounded from below, i.e. there exists* $A \in R_{\ge 0}$, *such, that:*

$$ess\inf\{a(t),\ t \ge t_0\} \ge -A.$$

Then solution of system (8) is defined for all $t \ge t_0$ *and it admits an upper estimate:*

$$p(t) \le \begin{cases} \left[|p(t_0)| + \|b\| e^{A(t_0+\Delta)}\right] e^{A\Delta} & \text{if } t \le t_0 + \Delta; \\ p(t_0) e^{-\mu(t-t_0)} + \mu^{-1}\|b\| e^{-\mu t_0} & \text{if } t \ge t_0 + \Delta. \end{cases}$$

Additionally, if $b(t) \to 0$ *for* $t \to +\infty$, *then also* $p(t) \to 0$ *asymptotically.* ■

Properties of designed adaptive observer without noise are substantiated in the following theorem.

Theorem 1 [4]. *Let Assumptions 1 – 3 hold and* $\mathbf{d}(t) \equiv 0$, $t \ge 0$, *then closed-loop system consisting of transmitter (1), adjustable receiver (4), scheme of augmentation (5), (6) and adaptation algorithm (7) possesses properties:*

1. *For any initial conditions and any* $\gamma > 0$ *solution of the system is bounded and*

$$\lim_{t\to+\infty} \mathbf{y}(t)-\hat{\mathbf{y}}(t)=0\,;$$

2. *If function* $\left|\mathbf{C}\,\mathbf{\Omega}(t)\right|^2$ *satisfies* (μ,Δ)*–PA condition for some* $\mu>0$, $\Delta>0$*, then additionally*

$$\lim_{t\to+\infty} \mathbf{\theta}-\hat{\mathbf{\theta}}(t)=0\,. \qquad ■$$

Note, that second condition of the Theorem is fulfilled, if matrix function $\mathbf{B}(\mathbf{y})$ satisfies PE condition as in [9]. The result of Theorem 1 is based on existence of special kind Lyapunov function, like stated in Assumption 2. But in general it is a hard task to construct such Lyapunov function for the system. Due to Assumption 2, such Lyapunov function should depend on whole state space vector $\mathbf{s}$ of system (3), but it is much easy to find a Lyapunov function, which is dependent only on signal $\mathbf{Cs}$ like the following one.

A s s u m p t i o n 4. *There exists continuous matrix function* $\mathbf{K}:R^m\to R^{n\times m}$*, such, that there exists function* $V:R^n\to R_{\geq 0}$,

$$\alpha_1(|\mathbf{C}\mathbf{x}|)\leq V(\mathbf{x})\leq\alpha_2(|\mathbf{C}\mathbf{x}|),\ |\partial V/\partial\mathbf{s}|\leq\rho(|\mathbf{s}|);$$
$$\partial V(\mathbf{x})/\partial\mathbf{x}\,\mathbf{G}(\mathbf{y})\mathbf{x}\leq-\alpha_3(|\mathbf{C}\mathbf{x}|)+\alpha_4(|\mathbf{x}|),$$

for any bounded values of $\mathbf{y}\in R^m$ *and* $\mathbf{x}\in R^n$*, where* α_1, α_2, α_3 *are some functions from class* $\mathcal{K}_\infty$ *and* α_4,ρ *are functions from class* $\mathcal{K}$. ■

R e m a r k 1. By itself Assumption 4 means nothing, but with combination with Assumption 3 they provide for system (3) ultimate boundedness of signal $\mathbf{Cs}$ for $\mathbf{r}=0$. Indeed, when Assumption 3 implies existence of finite norm $\|\mathbf{s}\|$ for state vector of system (3), inequality for time derivative of function V from Assumption 4 takes form:

$$\dot{V}\leq-\alpha_3\circ\alpha_2^{-1}(V(\mathbf{s}))+\alpha_4\circ\sigma_1(|\mathbf{s}_0|).$$

