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Robust Synchronization of Linear Networks with Compensation of Disturbances[1]

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Abstract: The problem of robust synchronization of a network of interconnected dynamical subsystems with a leader is considered. Each local subsystem of network is described by a linear time-varying parametrically and functionally uncertain differential equation. Only scalar inputs and outputs of local subsystems are supposed to be measured. A simple decentralized controller ensuring the tracking of the local subsystems by the leader under matching conditions is designed. The method is illustrated by an example: synchronization of the network with four nodes.

Keywords: Networks, robust control, decentralized control.

1. INTRODUCTION

The compensation of uncontrolled disturbances is an important problem in control theory. There are many publications proposing various solutions. Bobtsov (2003), Nikiforov (2003) the method of internal model of disturbances and adaptive, robust approaches are used for compensation of unknown perturbation. Bobtsov et. al. (2010), Bobtsov et. al. (2011), Pyrkin (2010) the method of adaptive and robust control are used for compensation of multi-harmonic disturbances. Control systems based on compensation of disturbance estimates is considered by Tsykunov (2007). It is proposed to introduce the auxiliary plant which allows to identify, estimate and compensate disturbances. On the basis of the algorithm of Tsykunov (2007) in Furtat and Tsykunov (2008) a compensation of parametric, functional and structural uncertainties is proposed. The principles of the suboptimal compensation of disturbances of uncertain plants are proposed by Bukov (2006). It was proposed to decompose the input of plant as a sum of the optimal and compensation signals for solving. The problem of suboptimal control. The idea of creating the optimal control (Bukov (2006)) and the compensation signal (Tsykunov (2008)) allowed to solve the problem of suboptimal control for time-varying parametrically and functionally uncertain plant in Furtat (2009).

An interest in control of networks has increased recently, see Lu and Chen (2005), Ren and Beard (2005), Yao et al. (2006), Zhou at al. (2006). One of the approaches to solving these problems is the decentralized control (Ioannou (1986), solved when the state vector of plant was available for measurement. The relative degree of subsystems is assumed to be Gavel and Siljak (1989), Mirkin (1999), Zhong et al. (2007), Das and Lewis (2010)). In most papers the problem was equal to one in the problems of network synchronization on outputs (Dzhunusov and Fradkov (2009)). Moreover, the goal is to make state vector of the local subsystem close to the state of the leader when the time tends to infinity. Therefore, there is an interest to solve the problem of synchronization of the network by an output feedback with compensation of disturbances if each local subsystem has an arbitrary relative degree.

In this paper an algorithm for synchronization of network of non-identical time-varying parametrically and functionally uncertain nodes is proposed. Only scalar outputs of the local subsystems are available to measurement. The goal condition is synchronization of network with leader. The auxiliary plant allows to estimate disturbances and guarantee the closeness of outputs of each local subsystems to the output of the leader and the output of the leader reaches the output of the nominal plant. The behavior of this nominal plant is optimal according to integral performance index. The results of simulations illustrate the efficiency of the proposed algorithm.

2. PROBLEM STATEMENT

Consider a network S with a nonempty finite set of k nodes $V=\{v_1, ..., v_k\}$ and set of links $E \subseteq V \times V$. Let the digraph $G=(V, E)$ of the networks be simple, e.g. there is no repeated arcs and $(v_i, v_i) \notin E$ (no self loops). In the graph the node i corresponds to the i-th local subsystem. If the arc of

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the node j is directed to the node i then $c_{ij}(t) \neq 0$, where $c_{ij}(t)$ are the directed couplings from nodes i to j, the matrix $C(t) = \left(c_{ij}(t)\right) \in R^{k\times k}$ is the matrix of the network. The sum $\sum_{j=1,\, i\neq j}^{k} \left|c_{ij}(t)\right|$ defines the weighted incoming degree of a node i.

Let the dynamics of the node i of network S be defined by the following differential equation

$$\begin{aligned} &\dot{x}_i(t) = A_i(t)x_i(t) + B_i(t)u_i(t) + D_i(t)f_i(t) + \\ &+ \sum_{j=1,\, i\neq j}^{k} c_{ij}(t)x_j(t), \quad y_i(t) = Lx_i(t), \\ &x_i(0) = x_{0i}, \quad i = \overline{1,k}, \end{aligned} \tag{1}$$

where $x_i(t) \in R^n$ is state vector of the node i, $u_i(t)$, $f_i(t)$ and $y_i(t)$ are the scalar input, uncontrollable disturbance and output respectively, the elements of matrix $A_i(t) \in R^{n\times n}$ and vectors $B_i(t) \in R^n$, $D_i(t) \in R^n$ are unknown functions, $L = [1, 0, \ldots, 0] \in R^{1\times n}$, x_{0i} are unknown initial conditions.

