FORMATION OF AN UNKNOWN DISCRETE SPECTRUM IN A GENERAL SPECTRUM OF STRUCTURES, INTERACTING WITH FLUID

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Abstract During investigation of noise levels from large structures, such as ship hulls, attention is usually drawn to obvious strong vibration mechanisms. However, unexpected noise levels at certain frequencies may be due to radiation from structural details, e.g. joints or walls.

As a consequence some radiation at an unexpected frequencies occurs when the structure is operating. Three possible reasons causing the existence of such frequencies have been investigated.

1. LOCALIZED MODES OF ELASTIC STRUCTURES

Infinite length structures with elastic mass inclusions have both continuous and real discrete spectrums [1]. A real discrete spectrum can be found above a cut-off frequency of a system as well as below it. Location of the discrete spectrum depends on the parameters of the structure and its inclusions. The real discrete spectrum corresponds to localized modes containing finite energy (trapped modes as they were called in [2]).

It has been found in [1] that the phenomenon of trapped modes explains some cases of noise levels at certain frequencies in finite length structures. Author has sequentially solved the problems of the evaluation of the spectrums for the following hinged structures:

- a finite length beam on the elastic foundation with two similar elastic supports
- a circular cylindrical shell supported by two circular frames

To solve these problems, the solutions for finite length structures were expressed in terms of Green function for the similar infinite length structures. Found solutions lead to the following results:
1. There is a real dicsrete spectrum above the cut-off frequency $\omega^2 = \frac{k}{m}$ (k — elastic foundation rigidity; m — mass per unit length of the beam). The spectrum frequencies can be found from the expression:

$$\omega_n^2 = D(k + (\pi n/l_1)^4)/m, \quad n = 1, 3, \dots \tag{1.1}$$

where D — beam rigidity; l_1 — half of the distance between the spring supporters. Obtained values of these frequencies coincide with the trapped frequencies of the infinite length system.
2. For the frequencies from (1.1) being natural, it is necessary to find the corresponding rigidity spectrum of the supporters.
3. Trapped frequencies become multiple the 2nd order natural frequencies of $2l$ length beam under the following condition: $l/l_1 = (2j + 1)/n (j = 1, 2, ...)$.
4. Multiple frequency corresponds to two modes: the first mode has nodes in the points of the springs attachment to the beam, the second mode is localized between the springs. Diagrams of the modes are given in fig. 1.

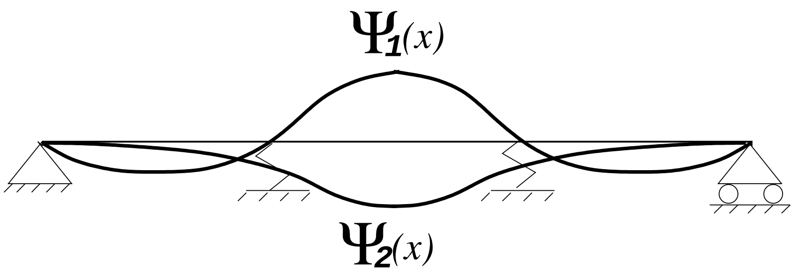

FIG 1

A related but more complicated solution has been found for a cylindrical shell. For this case, the spectrum of frequencies corresponding to the trapped modes lies in the range $0 < \omega_{trapped} < c_0/r$ (c_0 — the speed of sound in shell material, r — shell radius).

At the found trapped frequencies the longitudinal forces cause the vibration in both longitudinal and transverse directions. Modes of such vibration are shown in fig. 2.

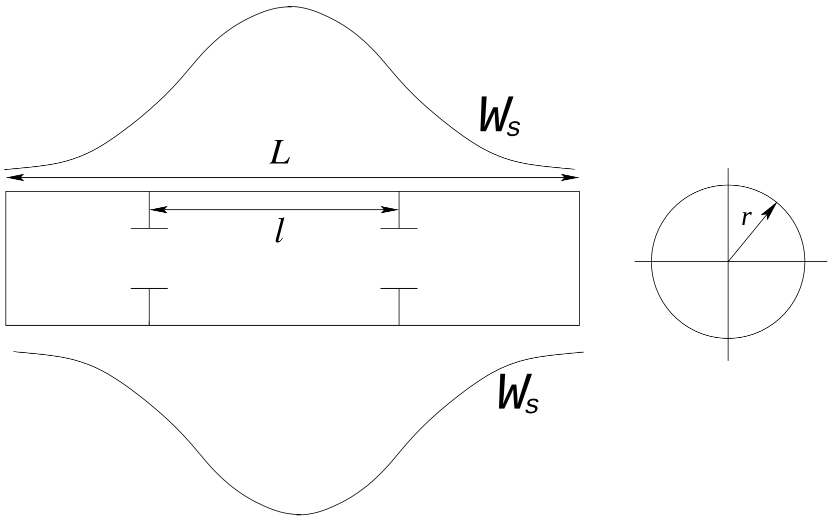

FIG 2

Both above mentioned systems have the following peculiarity in common: under harmonic force loading the main radiating mode becomes a localized one. The reason is that the generalized force corresponding to this mode is much greater than the generalized forces corresponding to the multinode mode. The trapped frequency in the general spectrum is not the lowest and usually it is not taken into account in calculations.

