On the waveless statement of the two-dimensional Neumann–Kelvin problem for a surface-piercing body

By N. KUZNETSOV AND O. MOTYGIN

Laboratory on Mathematical Modelling in Mechanics,
Institute of Problems of Mechanical Engineering, Russian Academy of Sciences,
V.O., Bol'shoy pr., 61, St. Petersburg, 199178, Russia

(Received March 1996, Revised January 1997)

New supplementary conditions are proposed for the two-dimensional Neumann–Kelvin problem describing the forward motion of a surface-piercing body. They lead to the absence of waves behind the body, and hence, the wave resistance is equal to zero. Moreover, for the body with symmetry about vertical axis the total resistance to the forward motion (the sum of wave and spray resistance) also vanishes, when these conditions are imposed. The behaviour of spray resistance is investigated numerically for a family of asymmetric contours.

---

1. Introduction

The present paper is concerned with a new well-posed statment of the two-dimensional linear boundary value problem (usually referred to as the Neumann–Kelvin problem), which is related to the uniform forward motion of a surface-piercing cylinder in an infinite depth fluid. The latter is assumed to be inviscid, incompressible and heavy.

Almost exhaustive results are obtained for the case of a totally submerged cylinder. About 60 years ago Kochin (1937) proved the first result on the solvability of this problem. Using the integral equation techniques he demonstrated that a solution exists for sufficiently large and sufficiently small values of the cylinder's forward velocity U. Vainberg & Maz'ya (1973) noticed that Kochin's result yield solvability of the problem for all values of U with possible exception for a finite number of values. Moreover, the uniqueness theorem is true under the same restriction on the parameter U. That can be proved in the same way as for a surface-piercing cylinder (see Kuznetsov & Maz'ya 1989).

Ursell (1981) investigated the Neumann–Kelvin problem for a single surface-piercing semicircle and found this problem to be sub-definite. He proved that the problem has a two-parameter set of solutions. Earlier, this fact was discovered numerically (see Suzuki (1982) and references cited therein). In particular, Suzuki points out that as early as 1963 Bessho and Mizuno solved numerically the Neumann–Kelvin problem for a

semi-submerged circular cylinder and found, that there are infinitely many solutions, and hence, the calculated wave resistance can have any value. Recently, Bessho (1994) in his G. Weinblum Memorial Lecture presented results on this problem (for the most part numerical) obtained during the last decades in Japan. Eggers (1976) in his discussion to Bessho (1976) had suggested that the homogeneous problem possesses non-trivial solutions. However, the first mathematical proof of this assertion was given only by Ursell (1981). In the same paper he proposed a well-posed statement of the problem which leads to the so-called least singular solution.

There are a number of other well-posed statements of the problem. Lenoir (1982) proposed a statement in terms of stream function with a condition of Kutta–Zhoukovskiy type at the stern point. Kuznetsov & Maz'ya (1989) introduced two versions of supplementary conditions for an arbitrary surface-piercing body. The version of supplementary conditions proposed in Kuznetsov (1992) (see also the abstract by Motygin & Kuznetsov (1995)) provides the resistance to be purely wave-making, i.e. there are no splashes at bow and stern points, and to be expressed by just the same formula as for a totally submerged cylinder (see e.g. Kochin 1937).

The aim of the present paper is to consider new supplementary conditions giving a wave-free solution and, hence, vanishing the wave resistance. The corresponding disturbance has finite energy. The other component of resistance, the so-called spray resistance is not zero under these conditions, generally speaking. Nevertheless, the total resistance vanishes if the body contour is symmetric about its mid-section. In a forthcoming paper (see enlarged abstract by Kuznetsov & Motygin (1995) in Russian) the authors will consider a similar waveless statement of the Neumann–Kelvin problem for a tandem of surface-piercing cylinders. In the latter case there exist four supplementary conditions cancelling both the wave resistance and the spray resistance, but giving well-posed statement of the problem, i.e. there exists a unique solution for all U except for a sequence tending to zero. At the same time, for exceptional U examples of non-uniqueness are constructed, i.e. a couple of surface-piercing contours for which the homogeneous problem augmented with the same four waveless supplementary conditions has a non-trivial solution. The construction of these examples is based on the inverse procedure proposed by McIver (1996) for a non-uniqueness example in the sea-keeping problem. Thus, there are two different kinds of non-uniqueness in the Neumann–Kelvin problem. The non-uniqueness of the first kind can be removed by imposing proper supplementary conditions. The non-uniqueness of the second kind is intrinsic to the problem with supplementary conditions but requires more than one surface-piercing body.