The leading node of network S is described by the equation

$$\begin{aligned} &\dot{x}_L(t) = A_L x_L(t) + B_L u_L(t), \quad y_L(t) = Lx_L(t), \\ &x_L(0) = 0. \end{aligned} \tag{2}$$

Here $x_L(t) \in R^n$ is the state vector, $u_L(t)$ is reference input, $A_L \in R^{n\times n}$ и $B_L \in R^n$ are known matrices and A_L is Hurwitz. The signal $y_L(t)$ is known reference output.

Assumptions:

A1. Elements of the matrices $A_i(t)$, $B_i(t)$, $D_i(t)$, function $c_{ij}(t)$ and initial condition are unknown. Size of matrices $A_i(t)$, $B_i(t)$, G, $D_i(t)$ and L_i are known and $\text{rank}B_i(t) = m$, $\text{rank}D_i(t) = l$, $m < n$, $l < n$. The set Ξ of possible values of elements of the matrices $A_i(t)$, $B_i(t)$, $D_i(t)$ and functions $c_{ij}(t)$ are known. The elements of the matrices $A_i(t)$, $B_i(t)$, $D_i(t)$ and functions $c_{ij}(t)$ are bounded functions.

A2. The matching conditions $A_i(t) = A_L + B_L c_i^{\mathrm{T}}(t)$, $B_i(t) = B_L + B_L \tau_i(t)$, $D_i(t) = B_L k_i(t)$, $c_{ij}(t) = B_L \theta_{ij}^T(t)$ hold, where $c_i(t) \in R^n$, $\tau_i(t) \in R$, $k_i(t) \in R$ and $\theta_{ij}(t) \in R^n$ are vectors and functions of the unknown parameters.

A3. External disturbances $f_i(t)$ are unknown bounded functions.

A4. Derivatives of the signals $y_i(t)$ and $u_i(t)$ are not available to measurement.

Also consider nominal model for the tracking error $\bar{y}_i(t) = y_i(t) - y_L(t)$:

$$\begin{aligned} &\dot{\bar{x}}_{Ni}(t) = A_L \bar{x}_{Ni}(t) + B_L u_{0i}(t), \\ &\bar{y}_{Ni}(t) = L\bar{x}_{Ni}(t), \; \bar{x}_{Ni}(0) = x_{0Ni}. \end{aligned} \tag{3}$$

Assume, that in (3) the state vector is available to measurement. Introduce performance index and the optimal control (Athans and Falb (1966))

$$\begin{aligned} &J_i = \int_0^\infty \left[\bar{x}_i^T(t)\,\tilde{Q}_i\,\bar{x}_i(t) + \tilde{r}_i u_{0i}^2(t)\right]dt, \\ &u_{0i}(t) = -K_{0i}\bar{x}_i(t), \end{aligned} \tag{4}$$

where $\tilde{Q}_i = diag\{\tilde{q}_i, 0, \ldots, 0\}$, $\tilde{q}_i > 0$, $\tilde{r}_i > 0$, $K_{0i} = \tilde{r}_i^{-1} B_L^{\mathrm{T}} F_i$, matrix $F_i = F_i^T > 0$ is the solution of the Riccati equation $A_L^{\mathrm{T}} F_i + F_i A_L - \tilde{r}_i^{-1} F_i B_L B_L^{\mathrm{T}} F_i = -\tilde{Q}_i$.

The problem is to design the law which provides the tracking error $\bar{y}_i(t)$ close enough to the output of the nominal model (3). The behavior of the model (3) should be optimal according to the performance index (4), i.e.

$$\left|\bar{y}_i(t) - \bar{y}_{Ni}(t)\right| < \delta_1, \tag{5}$$

where δ_1 is small enough number. With respect to additionally the following conditions should be satisfied:

$$\overline{\lim_{t\to\infty}}\left|\bar{y}_i(t)\right| < \delta_2. \tag{6}$$

Here δ_2 is a small number.

3. MAIN RESULT

Using the matching conditions (A2) rewrite the plant (1) as

$$\begin{aligned} &\dot{x}_i(t) = A_L x_i(t) + B_L u_i(t) + B_L u_L(t) + B_L \varphi_i(t), \\ &y_i(t) = Lx_i(t), \quad x_i(0) = x_{0i}, \end{aligned} \tag{7}$$

where $\varphi_i(t) = c_i^T(t)x_i(t) - u_L(t) + \sum_{j=1,\, i\neq j}^{k} \theta_{ij}^T(t)x_j(t) + \tau_i(t)u_i(t) + k_i(t)f_i(t)$ depends on the parametric and functional disturbances of the plant (1).