From the last inequality the following output asymptotic gain can be obtained ($\lambda(s)=\alpha_1^{-1}\circ\alpha_2\circ\alpha_3^{-1}\circ\alpha_4\circ\sigma_1(s)$):

$$|\mathbf{Cs}(t)|\leq\lambda(|\mathbf{s}_0|)\ \text{for}\ t\geq T,$$

for some $T>0$. It is clear, that gain function α_3^{-1} can be "decreased" by appropriate choice of design matrix function $\mathbf{K}$, thus desired asymptotic bound for signal $\mathbf{Cs}$ can be assigned. Additionally, Assumptions 3 and 4 also ensure boundedness of the signal $\mathbf{Cs}$ for system (3) with additive input $\mathbf{r}\in R^n$:

$$|\mathbf{Cs}(t)|\leq\lambda_1(|\mathbf{s}_0|)+\lambda_2(\|\mathbf{r}\|)\ \text{for}\ t\geq T,$$
$$\lambda_1(s)=\alpha_4(2\sigma_1(s))+0.5\rho(2\sigma_1(s))^2,$$
$$\lambda_2(s)=0.5s^2+\alpha_4(2\sigma_1(s))+\rho(2\sigma_1(s))^2 s,$$

this result can be obtained by analogous line of consideration. ■

T h e o r e m 2 [4]. *Let Assumptions 1, 3 and 4 hold and function* $\left|\mathbf{C}\,\mathbf{\Omega}(t)\right|^2$ *satisfies* (μ,Δ)*–PA condition for some* $\mu>0$, $\Delta>0$*;* $\mathbf{d}(t)\equiv 0$, $t\geq 0$*. Then closed-loop system consisting of transmitter (1), adjustable receiver (4), scheme of augmentation (5), (6) and adaptation algorithm (7) provides boundedness property of the system solution for any initial conditions and any* $\gamma>0$*, additionally*

$$\lim_{t\to+\infty}\left|\mathbf{\theta}-\hat{\mathbf{\theta}}(t)\right|\leq\sqrt{\mu^{-1}e^{-\gamma\mu\Delta}}\ \lambda(|\mathbf{\delta}_0|). \qquad ■$$

Opposite to case of Theorem 1, the result of the last Theorem does not guarantee attractiveness property of output estimation error $\mathbf{\varepsilon}$ for an adaptive observer. In general only asymptotic convergence of parameter error $\mathbf{\theta}-\hat{\mathbf{\theta}}(t)$ to some compact set is provided. In fact, for information transmitting purposes only parameter error convergence is necessary, and output error estimation is not needed.

III. INVESTIGATION OF ROBUST PROPERTIES OF OBSERVER

Both theorems in the previous section used Assumptions 1 and 3 (boundedness of solutions of systems (1) and (3)) and PA property. Thus, in this section we will prove robust stability property for the observer system basing only on these conditions.

T h e o r e m 3. *Let Assumptions 1 and 3 hold and function* $\left|\mathbf{C}\,\mathbf{\Omega}(t)\right|^2$ *satisfies* (μ,Δ)*–PA condition for some* $\mu>0$, $\Delta>0$*, then the closed-loop system consisting of transmitter (1), adjustable receiver (4), scheme of augmentation (5), (6) and adaptation algorithm (7) provides boundedness of the system solutions for any initial conditions and any* $\gamma>0$.

P r o o f. The dynamics of state estimation error $\mathbf{e}=\mathbf{x}-\mathbf{z}$ can be written as follows:

$$\dot{\mathbf{e}}=\mathbf{G}(\mathbf{y}_d)\mathbf{e}+\varphi(\mathbf{y})-\varphi(\mathbf{y}_d)+\mathbf{B}(\mathbf{y})\mathbf{\theta}-\mathbf{B}(\mathbf{y}_d)\hat{\mathbf{\theta}}-$$
$$-\mathbf{K}(\mathbf{y}_d)\mathbf{d}(t)+[\mathbf{A}(\mathbf{y})-\mathbf{A}(\mathbf{y}_d)]\mathbf{x},$$

