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Stability of particle triangle under compression

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Abstract

It is known that some solid materials, in particular metals, being subjected to a big external pressure behave as a liquid. This tells about stability loss of the material internal structure. The purpose of the presented research is to study the nature of this phenomenon on a simple model of three particles, interacting via pair interatomic forces.

1 Statement of the problem

The system consists of three equal particles subjected to an external loading. Force of interaction inside the system is defined by a pair potential. External forces are follower or dead. In the equilibrium state they are directed to the mass center of the system.

2 Analysis of equilibrium

Let us find the equilibrium state of the system and investigate its stability. The equilibrium position for the particles is an equilateral triangle. The geometry of the problem is represented in figure 1. In the figure C — the center of masses; $\underline{F}_i$ —

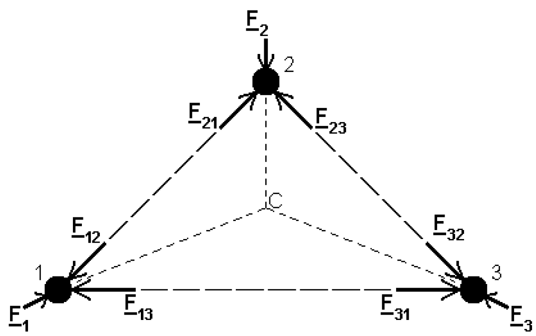

Figure 1: the equilibrium state of the system

the external force ($F_i = F$); $\underline{F}_{ij}$ — the force of interaction between particles; $\underline{R}_i$ — the radius vector of the particle (in equilibrium $R_i = R$); $i, j = \overline{1, 3}$.

In the equilibrium the following equations are fulfilled

$$\underline{R}_i = R\underline{e}_i; \quad \underline{R}_C = \frac{1}{3}\left(\underline{R}_1 + \underline{R}_2 + \underline{R}_3\right); \tag{1}$$

$$\underline{F}_i = F\underline{e}_i^F = F\frac{\underline{R}_C - \underline{R}_i}{|\underline{R}_C - \underline{R}_i|}; \quad \underline{F}_{ij} = F_{ij}\underline{e}_{ij} = F_{ij}\frac{\underline{R}_i - \underline{R}_j}{|\underline{R}_i - \underline{R}_j|}. \tag{2}$$

The final system of the equilibrium equations is the following

$$\begin{cases} F\underline{e}_1^F + F_{12}\underline{e}_{12} + F_{13}\underline{e}_{13} = \underline{0} \\ F\underline{e}_2^F + F_{21}\underline{e}_{21} + F_{23}\underline{e}_{23} = \underline{0} \\ F\underline{e}_3^F + F_{31}\underline{e}_{31} + F_{32}\underline{e}_{32} = \underline{0} \end{cases} \tag{3}$$

From it one can obtain

$$F_{12} = F_{21} = F_{13} = F_{31} = F_{32} = F_{23} = \Phi; \quad F = \sqrt{3}\Phi. \tag{4}$$

3 Analysis in the case of follower external forces

The system of the dynamics equation is

$$\begin{cases} m\ddot{\underline{R}}_1 = F\frac{\underline{R}_C - \underline{R}_1}{|\underline{R}_C - \underline{R}_1|} + F_{12}\frac{\underline{R}_1 - \underline{R}_2}{|\underline{R}_1 - \underline{R}_2|} + F_{13}\frac{\underline{R}_1 - \underline{R}_3}{|\underline{R}_1 - \underline{R}_3|} \\ m\ddot{\underline{R}}_2 = F\frac{\underline{R}_C - \underline{R}_2}{|\underline{R}_C - \underline{R}_2|} + F_{21}\frac{\underline{R}_2 - \underline{R}_1}{|\underline{R}_2 - \underline{R}_1|} + F_{23}\frac{\underline{R}_2 - \underline{R}_3}{|\underline{R}_2 - \underline{R}_3|} \\ m\ddot{\underline{R}}_3 = F\frac{\underline{R}_C - \underline{R}_3}{|\underline{R}_C - \underline{R}_3|} + F_{31}\frac{\underline{R}_3 - \underline{R}_1}{|\underline{R}_3 - \underline{R}_1|} + F_{32}\frac{\underline{R}_3 - \underline{R}_2}{|\underline{R}_3 - \underline{R}_2|} \end{cases} \tag{5}$$