The contents of the paper is as follows. In § 2 we formulate the Neumann–Kelvin problem and give a summary of auxiliary results obtained for its solutions in Kuznetsov & Maz'ya (1989). After that the new supplementary conditions are defined. Section 3 is devoted to the proof that the waveless statement of the problem is solvable for all $U > 0$ with possible exception for a tending to zero sequence of values. In § 4 the uniqueness of the solution is proved under the assumption that cylinder's contour is symmetric about its

[h]

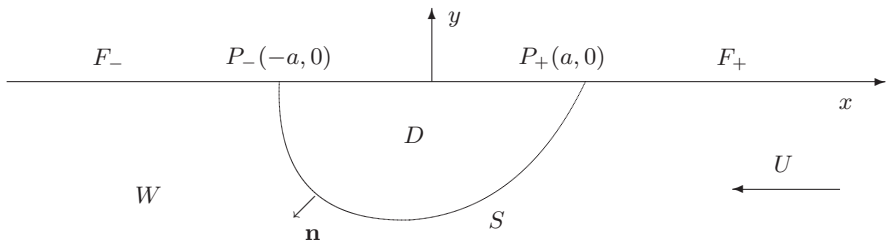

FIGURE 1. A sketch of body's geometry and some notations.

mid-section. In § 5 we consider the effect of waveless supplementary conditions on the total resistance R to the forward motion. First we prove that $R = 0$ for a symmetric contour under assumption that the waveless potential is unique. Then a numerical example is considered for a family of contours. Appendix A contains some necessary properties of the Green function. Appendix B is concerned with the solvability theorems for two auxiliary problems. Their solutions are used in § 4.

2. Formulation of the problem and some auxiliary results

By $\mathbb{R}^2_-$ we denote the half-plane $\{(x, y) : \ y < 0)\}$. Let the cross-section of moving cylinder be bounded, simply connected domain D in $\mathbb{R}^2_-$, such that ∂D consists of a segment $\{x \in [-a, a] \ , \ y = 0\}$ $(a > 0)$ and of a simple closed C^2-arc S with the endpoints $P_\pm = (\pm a, 0)$. Let $W = \mathbb{R}^2_- \setminus \overline{D}$ be a region occupied by the fluid, and let $F_\pm = \{a < \pm x < +\infty, \ y = 0\}$ be two parts of the fluid's free surface (see figure 1). We assume that the unilateral tangent to S at the point $P_\pm$ forms angle $\beta_\pm \neq 0, \pi$ with the vector $\pm\mathbf{i}$ ($\mathbf{i}$ is the unit vector directed along the x-axis).

The velocity potential u describing the fluid motion must satisfy the following boundary value problem:

$$\nabla^2 u = 0 \text{ in } W, \tag{2.1}$$

$$u_{xx} + \nu u_y = 0 \text{ on } F = F_+ \cup F_-, \tag{2.2}$$

$$\partial u/\partial n = U \cos(n, x) \text{ on } \operatorname{int} S = S \setminus \{P_+, P_-\}, \tag{2.3}$$

$$\lim_{x \to +\infty} |\nabla u| = 0, \tag{2.4}$$

$$\sup\{|\nabla u| : (x, y) \in W \setminus E\} < \infty, \tag{2.5}$$

$$\int_{W \cap E} |\nabla u|^2 \, \mathrm{d}x \, \mathrm{d}y < \infty. \tag{2.6}$$

Here $\nu = g\,U^{-2}$; g is the acceleration due to gravity and U is the constant speed of the cylinder. Furthermore, $\mathbf{n}$ is the unit normal directed into W, E is an arbitrary compact set in $\overline{\mathbb{R}^2_-}$ such that $\overline{D} \subset E$ and $F_\pm \cap E \neq \emptyset$. Clearly, u is defined up to an arbitrary constant term.