The equation of the tracking error $\bar{x}_i(t) = x_i(t) - x_L(t)$ are

$$\begin{aligned} &\dot{\bar{x}}_i(t) = A_L \bar{x}_i(t) + B_L u_i(t) + B_L \varphi_i(t), \\ &\bar{y}_i(t) = L\bar{x}_i(t), \quad \bar{x}_i(0) = x_{0i}. \end{aligned} \tag{8}$$

Consider an isolated subsystem (8) which does not contain unknown uncertainties, i.e. $\varphi_i(t) \equiv 0$. Then we get the nominal model (3) . Adding to and subtracting from the right hand side of (8) the optimal control $u_{0i}(t)$ from (4), transform equation (8) to the form

$$\dot{\bar{x}}_i(t) = A_{0i}\bar{x}_i(t) + B_L u_i(t) + B_L \psi_i(t), \quad \bar{y}_i(t) = L\bar{x}_i(t), \tag{9}$$

where $A_{0i} = A_L - B_L K_{0i}$, $\psi_i(t) = \varphi_i(t) - K_{0i}\bar{x}(t)$.

The optimal control (4) minimizes the functional (4) if $\varphi_i(t) \equiv 0$ in (8). However, in the nodes (1) $\varphi_i(t) \neq 0$. Consequently, it is necessary to introduce some signal requiring to compensate these uncertainties. As in Furtat (2009), consider the auxiliary plant

$$\dot{x}_{ai}(t) = A_{0i}x_{ai}(t) + \beta_i B_L u_i(t), \quad y_{ai}(t) = Lx_{ai}(t), \quad x_{ai}(0) = x_{0i}, \tag{10}$$

$\beta_i > 0$. Considering the equation (9) and (10), form the mismatch function $\zeta_i(t) = \bar{y}_i(t) - y_{ai}(t)$:

$$\dot{\sigma}_i(t) = A_{0i}\sigma_i(t) + B_L\phi_i(t), \quad \zeta_i(t) = L\sigma_i(t), \quad \sigma_i(0) = 0. \tag{11}$$

Here $\sigma_i(t) \in R^n$, $\phi_i(t) = (1-\beta_i)u_i(t) + \psi_i(t)$. Transform (11) to the input-output form

$$Q_{0i}(p)\zeta_i(t) = R_{Li}(p)\phi_i(t), \tag{12}$$

where $Q_{0i}(p)$, $R_{Li}(p)$ are linear stationary differential operators obtained in the transformation from (11) to (12).

If the derivatives of the signal $\zeta_i(t)$ were available for the measurement then the signal $u_i(t) = -\beta_i^{-1}R_{Li}^{-1}(p)Q_{0i}(p)\zeta_i(t) = -\beta_i^{-1}\phi_i(t)$ would provide the closed-loop system $\dot{\bar{x}}_i(t) = A_{0i}\bar{x}_i(t)$, $\bar{y}_i(t) = L\bar{x}_i(t)$.

However it follows from A5 that the derivatives of the signal $\zeta_i(t)$ are not available to measurement. For compensation of the uncertainties in (1) define the signal $u_i(t)$ in the form

$$u_i(t) = -\beta_i^{-1}R_{Li}^{-1}(p)Q_{0i}(p)\bar{\zeta}_i(t) = -\beta_i^{-1}\bar{\phi}_i(t), \tag{13}$$

where $\bar{\phi}_i(t)$ and $\bar{\zeta}_i(t)$ are estimation of the functions $\phi_i(t)$ and $\zeta_i(t)$ respectively. Add and subtract the ideal control $u_i(t) = -\beta_i^{-1}R_{Li}^{-1}(p)Q_{0i}(p)\zeta_i(t)$ and substitute (13) in (9). Then the equation of the closed-loop system is:

$$\dot{\bar{x}}_i(t) = A_{0i}\bar{x}_i(t) + \beta_i^{-1}B_L\bar{\Delta}_i(t), \quad \bar{y}_i(t) = L\bar{x}_i(t), \tag{14}$$

$\bar{\Delta}_i(t) = \bar{\zeta}_i(t) - \zeta_i(t)$ is the estimation error.