The oscillation mode is a priori assumed to be multimode, and the liquid to be noncompressible (effect of an added mass). The trapped mode causes considerable radiation for its localized shape.

2. CHOICE OF RHEOLOGICAL MODEL OF THE LIQUID

A choice of a rheological model of the liquid greatly influences a structure radiation. Let us consider the axisymmetric oscillation of a system consisting of two coaxial cylindrical shells submerged into a compressible nonviscous liquid. Vibration is caused by kinematic excitation of the inner shell: $w(x,t) = w_0 \exp(\alpha x - \omega t)$. If H is the distance between the shells, r_1 is the radius of the inner shell and it satisfies the inequality $H/r_1 \ll 1$, the liquid layer between the shells can be considered as a plane one. The analytical solution gives us an expression for the liquid pressure in the space between the shells and outside of the system.

Module of the system transfer function $|K(i\omega)| = |w_{20}/w_{10}|$ (ω — the frequency of excitation, w_{10}, w_{20} — the shells displacements) determines the influence of the outer shell on the radiation pressure. Fig. 3 shows the $|K(i\omega)|$ dependence of the excitation frequency for the models of compressible and fig. 4 of noncompressible liquids of the plane layer:

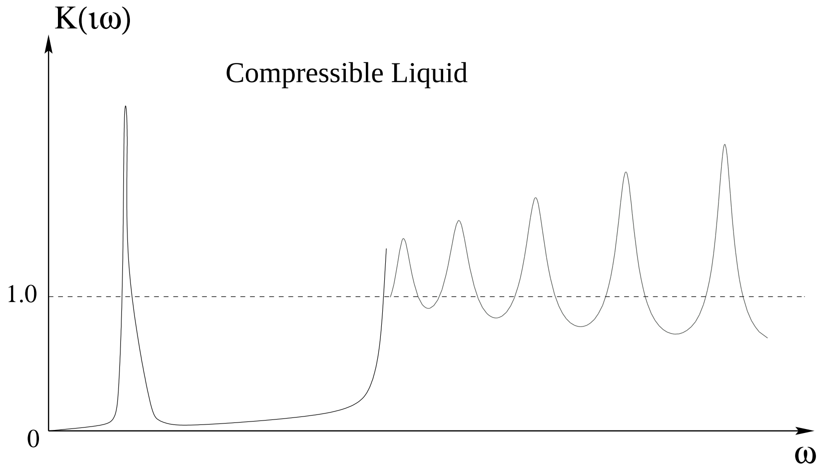

FIG 3

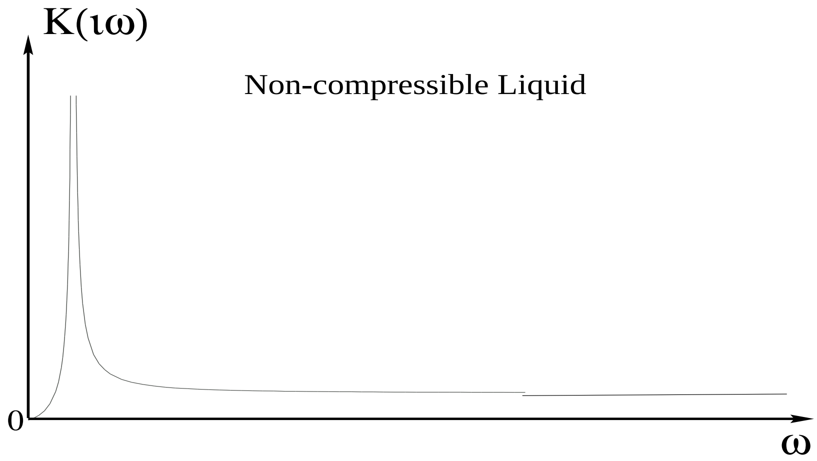

FIG 4

Let us study $|K(i\omega)|$ in three frequency bands. The first band is located between 0 and the resonance frequency of the liquid layer. The liquid layer behaves itself as a damper. The same behaviour is seen in the model of the noncompressible liquid in this frequency range. The second frequency band lies between the resonance frequency and the cut-off frequency of the system (the cut-off frequency is the frequency with which the outer shell radiation starts).