The Laplace equation follows from the assumptions that the fluid is incompressible and its motion is

irrotational. The boundary condition (2.2) is a consequence of the linearized kinematic and dynamic conditions on the free surface of the fluid. The relation (2.3) means that S is a rigid impermeable surface, (2.4) expresses the absence of waves at infinity upstream. The condition (2.5) means that the induced velocity field is bounded everywhere except for a vicinity of corner points $P_\pm$, (2.6) is the condition of local finiteness of the kinetic energy. The name "Neumann–Kelvin problem" is usually attributed to (2.1)–(2.6).

Now we remind some auxiliary results on the behavior of any solution to (2.1)–(2.6) near the points $P_\pm$ and at infinity. One can find the details in Kuznetsov & Maz'ya (1989) (see also Kuznetsov 1995). The formulation of our supplementary conditions is based on these properties.

The condition (2.6) implies that there are no strong singularities at $P_\pm$. This follows from the local asymptotics of u at a corner point, which shows that the finite limits along the free surface exist despite the velocity vector ∇u can be singular when approaching $P_\pm$ along all non-horizontal directions. We put $u_x(P_\pm) = \lim_{x\to\pm a\mp 0} u_x(x,0)$.

Any solution of (2.1)–(2.6) has the following asymptotics as $|z|\to\infty$ ($z = x+iy$):

$$u(x,y) = C + Q\log|z| + H(-x)\mathrm{e}^{\nu y}(\mathcal{A}\sin\nu x + \mathcal{B}\cos\nu x) + \psi(x,y). \tag{2.7}$$

Here C is an arbitrary constant, H is the Heaviside function, and the estimates $\psi = O(|z|^{-1})$, $|\nabla\psi| = O(|z|^{-2})$ hold. The constants Q, $\mathcal{A}$, $\mathcal{B}$ are determined as follows:

$$\pi\nu Q + u_x(P_+) - u_x(P_-) = \nu\int_S \partial u/\partial n\,\mathrm{d}s, \tag{2.8}$$

$$\begin{aligned}\mathcal{A} = -2\Big\{\int_S \big[u\frac{\partial}{\partial n}(\mathrm{e}^{\nu y}\cos\nu x) - \frac{\partial u}{\partial n}\mathrm{e}^{\nu y}\cos\nu x\big]\,\mathrm{d}s\\ +\nu^{-1}\cos\nu a\big[u_x(P_+) - u_x(P_-)\big] + \sin\nu a\big[u(P_+) + u(P_-)\big]\Big\},\end{aligned} \tag{2.9}$$

$$\begin{aligned}\mathcal{B} = 2\Big\{\int_S \big[u\frac{\partial}{\partial n}(\mathrm{e}^{\nu y}\sin\nu x) - \frac{\partial u}{\partial n}\mathrm{e}^{\nu y}\sin\nu x\big]\,\mathrm{d}s\\ +\nu^{-1}\sin\nu a\big[u_x(P_+) + u_x(P_-)\big] - \cos\nu a\big[u(P_+) - u(P_-)\big]\Big\}.\end{aligned} \tag{2.10}$$

Remark 2.1. The coefficients in (2.7) have clear physical meaning: $\mathcal{A}$ and $\mathcal{B}$ are proportional to the amplitudes of sine and cosine waves at infinity downstream, $-Q\pi/2$ is equal to the supplementary flux of fluid at infinity due to the presence of cylinder. If S is rigid, then by (2.3) the right hand side in (2.8) vanishes. In this case Q is defined by the difference $u_x(P_+) - u_x(P_-)$, and hence, is connected with sprays that can occur at points $P_\pm$.

Definition 2.1. We say that u is a waveless potential (or a solution of Problem (L)), if it satisfies (2.1)–

(2.6), and the following supplementary conditions

$$\mathcal{A} = \mathcal{B} = 0 \tag{2.11}$$

hold. Here $\mathcal{A}$ and $\mathcal{B}$ are considered as linear functionals of u given by (2.9) and (2.10).

According to this definition and Remark 2.1 there are no waves at infinity downstream, and hence, the wave resistance vanishes. The effect of the supplementary conditions (2.11) on the total resistance to the forward motion (it contains the spray component apart from the wave resistance) is considered in § 5.