For the implementation of the control (13) introduce the observer (Atassi and Khalil (1999))

$$\dot{\xi}_i(t) = G_{0i}\xi_i(t) + D_{0i}(\bar{\zeta}_i(t) - \zeta_i(t)), \quad \zeta_i(t) = L\xi_i(t). \tag{15}$$

In the equations (15): $\xi_i(t) \in R^n$, $G_{0i} = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}$, I_{n-1} is square identity matrix of order $n-1$, $D_{0i} = -[d_{1i}\mu^{-1}, d_{2i}\mu^{-2}, \dots, d_{ni}\mu^{-n}]^{\mathrm{T}}$, coefficients $d_{1i}\mu^{-1}, d_{2i}\mu^{-2}, \dots, d_{ni}\mu^{-n}$ are chosen from condition that the matrix $G_i = G_{0i} - D_i L_i$ is Hurwitz, $D_i = [d_{1i}, d_{2i}, \dots, d_{ni}]^{\mathrm{T}}$, $\mu > 0$ is a small value.

For estimation of accuracy of observation introduce the vector $\bar{\eta}_i(t) = \Gamma_i^{-1}(\xi_i(t) - \theta_i(t))$, where $\Gamma_i = \mathrm{diag}\{\mu^{n-1}, \dots, \mu, 1\}$, $\theta_i(t) = [\zeta_i(t), \dot{\zeta}_i(t), \dots, \zeta_i^{(n)}(t)]^{\mathrm{T}}$. Takin into account (15) differentiate $\bar{\eta}_i(t)$: $\dot{\bar{\eta}}_i(t) = \mu^{-1}G_i\bar{\eta}_i(t) + \bar{b}_i\zeta_i^{(n+1)}(t)$, $\bar{\Delta}_i(t) = \mu^{n-1}L\bar{\eta}_i(t)$. Transform the last equation to the equivalent relative output $\bar{\Delta}_i(t)$

$$\dot{\eta}_i(t) = \mu^{-1}G_i\eta_i(t) + b_i\dot{\zeta}_i(t), \quad \bar{\Delta}_i(t) = \mu^{n-1}L_i\eta_i(t). \tag{16}$$

Here $\eta_i^l(t) = \bar{\eta}_i^l(t) - \mu^{1+l-n}\zeta_i^{(l-1)}(t)$, $l = \overline{2, n}$, $\eta_i^l(t)$ and $\bar{\eta}_i^l(t)$ – l-th components of vectors $\eta_i(t)$ and $\bar{\eta}_i(t)$, $\eta_i^1(t) = \bar{\eta}_i^1(t)$, $b_i = [\mu^{2-n}, 0, \dots, 0]^{\mathrm{T}}$.

Considering (16) transform (14) to the form

$$\dot{\bar{x}}_i(t) = A_{0i}\bar{x}_i(t) + \beta_i^{-1}\mu^{n-1}\bar{b}_i g_i \Delta_i(t), \quad \bar{y}_i(t) = L\bar{x}_i(t), \tag{17}$$

where $\Delta(t) = \left[\eta_1^i(t), \dot{\eta}_1^i(t), \dots, (\eta_1^i(t))^{(n)}\right]^{\mathrm{T}}$, g_i is the vector of coefficients of $Q_{0i}(\lambda)$.

Our main result is as follows.

Theorem. *Let the assumptions A1-A4 hold and the positive definite matrices P_i and H_i be the solutions of the matrix equations*

$$A_{0i}^{\mathrm{T}}P_i + P_iA_{0i} = -Q_{1i}, \quad G_i^{\mathrm{T}}H_i + H_iG_i = -Q_{2i},$$
$$Q_{1i} = Q_{1i}^{\mathrm{T}} > 0, \quad Q_{2i} = Q_{2i}^{\mathrm{T}} > 0.$$

Then there exist numbers $\beta_i > 0$ and $\mu_0 > 0$ such that for $\mu \le \mu_0$ and $\sum_{j=1, i\neq j}^{k} |c_{ij}(t)| \le \sqrt{0.5\mu_0^{-1}\lambda_{\min}(Q_{3i})}$, where $Q_{3i} = Q_{1i} - 2\beta_i^{-1}\mu_0^{n-1}P_i\bar{b}_i g_i (P_i\bar{b}_i g_i)^{\mathrm{T}}$ *the control system (10), (13), (15), (17) provides goal conditions (5) and (6).*

4. EXAMPLE

Consider the network S consisting of four nodes one of which being the leader. Let the dynamic of node i be given by

$$\dot{x}_i(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_{1i}(t) & a_{2i}(t) & a_{3i}(t) \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 0 \\ b_i(t) \end{bmatrix} u_i(t) + [0 \ \ 0 \ \ d_i(t)]^T f_i(t) + \sum_{j=1, i\neq j}^{4} c_{ij}(t)x_j(t), \tag{18}$$
$$y_i(t) = [1 \ \ 0 \ \ 0]x_i(t), \quad x_i(0) = x_{0i}, \quad i = 1, 2, 3.$$

The class of uncertainty Ξ is given by the inequalities: $|a_{li}(t)| \le 10$, $0 < b_i(t) \le 10$, $|d_i(t)| \le 10$, $|s_j(t)| \le 20$, $|f_i(t)| \le 10$, $l = 1, 2, 3$.