$\mathbf{\varepsilon}=\mathbf{Ce}$. The dynamics of auxiliary signal $\mathbf{\delta}=\mathbf{e}+\mathbf{\eta}-\mathbf{\Omega}(\mathbf{\theta}-\hat{\mathbf{\theta}})$ takes form :

$$\dot{\mathbf{\delta}}=\mathbf{G}(\mathbf{y}_d)\mathbf{\delta}+\varphi(\mathbf{y})-\varphi(\mathbf{y}_d)+[\mathbf{B}(\mathbf{y})-\mathbf{B}(\mathbf{y}_d)]\mathbf{\theta}-$$
$$-\mathbf{K}(\mathbf{y}_d)\mathbf{d}(t)+[\mathbf{A}(\mathbf{y})-\mathbf{A}(\mathbf{y}_d)]\mathbf{x}.$$

Let us denote external input in the last dynamical system as

$$\mathbf{D}(t)=\varphi(\mathbf{y}(t))-\varphi(\mathbf{y}_d(t))+[\mathbf{B}(\mathbf{y}(t))-\mathbf{B}(\mathbf{y}_d(t))]\mathbf{\theta}-$$
$$-\mathbf{K}(\mathbf{y}_d(t))\mathbf{d}(t)+[\mathbf{A}(\mathbf{y}(t))-\mathbf{A}(\mathbf{y}_d(t))]\mathbf{x}(t)\ ,$$

according to definitions of functions φ, $\mathbf{B}$, $\mathbf{K}$ and $\mathbf{A}$, they are locally Lipschitz continuous all. Hence, the following estimates hold:

$$\varphi(\mathbf{y})-\varphi(\mathbf{y}_d)\leq\mathbf{L}_\varphi\mathbf{d},$$
$$\mathbf{B}(\mathbf{y})-\mathbf{B}(\mathbf{y}_d)\leq\mathbf{L}_B\mathbf{d},\ \mathbf{A}(\mathbf{y})-\mathbf{A}(\mathbf{y}_d)\leq\mathbf{L}_A\mathbf{d},$$

where $\mathbf{L}_\varphi$, $\mathbf{L}_B$, $\mathbf{L}_A$ are corresponding Lipschitz constants for arguments are owned in compact set

$$\Xi = \{\mathbf{y} : |\mathbf{y}| \le |\mathbf{C}|\sigma_0(|\mathbf{x}_0|)\}.$$

Output $\mathbf{y}$ belongs to set Ξ due to Assumption 1. Thus, the following upper estimate is satisfied for signal $\mathbf{D}$:

$$|\mathbf{D}(t)| \le \left[|\mathbf{L}_\varphi| + |\mathbf{L}_B||\boldsymbol{\theta}| + K_{\max} + |\mathbf{L}_A|\sigma_0(|\mathbf{x}_0|)\right]\|\mathbf{d}\|,$$

where $K_{\max} = \max_{|\mathbf{y}_d| \le |\mathbf{C}|\sigma_0(|\mathbf{x}_0|) + \|\mathbf{d}\|}\{|\mathbf{K}(\mathbf{y}_d)|\}$ and value of vector $\boldsymbol{\theta}$ belongs to some compact set Ω_θ, then it is possible to conclude, that signal $\mathbf{D}$ is bounded. Dynamical behavior of variables $\mathbf{e}$ and $\boldsymbol{\delta}$ can be transformed to

$$\dot{\mathbf{e}} = \mathbf{G}(\mathbf{y}_d)\mathbf{e} + \mathbf{D}(t) + \mathbf{B}(\mathbf{y}_d)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}), \quad (9)$$

$$\dot{\boldsymbol{\delta}} = \mathbf{G}(\mathbf{y}_d)\boldsymbol{\delta} + \mathbf{D}(t). \quad (10)$$