Provided that the liquid compressibility is taken into account, the outer shell behaves itself as a damper. Application of uncompressible liquid model leads to the conclusion that the outer shell is a damper when $\alpha < \omega^2/c_0^2$ (c_0 — the sound speed in liquid), when $\alpha > \omega^2/c_0^2$ both models suggest that the outer shell behaves as a damper. In the third frequency band above the cut-off frequency, application of compressible liquid model leads to describe components in the radiation spectrum, which can not be found in the case of noncompressible liquid. The outer shell behaves itself as an oscillation amplifier. Noise radiation of such a structure should be determined in view of a liquid layer compressibility.

3. GEOMETRIC NONLINEARITY OF A STRUCTURE INTERACTING WITH A LIQUID

Let us study a problem of a nonlinear parametric oscillation of a beam contacting with a semi-infinite channel filled with compressible liquid. In the two-mode Galerkin approximation equations describing the structure have the form obtained in [3]:

$$\begin{gathered}\ddot{w}_1 + \Omega_1^2(1 - k_1\cos\omega t)w_1 + \varepsilon_1\dot{w}_1 + k_1 w_1^3 + k_2 w_1 w_2^2 = f_1\cos\omega t,\\ \ddot{w}_2 + \Omega_2^2(1 - k_2\cos\omega t + \gamma)w_2 + \varepsilon_2\dot{w}_2 + k_3 w_2^3 + k_4 w_2 w_1^2 = f_2\cos\omega t,\end{gathered} \tag{1.2}$$

where $\varepsilon_1, \varepsilon_2, \gamma$ — magnitudes corresponding to the liquid. System (1.2) may be solved only numerically. Computer calculation gives us the possibility to analyze in the paramater space the time evolution of the system and corresponding transition between different regimes: determinated dynamics $\rightarrow$ quasiperiodic dynamics $\rightarrow$ chaotic dynamics.

At chaotic regimes the structure radiates minimum energy because these regimes contain infinite number of frequencies and each frequency possesses some amount of energy.

4. ELASTIC NONLINEAR SYSTEMS

Consider the following simplified model (the beam with nonlinear and linear inclusions)

$$Du_{xxxx} + mu_{tt} + Ku + f_\epsilon(x)u + \gamma g_\epsilon u^3 = P(t,x) \tag{1.3}$$

where $x \in (-\infty, \infty)$. To simplify, firstly let us assume the external force $P = 0$. It is well known that, in the linear case ($g = 0$), for appropriate coefficients f, there exist "localized" and almost localized modes describing special beam oscillation. For example, if we take two inclusions $f = \delta(x) + \delta(x - l)$, then one obtains the solution of the form

$$u = [A\cos(\omega t) + B\sin(\omega t)]\Psi(x, \omega) \tag{1.4}$$

where $a(\omega) = (m\omega^2 - K)^{\frac{1}{4}}$, $\Psi = \exp(-a|x|) + \exp(-a|x-l|) + \lambda(\omega)(\cos(ax) + \cos(a(x-l)))$ and λ is small, if the "resonance condition" is satisfied: $la \approx \pi(2n+1), n \in \mathbf{Z}$. This mode is the sum of exponentially damping contribution and the small harmonic tail $\cos(ax) + \cos(a(x-l))$. The pure localized mode corresponds to the case $\lambda = 0$.

The aim of this subsection is to investigate nonlinear evolution of such modes and show the possibility of the coherent structure formation.

5. CASE OF SINGLE MODE

In this section a case of a single almost localized mode is considered (1.4). The natural and classical approach for small nonlinearities, (assume, γ is a small parameter), is to suppose that these amplitudes depend on slow time $\tau = \gamma t$. Then one can apply the well known Whitham principle. For the localized modes a simple approach leading to *ordinary* differential equations for slowly oscillating amplitudes $A(\tau)$ and $B(\tau)$ exists.