The equation of the leader is given as

$$\dot{x}_L(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -3 & -3\end{bmatrix}x_L(t)+\begin{bmatrix}0\\0\\1\end{bmatrix}(1+\sin t),$$
$$y_L(t)=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x_L(t),\quad x_L(0)=0.$$

The graph of the network S is shown in Fig. 1.

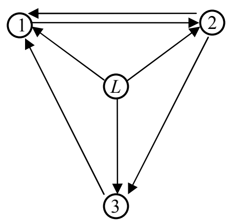

Fig. 1. The graph of network S.

The goal conditions are (5) and (6). Assume $\tilde{q}_i=1$ and $\tilde{r}_i=1$ in (4). For the nominal model (3) the performance index and the optimal control law (4) are defined as

$$J_i=\int_0^\infty\left[\bar{x}_i^T(t)\,\mathrm{diag}\{1,0,0\}\bar{x}_i(t)+u_{0i}^2(t)\right]dt,$$
$$u_{0i}(t)=-\begin{bmatrix}0.4142 & 0.4495 & 0.1463\end{bmatrix}\bar{x}(t). \tag{19}$$

Introduce the auxiliary plant (10) with $\beta_i=0.5$. Then

$$\dot{x}_{ai}(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1.4142 & -3.4495 & -3.1463\end{bmatrix}x_{ai}(t)+\begin{bmatrix}0\\0\\1\end{bmatrix}u_i(t)+$$
$$+0{,}5\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T u_i(t),\ y_{ai}(t)=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x_{ai}(t),$$
$$x_{ai}^{\mathrm{T}}(0)=0,\quad i=1,2,3.$$

For estimation of derivatives of the signal $\zeta_i(t)$ use the filter (15), where $\xi_i(t)\in R^3$, $\overline{D}_i=\begin{bmatrix}d_{1i} & d_{2i} & d_{3i}\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}3 & 3 & 1\end{bmatrix}^{\mathrm{T}}$, $\mu=0.01$. Then the observer is defined as

$$\dot{\xi}_{1i}(t)=-\xi_{2i}(t)-3\cdot100\left(\xi_{1i}(t)-\zeta_i(t)\right),$$
$$\dot{\xi}_{2i}(t)=-\xi_{3i}(t)-3\cdot100^2\left(\xi_{1i}(t)-\zeta_i(t)\right),$$
$$\dot{\xi}_{3i}(t)=-100^3\left(\xi_{1i}(t)-\zeta_i(t)\right),\ \xi_i(0)=0,\ i=1,2,3.$$

Using these equations the signal of compensation (13) can be written as

$$u_i(t)=-2\left(\dot{\xi}_{3i}+3.1463\xi_{3i}+3.4495\xi_{2i}+1.4142\xi_{1i}\right).$$

Let the parameters of the plant (18) be as follows:

- in the first subsystem $a_{11}(t)=2+\sin t$, $a_{21}(t)=3+5\sin 2t$, $a_{31}(t)=-3+5\sin 3t$, $b_1(t)=5+2\sin 5t$, $d_1(t)=10\sin t$, $f_1(t)=2+\sin t+P_1(t)$;
- in the second subsystem $a_{12}(t)=2+\cos t$, $a_{22}(t)=3+5\cos 2t$, $a_{32}(t)=1+2\cos 3t$, $d_2(t)=10\cos t$, $b_2(t)=3+\cos 2t$, $f_2(t)=1+\sin 1{,}5t+P_2(t)$;
- in the third subsystem $a_{13}(t)=2+7\sin t$, $a_{23}(t)=-2+4\sin 2t$, $a_{33}(t)=-5+5\sin t$, $b_1(t)=3+\sin t$, $d_1(t)=10\sin 2t$, $f_3(t)=\sin 2t+P_3(t)$; where $P_1(t)$, $P_2(t)$ and $P_3(t)$ are rectangular pulses with the amplitudes 1, periods 3, 5 and 7 sec, widths 1, 1.5 and 2 sec. The initial conditions in the nodes (18) are assumed equal to $x_i^{\mathrm{T}}(0)=\begin{bmatrix}2 & 2 & 1.5\end{bmatrix}$.