Both systems have form of system (3), hence, according to Assumption 3 solutions of these systems should be bounded if their inputs are. For system (10) boundedness of inputs $\mathbf{y}_d$ and $\mathbf{D}$ has been already established. For system (9) boundedness of parameter estimation error $\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ should be substantiated. To solve this problem note, that system (6) is also of form (3) with bounded inputs $\mathbf{y}_d$ and $\mathbf{B}(\mathbf{y}_d)$, hence its solution $\boldsymbol{\Omega}(t)$ is bounded. Let us investigate time derivative of function

$$W(\hat{\boldsymbol{\theta}}) = \gamma^{-1}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}),$$

for system (7) it takes form:

$$\dot{W} = -2(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T \boldsymbol{\Omega}^T \mathbf{C}^T\left[\mathbf{C}(\boldsymbol{\delta} + \boldsymbol{\Omega}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})) + \mathbf{d}\right] \le$$

$$\le -|\mathbf{C}\,\boldsymbol{\Omega}|^2|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}|^2 + |\mathbf{C}\boldsymbol{\delta} + \mathbf{d}|^2 = -\gamma|\mathbf{C}\,\boldsymbol{\Omega}|^2 W + |\mathbf{C}\boldsymbol{\delta} + \mathbf{d}|^2.$$

Signal $|\mathbf{C}\,\boldsymbol{\Omega}(t)|^2$ is PA and signal $\mathbf{C}\boldsymbol{\delta} + \mathbf{d}$ is bounded (it is corollary of Assumption 3). Thus, parameter estimation error $\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ is bounded due to result of Lemma 1. Further, boundedness of error $\mathbf{e}$ also is proven (in system (9) all inputs are bounded). And finally, signal $\boldsymbol{\eta}$ is bounded due to it forms bounded signal $\boldsymbol{\delta}$, but all other signals, which also form signal $\boldsymbol{\delta}$, are bounded. So, boundedness of the system solution is obtained. ■

It is worth to note, that in robust and non robust sections the same structure of adaptive observer was used, thus result of Theorem 3 can be combined with results of Theorem 1 and 2 in the following sense.

C o r o l l a r y 1 . *Let all conditions of Theorem 3 hold.*

A. *If Assumption 2 is valid, then for case of noise* $\mathbf{d}$ *absence the following asymptotic relations are satisfied*:

$$\lim_{t\to+\infty} \mathbf{y}(t) - \hat{\mathbf{y}}(t) = 0, \quad \lim_{t\to+\infty} \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(t) = 0.$$

B. *If Assumption 4 holds, then there exists* $T > 0$:

$$\left|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(t)\right| \le \sqrt{\mu^{-1} e^{-\gamma\mu\Delta}}\left[\lambda_1(|\boldsymbol{\delta}_0|) + \lambda_2(\|\mathbf{d}\|)\right] \textit{ for } t \ge T.$$

P r o o f . Point A of the Corollary clearly follows from Theorem 1 for case $\mathbf{y}_d = \mathbf{y}$, and point B can be deduced by analogues with Theorem 2 and Remark 1. ■

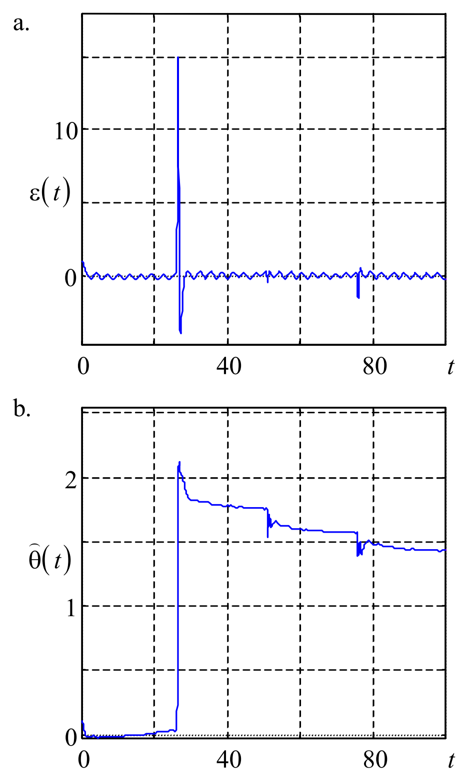

Fig. 1. Simulation result for brusselator model with noise presence.