We carry out a variation of the dynamics equations. Considering that the variation of the absolute value of the external force is equal to zero and using the system of the equilibrium equations we get

$$\begin{cases} m\delta\ddot{\underline{R}}_1 = \left[\Phi'\left(2\underline{\underline{E}} - \underline{e}_2\underline{e}_2 - \underline{e}_3\underline{e}_3\right) + \frac{\Phi}{\sqrt{3}R}\left(2\underline{e}_1\underline{e}_1 + \underline{e}_2\underline{e}_2 + \underline{e}_3\underline{e}_3 - 2\underline{\underline{E}}\right)\right]\cdot\delta\underline{R}_1 + \\ + \left[\Phi'\left(\underline{e}_3\underline{e}_3 - \underline{\underline{E}}\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{\underline{E}} - \underline{e}_1\underline{e}_1 - \underline{e}_3\underline{e}_3\right)\right]\cdot\delta\underline{R}_2 + \\ + \left[\Phi'\left(\underline{e}_2\underline{e}_2 - \underline{\underline{E}}\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{\underline{E}} - \underline{e}_1\underline{e}_1 - \underline{e}_2\underline{e}_2\right)\right]\cdot\delta\underline{R}_3 \\ m\delta\ddot{\underline{R}}_2 = \left[\Phi'\left(\underline{e}_3\underline{e}_3 - \underline{\underline{E}}\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{\underline{E}} - \underline{e}_2\underline{e}_2 - \underline{e}_3\underline{e}_3\right)\right]\cdot\delta\underline{R}_1 + \\ + \left[\Phi'\left(2\underline{\underline{E}} - \underline{e}_1\underline{e}_1 - \underline{e}_3\underline{e}_3\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{e}_1\underline{e}_1 + 2\underline{e}_2\underline{e}_2 + \underline{e}_3\underline{e}_3 - 2\underline{\underline{E}}\right)\right]\cdot\delta\underline{R}_2 + \\ + \left[\Phi'\left(\underline{e}_1\underline{e}_1 - \underline{\underline{E}}\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{\underline{E}} - \underline{e}_2\underline{e}_2 - \underline{e}_1\underline{e}_1\right)\right]\cdot\delta\underline{R}_3 \\ m\delta\ddot{\underline{R}}_3 = \left[\Phi'\left(\underline{e}_2\underline{e}_2 - \underline{\underline{E}}\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{\underline{E}} - \underline{e}_3\underline{e}_3 - \underline{e}_2\underline{e}_2\right)\right]\cdot\delta\underline{R}_1 + \\ + \left[\Phi'\left(\underline{e}_1\underline{e}_1 - \underline{\underline{E}}\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{\underline{E}} - \underline{e}_3\underline{e}_3 - \underline{e}_1\underline{e}_1\right)\right]\cdot\delta\underline{R}_2 + \\ \left[\Phi'\left(2\underline{\underline{E}} - \underline{e}_1\underline{e}_1 - \underline{e}_2\underline{e}_2\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{e}_1\underline{e}_1 + \underline{e}_2\underline{e}_2 + 2\underline{e}_3\underline{e}_3 - 2\underline{\underline{E}}\right)\right]\cdot\delta\underline{R}_3 \end{cases} \tag{6}$$

Let us search for the solution in the form $\delta\underline{R}_j = \underline{A}_j e^{i\lambda}, j = \overline{1,3}$. Characteristic determinant of the system is

$$\lambda^6\left(m\lambda^2 + 3\Phi'\right)\left(2m\lambda^2 + 3\Phi'\right)^2 = 0. \tag{7}$$

Stability will be in the case of real and positive λ. From the above equation it follows that this is fulfilled in the case $\Phi' < 0$. Thus the system loaded with the follower forces under compression and tension is stable for $r < b$, where b is the value of r where $\Pi''(r) = 0$.