Let the matrix $C(t)$ be $C(t)=\left(c_{ij}(t)\right)=$

$$=\begin{pmatrix}0 & 0.5+0.1\sin 2t & 0.25+0.3\sin t\\ 0.65+0.1\sin t & 0 & 0\\ 0 & 0.6-0.2\cos t & 0\end{pmatrix}.$$

In the Fig. 2 the simulation results for the tracking errors $\bar{y}_i(t)$ and the output $y_{Ni}(t)$, $i=1,2,3$ of the nominal model (3) are shown. Moreover, the optimal control (4) is fed on the input of (3). On the fig. 3 the outputs $y_i(t)$, $i=1,2,3$ of the plant (18) and the output of the leader $y_L(t)$ are given.

Simulation showed that the results of transient processes depend on the choice of the values β_i in the auxiliary plant (10), the control law (13) and value μ of observer (15).

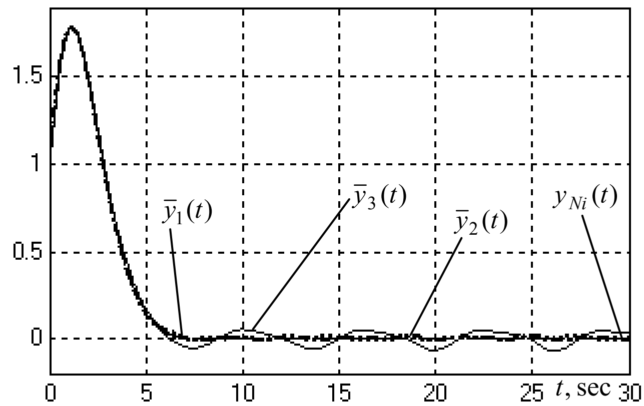

Fig. 2. Transient processes on $\bar{y}_i(t)$, $i=1,2,3$ and $y_{Ni}(t)$.

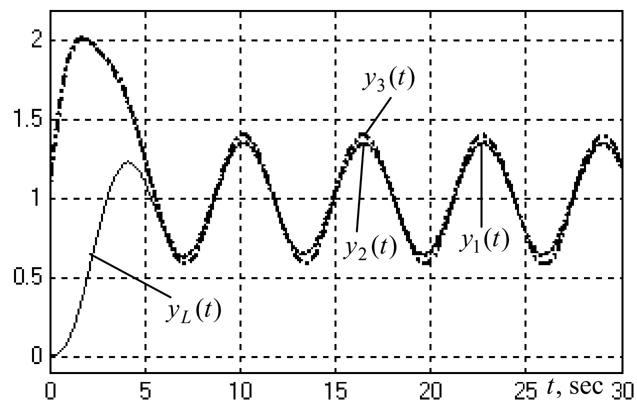

Fig. 3. Transient processes on $y_i(t)$, $i=1,2,3$ and $y_L(t)$.

5. CONCLUSIONS

In contrast with existing works in the paper robust suboptimal synchronization of the network was considered. Each node of the network is described of parametrically and functionally uncertain linear time-varying differential equation. Only local outputs of subsystems are available to measurement. It is enough to know the set of possible values of parameters of the subsystems to solve the problem. Law of control was proposed to represent as the sum of virtual optimal control and the signal of compensation of uncertain. Auxiliary subsystem, introduced parallel to the plant. It allows to identify disturbances and ensure closeness of the difference between outputs of each subsystems and leader to the nominal plant. The analytical results are confirmed by simulations.

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Appendix

Proof of the theorem. Let us prove that the system (10), (13), (15), (17) ensures the inequalities (5) and (6). Introduce the new variable $\varepsilon_i(t) \in R^n$ which characterizes closeness of the phase variables error (9) and the nominal model (3). Consider the difference between the state vectors (17) and (3). Obtain:

$$\dot{\varepsilon}_i(t) = A_{0i}\varepsilon_i(t) + \beta_i^{-1}\mu^{n-1}\bar{b}_i g_i \Delta_i(t).$$