An adaptive observer scheme properties are investigated for two different set of conditions. The main difference between these conditions consists in requirement imposed on Lyapunov function candidate V, which should be found for transmitter system under feedback gain $\mathbf{K}$. The character of function V forms asymptotic bounds for output and parameter estimation errors.

IV. Applications

In this section we will consider two examples of transmitter systems, which are useful from practical point of view. Both examples satisfy conditions of Theorems 2 and 3 [4], computer simulations demonstrate workability of proposed results.

4.1. Forced brusselator model

The equations of this model can be written in the following form [19]:

$$\dot{s}_1 = A + a\cos(\omega t) - (B + 1 + \theta)s_1 + s_1^2 s_2;$$

$$\dot{s}_2 = (B + \theta)s_1 - s_1^2 s_2; \; y = s_1,$$

where s_1 and s_2 are state variables with positive real values; y as usually is on-line measured output; parameters $A = 0.4$, $a = 0.05$, $\omega = 0.81$ and $B = 1.2$; unknown or "transmitted" parameter θ belongs to set $\Omega_\theta = [0, 2]$ (during simulation it will be taken $\theta = 1$). Equations of adaptive observer for this system can be found in [4]. Results of computer simulation of the system with adaptive observer are presented in Fig. 1 for noise $d(t) = 0.5\sin(2t)$, in Fig. 1,a and 1,b time graphics for output and parameter estimation errors are presented.

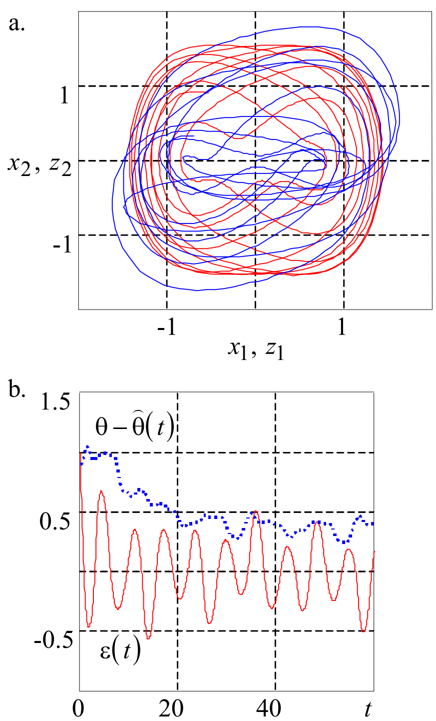

Fig. 2. Simulation result for Duffing's model with noise presence.

4.2. Forced Duffing's model

This well known model has form:

$$\dot{x}_1 = x_2; \; y = x_1;$$
$$\dot{x}_2 = -a\,x_1 + \theta x_1^3 + B\cos(\omega t),$$

where x_1 and x_2 are state variables; y is measured output; $\theta \in \Omega_\theta = [0.5, 1.5]$ is "transmitted" parameter; model parameters $a = B = \omega = 1$. For some bounded set of initial conditions $x_1(0) \le 1$ and $x_2(0) \le 1$ this system produces bounded solution and for such initial conditions system satisfies to Assumption 1. Equations of adaptive observer can be taken from [4].

The result of computer simulation of this system is shown in Fig. 2 for $K = \gamma = 1$ and noise $d(t) = 0.2\sin(t)$. In Fig. 2,a and 2,b state space trajectories and time graphics for output and parameter estimation errors are presented. The presence of noise d as in the first example does not cause unboundedness of solution of adaptive observer.

V. Conclusion

Robust properties with respect to any bounded noise in measurement channels of the observer from [4] are substantiated. Additionally two examples of computer simulation are included, which demonstrate potentialities and workability of developed observer scheme for case of noise presence.

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