4 Analysis in the case of dead external forces

Variation of the dynamic equations gives

$$\begin{cases} m\delta\ddot{\underline{R}}_1 = \left[\Phi'\left(2\underline{\underline{E}} - \underline{e}_2\underline{e}_2 - \underline{e}_3\underline{e}_3\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{e}_2\underline{e}_2 + \underline{e}_3\underline{e}_3\right)\right]\cdot\delta\underline{R}_1 + \\ + \left[\Phi'\left(\underline{e}_3\underline{e}_3 - \underline{\underline{E}}\right) - \frac{\Phi}{\sqrt{3}R}\underline{e}_3\underline{e}_3\right]\cdot\delta\underline{R}_2 + \\ + \left[\Phi'\left(\underline{e}_2\underline{e}_2 - \underline{\underline{E}}\right) - \frac{\Phi}{\sqrt{3}R}\underline{e}_2\underline{e}_2\right]\cdot\delta\underline{R}_3 \\ m\delta\ddot{\underline{R}}_2 = \left[\Phi'\left(\underline{e}_3\underline{e}_3 - \underline{\underline{E}}\right) - \frac{\Phi}{\sqrt{3}R}\underline{e}_3\underline{e}_3\right]\cdot\delta\underline{R}_1 + \\ + \left[\Phi'\left(2\underline{\underline{E}} - \underline{e}_1\underline{e}_1 - \underline{e}_3\underline{e}_3\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{e}_1\underline{e}_1 + \underline{e}_3\underline{e}_3\right)\right]\cdot\delta\underline{R}_2 + \\ + \left[\Phi'\left(\underline{e}_1\underline{e}_1 - \underline{\underline{E}}\right) - \frac{\Phi}{\sqrt{3}R}\underline{e}_1\underline{e}_1\right]\cdot\delta\underline{R}_3 \\ m\delta\ddot{\underline{R}}_3 = \left[\Phi'\left(\underline{e}_2\underline{e}_2 - \underline{\underline{E}}\right) - \frac{\Phi}{\sqrt{3}R}\underline{e}_2\underline{e}_2\right]\cdot\delta\underline{R}_1 + \\ + \left[\Phi'\left(\underline{e}_1\underline{e}_1 - \underline{\underline{E}}\right) - \frac{\Phi}{\sqrt{3}R}\underline{e}_1\underline{e}_1\right]\cdot\delta\underline{R}_2 + \\ + \left[\Phi'\left(2\underline{\underline{E}} - \underline{e}_1\underline{e}_1 - \underline{e}_2\underline{e}_2\right) + \frac{\Phi}{\sqrt{3}R}\left(\underline{e}_1\underline{e}_1 + \underline{e}_2\underline{e}_2\right)\right]\cdot\delta\underline{R}_3 \end{cases} \quad (8)$$

Let us search for the solution in the form $\delta\underline{R}_j = \underline{A}_j e^{i\lambda}, j = \overline{1,3}$. Characteristic determinant of the system is

$$\lambda^4\left(m\lambda^2 + 3\Phi'\right)\left(m\lambda^2 + \sqrt{3}\frac{\Phi}{R}\right)\left(2m\lambda^2 + 3\Phi' + \sqrt{3}\frac{\Phi}{R}\right)^2 = 0. \quad (9)$$

Stability will be in the case of real and positive λ. From the equation it follows that this is fulfilled in the case $\Phi' < 0$ and $\Phi < 0$. Thus the system loaded with the dead forces under tension is stable for $r < b$, where b is the value of r where $\Pi''(r) = 0$, and unstable under compression.

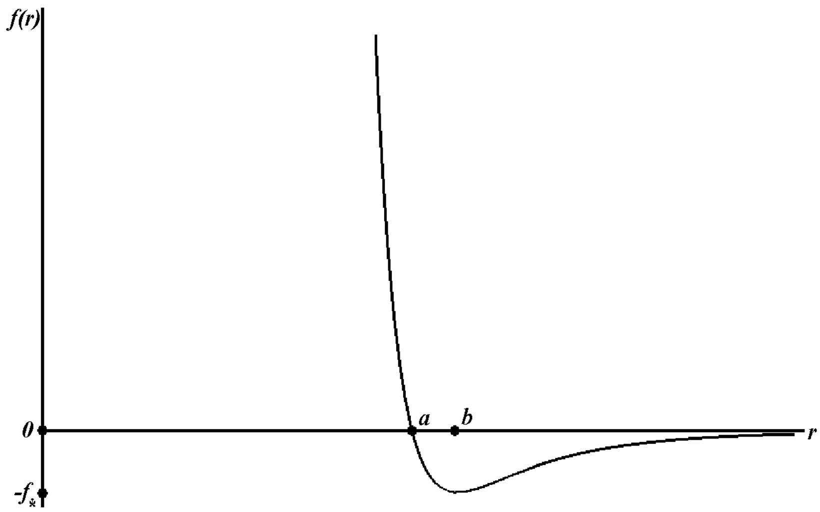

Figure 2: Interaction force via interparticle distance

5 Conclusions

Dependence of the interaction force on the distance is represented in figure 2. The critical range is determined by the equation $f'(r) = 0$. The conclusion of the research is represented in table 1. From the table it follows that the dead forces under compression destabilize the configuration, while the action of the follower forces preserve the stability. Under tension in both cases the stability loss is due to the breakage of the interparticle bond. These results are quite reasonable, but unfortunately they do not explain the stability loss of the material internal structure. Probably the desired phenomenon can be described by more complicated model, in which the properties of the particles and the absolute values of the forces are not exactly equal to each other.

	Follower forces	Dead forces
Compression	Stability	Instability
Tension	Stability until the critical range	Stability until the critical range

Table 1: Summary of the stability analysis.

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