Transform the last equation and (16) to the form

$$\begin{aligned} &\dot{\varepsilon}_i(t) = A_{0i}\varepsilon_i(t) + \beta_i^{-1}\mu_2^{n-1}\bar{b}_i g_i \Delta_i(t), \\ &\mu_1\dot{\eta}_i(t) = G_i\eta_i(t) + \mu_2 b_i \dot{\zeta}_i(t), \end{aligned} \tag{20}$$

where $\mu_1 = \mu_2 = \mu$. For further proof use the first lemma (Brusin (1995)). Consider the system (20) when $\mu_2 = 0$:

$$\dot{\varepsilon}_i(t) = A_{0i}\varepsilon_i(t), \quad \mu_1\dot{\eta}_i(t) = G_i\eta_i(t). \tag{21}$$

As matrices A_{0i} and G_i are Hurwitz then the solution of (21) is an asymptotically stable. According to the lemma, signals $\eta_i(t)$, $\varepsilon_i(t)$, $\Delta_i(t)$ and $\dot{\zeta}_i(t)$ are bounded. The proof of boundedness of the remaining functions in each subsystem is similar to Furtat and Tsykunov (2008). Therefore, according to lemma of Brusin (1995) the system (20) is asymptotically stable. Then all variables in closed-loop system are bounded.

However the stable (21) does not guarantee the asymptotically stable of (20) as singularly perturbed system. Let $\mu_1 = \mu_2 = \mu_0$. Choose the Lyapunov function

$$V(t) = \sum_{i=1}^{k} \left[\varepsilon_i^{\mathrm{T}}(t) P_i \varepsilon_i(t) + \eta_i^{\mathrm{T}}(t) H_i \eta_i(t) \right], \quad (22)$$

and differentiate it taking into account (20) and conditions of the theorem:

$$\begin{aligned} \dot{V}_i(t) = \sum_{i=1}^{k} \Big[& -\varepsilon_i^{\mathrm{T}}(t) Q_{1i} \varepsilon_i(t) + \\ & + 2\beta_i^{-1} \mu_0^{n-1} \varepsilon_i^{\mathrm{T}}(t) P_i \bar{b}_i g_i \Delta_i(t) - \\ & - \mu_0^{-1} \eta_i^{\mathrm{T}}(t) Q_{2i} \eta_i(t) + 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i \dot{\zeta}_i(t) \Big]. \end{aligned} \quad (23)$$

Estimate the left-hand side of (23):

$$\begin{aligned} & 2\beta_i^{-1} \mu_0^{n-1} \varepsilon_i^{\mathrm{T}}(t) P_i \bar{b}_i g_i \Delta_i(t) \le \\ & \le 2\beta_i^{-1} \mu_0^{n-1} \varepsilon_i^{\mathrm{T}}(t) P_i \bar{b}_i g_i \left(P_i \bar{b}_i g_i \right)^{\mathrm{T}} \varepsilon_i(t) + \\ & + 2\beta_i^{-1} \mu_0^{n-1} \left| \Delta_i(t) \right|^2 ; \end{aligned}$$

$$\begin{aligned} & 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i \dot{\zeta}_i(t) = 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L \dot{\sigma}_i(t) = \\ & = 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L A_{0i} \sigma_i(t) + 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L B_L \phi_i(t); \end{aligned}$$

$$\begin{aligned} & 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L A_{0i} \sigma_i(t) \le \\ & \le 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L A_{0i} \left(H_i b_i L A_{0i} \right)^T \eta(t) + 2\mu_0 \left| \sigma_i(t) \right|^2 ; \end{aligned}$$

$$\begin{aligned} & 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L B_L \phi_i(t) = \\ & = 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L B_L \left(\sum_{j=1,\, i \ne j}^{k} \theta_{ij}^T \varepsilon_j(t) + \omega_i(t) \right) = \\ & = 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L \sum_{j=1,\, i \ne j}^{k} c_{ij}(t) G \varepsilon_j(t) + \\ & + 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L B_L \omega_i(t); \end{aligned}$$

where $\omega_i(t) = \phi_i(t) - \sum_{j=1,\, i \ne j}^{k} \theta_{ij}^T \left(\bar{x}_j(t) + x_{Nj}(t) \right)$,

$$\begin{aligned} & 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L B_L \omega_i(t) \le \\ & \le 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L B_L \left(H_i b_i L B_L \right)^T \eta_i(t) + 2\mu_0 \omega_i^2(t) ; \end{aligned}$$

$$\begin{aligned} & 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L \sum_{j=1,\, i \ne j}^{k} c_{ij}(t) \varepsilon_j(t) \le \\ & \le 2\mu_0 \eta_i^{\mathrm{T}}(t) H_i b_i L \left(H_i b_i L \right)^T \eta_i(t) + \\ & + 2\mu_0 \sum_{j=1,\, i \ne j}^{k} c_{ij}^2(t) \varepsilon_j^T(t) \varepsilon_j(t). \end{aligned}$$