Let define the Lagrangian

$$L = \frac{1}{2}\int_0^1 (mu_t^2 - Du_{xx}^2 - Ku^2 - f_\epsilon(x)u^2 - \frac{1}{2}\gamma g_\epsilon u^4)dx.$$

Substituting (1.4) into this expression and integrating over "fast" variables x and t, the averaging Whitham lagrangian $\bar{L}$ is obtained. Up to small corrections, one finds

$$\bar{L} = c\frac{1}{2}\int_0^1 [\omega(dA/d\tau B - dB/d\tau A) + c_1(A^2+B^2)^2 + c_2(A^2+B^2)]d\tau.$$

The corresponding motion equations can be easily resolved and they describe slow oscillations of A and B:

$$A = \alpha\cos(\bar{\omega}(\tau - \tau_0)), \quad B = \alpha\sin(\bar{\omega}(\tau - \tau_0)).$$

The frequency $\bar{\omega}$ is proportional to the quantity $A^2 + B^2$ (that does not depend on time τ).

Finally, we see that, in this simplest case, the slow time evolution is defined by an integrable Hamiltonian system with the single degree of freedom.

6. CASE OF SOME LOCALIZED MODES

If the coexistence of some modes is possible in system (1.3), then the mathematical approach above holds but the results are more interesting. Suppose modes $\Psi_1, \Psi_2, ...\Psi_n$ coexist, with corresponding frequencies $\omega_1, \omega_2, ...$ Let $A_i(\tau)$ and $B_i(\tau)$ be the corresponding slowly oscillating modes. A nontrivial Whitham lagrangian (with nonlinear contributions that define the mode interactions) can occur if the time resonance condition $\sigma = \omega_{i_1} + \omega_{i_2} + \omega_{i_3} + \omega_{i_4} = 0$ or $\sigma = \omega_{i_1} + \omega_{i_2} - \omega_{i_3} - \omega_{i_4} = 0$ holds, for some ω_{i_k} from the frequency set.

Repeating the calculations from the previous point, we find that the Whitham lagrangian has the following general form

$$\begin{gathered}\bar{L} = \frac{1}{2}\int_0^1 \Big\{ (\sum_{i=1}^{n} c[\omega(dA_i/d\tau B_i - dB_i/d\tau A_i) + c_1(A_i^2 + B_i^2)^2 + \\ + c_2(A_i^2 + B_i^2)] + \lambda^4 \sum_{i_1,i_2,i_3,i_4:\sigma=0} c_{i_1,i_2,i_3,i_4} A_{i_1}A_{i_2}A_{i_3}A_{i_4} + ... \\ + d_{i_1,i_2,i_3,i_4} B_{i_1}B_{i_2}B_{i_3}B_{i_4}) \Big\}\, d\tau \end{gathered} \tag{1.5}$$

where the second sum describes different contributions connected with the 4-order resonances $\sigma = 0$. The corresponding equations for A_i and B_i, in general, are not integrable. They can be analyzed by the KAM theory which can be used due to the condition $\lambda << 1$. Due to classical results, we know that they can describe quasiperiodic motions and moreover, different chaotic occurrences (for example, Arnold's web, homoclinic structures and etc.)

To conclude this subsection, let us note the important point. We can change the form of $\bar{L}$ by a frequency and coefficient f, g choice. Thus, in a sense, these coherent structures are controllable.

7. NONLINEAR ELASTIC BODIES INTERACTING WITH FLUID

As a simplest model, the beam (1.3) in a fluid flow is considered. For simplicity this flow is taken in acoustic approximations

$$\Delta\phi(x, y) = c_0^{-2}\phi_{tt}, \quad \phi'_y(x, y)|_{y=0} = u_t,$$

and the force P in (1) has the form $P = \rho_0\phi_t(x, 0)$. The influence of the fluid can be easily taken into account in the limit of large frequencies $\omega_j >> 1$. Then for ϕ one has the following asymptotic expansion $\phi = \sum_j \phi_j(x, \omega, y, \tau)\cos(\omega_j t)$, where $\phi_j \approx c\Psi_j(x)\exp(i\omega_j y) + ...$ Thus, one can show, that for high frequencies, the term P in (1.3) can replaced to the usual simple dissipative term cu_t.

Repeating all usual procedure of two scale expansion, we can obtain some complicated equations for amplitudes A_i, B_i. They contains some dissipative contributions.

The investigation shows that the coherent structures lead to slowly damping quasiperiodic waves (with slowly oscillating amplitudes) which exist in the beam and the fluid.

The nontrivial structures occur as a result of the interaction of localized modes through their tails.

8. CONCLUSION

1. The reason of radiation at unexpected frequencies of a structure with elastic-mass inclusions could be the localized modes.
2. Localized modes correspond to the trapped frequencies of the infinite length system.
3. The choice of a liquid model essentially influence the estimation of the spectrum of a compound structure radiation.
4. Geometric nonlinearity causes quasiperiodic and chaotic regimes. Classification of possible dynamic regimes of a nonlinear systems allows us to predict frequencies which can not be found from linear analysis of the spectrum.

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