Using these estimates, rewrite (23) as:

$$\begin{aligned} \dot{V}(t) \le \sum_{i=1}^{k} \Big[& -\varepsilon_i^{\mathrm{T}}(t) Q_{3i} \varepsilon_i(t) - \eta_i^{\mathrm{T}}(t) Q_{4i} \eta_i(t) + \\ & + 2\mu_0 \sum_{j=1,\, i \ne j}^{k} c_{ij}^2(t) \varepsilon_j^T(t) G^T G \varepsilon_j(t) + \bar{\varphi} \Big], \end{aligned}$$

where $Q_{3i} = Q_{1i} - 2\beta_i^{-1} \mu_0^{n-1} P_i \bar{b}_i g_i \left(P_i \bar{b}_i g_i \right)^{\mathrm{T}}$, $Q_{4i} = Q_{2i} - 2\mu_0^2 H_i b_i L \left[A_{0i} A_{0i}^T L^T b_i^{\mathrm{T}} H_i + \left(H_i b_i L \right)^T \right]$, $\bar{\varphi} = 2\mu_0 \sup_t \left(\beta_i^{-1} \mu_0^{n-2} \left| \Delta_i(t) \right|^2 + \left| \sigma_i(t) \right|^2 + \omega_i^2(t) \right)$.

There are numbers $\mu_0 > 0$ and $\beta_i > 0$ which provide $Q_{3i} > 0$ and $Q_{4i} > 0$. Let $\varepsilon(t) = \left[\varepsilon_1(t), ..., \varepsilon_k(t) \right]$, $\eta(t) = \left[\eta_1(t), ..., \eta_k(t) \right]$, $Q_3 = \mathrm{diag}\{Q_{3i}\}$, $Q_4 = \mathrm{diag}\{Q_{4i}\}$, $\bar{c}(t) = \begin{bmatrix} 0 & c_{12}^2(t) & \cdots & c_{1k}^2(t) \\ c_{21}^2(t) & 0 & \cdots & c_{2k}^2(t) \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1}^2(t) & c_{k2}^2(t) & \cdots & 0 \end{bmatrix}$. Then the derivative of Lyapunov function can be rewritten as

$$\begin{aligned} \dot{V}(t) \le & -\varepsilon^T(t) Q_3 \varepsilon(t) - \eta^T(t) Q_4 \eta(t) + \\ & + 2\mu_0 \varepsilon^T(t) \bar{c}(t) \varepsilon(t) + k\bar{\varphi}. \end{aligned}$$

If $Q_5 = Q_3 - 2\mu_0 \bar{c}(t) \ge 0$ or $\sum_{j=1,\, i \ne j}^{k} \left| c_{ij}(t) \right| \le \sqrt{0.5 \mu_0^{-1} \lambda_{\min}(Q_{3i})}$ then

$$\dot{V}(t) \le -\varepsilon^T(t) Q_5 \varepsilon(t) - \eta^T(t) Q_4 \eta(t) + k\bar{\varphi}.$$

Rewrite the derivative of Lyapunov function as

$$\dot{V}(t) \le -\chi V(t) + k\bar{\varphi},$$

where $\chi = \min\left\{ \frac{\lambda_{\min}(Q_5)}{\lambda_{\max}(P)}, \frac{\lambda_{\min}(Q_4)}{\lambda_{\max}(H)} \right\}$, $P = \mathrm{diag}\{P_i\}$, $H = \mathrm{diag}\{H_i\}$. The solution of the last inequality is

$$V(t) \le e^{-\chi t} V(0) + \chi^{-1} \left(1 - e^{-\chi t} \right) k\bar{\varphi}.$$

Then from the structure (22) we have

$$\delta_1 \le \sqrt{e^{-\chi t} V(0) + \left(1 - e^{-\chi t} \right) \chi^{-1} k\bar{\varphi}},$$

$$\begin{aligned} \delta_2 & \le \overline{\lim_{t \to \infty}} \left[\sqrt{e^{-\chi t} V(0) + \left(1 - e^{-\chi t} \right) \chi^{-1} k\bar{\varphi}} - \left| \bar{y}_N(t) \right| \right] = \\ & = \sqrt{\chi^{-1} k\bar{\varphi}}. \end{aligned}$$

The right-hand sides of the last inequalitys depend on the values β_i and μ_0. Obviously, with decrease of numbers μ_0 and β_i the values δ_1, δ_2 in (5), (6) can be decreased. The last statment is confirmed by the simulation results.