3. On the solvability of Problem (L)

Here we prove that Problem (L) is solvable for all $\nu > 0$ except possibly for a sequence of values tending to infinity. The proof does not use the specific form of the right-hand side term in (2.3) and we replace it by the general Neumann condition

$$\partial u/\partial n = f \quad \text{on} \quad \operatorname{int} S \tag{3.1}$$

with arbitrary f from the Hölder space $C^{0,\alpha}(S),\ 0 < \alpha < 1$. The same can be done with the supplementary conditions. Any real numbers can be prescribed as the values of the functionals $\mathcal{A}$ and $\mathcal{B}$. However, in what follows we restrict ourselves by the homogeneous conditions (2.11) to make manipulations more simple.

Let us outline the strategy of the solvability proof which follows the scheme proposed in Kuznetsov & Maz'ya (1989) (see also Kuznetsov 1995). Applying the potential method we reduce Problem (L) to an integro-algebraic system and show that the Fredholm alternative holds for this system in a suitable Banach space. Then we demonstrate that the system has unique solution for sufficiently small ν. These properties of the system and its analytical dependence on the papameter ν allow to apply a general result from the operator theory in Banach space (see Trofimov (1968)). It yields that the system is uniquely solvable for all $\nu > 0$ except possibly for a sequence of values tending to infinity. The solvability of the system implies the solvability (but not uniqueness) of Problem (L).

We seek a solution in the form

$$u(z) = (\mathcal{U}\mu)(z) + \sum_{\pm} \mu_{\pm} G(z, \pm a). \tag{3.2}$$

Here $\sum_{\pm}$ denotes summation of two terms, $G(z,\zeta)$ is the Green function (see Appendix A), $\mu_{\pm}$ are unknown real numbers and potential

$$(\mathcal{U}\mu)(z) = \int_S \mu(\zeta) G(z,\zeta)\, \mathrm{d}s_\zeta, \quad z \in \overline{L}, \quad \zeta = \xi + i\eta,$$

has an unknown real density μ belonging to the class $C^{0,\alpha}(\operatorname{int} S)$, $0 < \alpha < 1$, and to a Banach space $C_\kappa(S)$.

The latter consists of continuous on $\operatorname{int} S$ functions and is supplied with a norm

$$\|\mu\|_\kappa = \sup\{|y|^{1-\kappa}|\mu(z)| : \ z \in \operatorname{int} S\}, \quad 0 < \kappa < 1.$$

For any such μ the properties of G guarantee that the function (3.2) satisfies all relations of Problem (L) except for (3.1) and (2.11). For example, Theorem 6 in Kantorovich & Akilov (1982, ch. 11, § 3) guarantees that for every finite $b > 0$

$$\int_{W_b} |\nabla \mathcal{U}\mu|^2 \, \mathrm{d}x \, \mathrm{d}y < \infty, \quad W_b = W \cap \{|x| < b\}.$$

This and (A.1)–(A.2) (see Appendix A) yield that (3.2) satisfies (2.6).

Further, by (A.1) we have

$$-2\pi(\mathcal{U}\mu)(z) = \int_{S\cup S'} \mu(\zeta) \log|z-\zeta| \, \mathrm{d}s_\zeta + \int_S \mu(\zeta) g(z,\zeta) \, \mathrm{d}s_\zeta, \tag{3.3}$$

where $S' = \{(x,y) : \ (x,-y) \in S\}$ and μ is extended to S' as a function odd with respect to y. The expansion (A.2) for $g(z,\zeta)$ and Theorem 4 in Kantorovich & Akilov (1982, ch. 11, § 3) yield that the second integral in (3.3) belongs to the class $C^{1,\beta}(\overline{L})$ for every $\beta \in (0,\kappa)$. By (3.3) we can apply the formula for a normal derivative of a single layer logarithmic potential on the exterior side of $S \cup S'$. This gives

$$\frac{\partial\,(\mathcal{U}\mu)}{\partial n_z} = \frac{1}{2}\left[-\mu(z) + (T\mu)(z)\right], \quad z \in \operatorname{int} S. \tag{3.4}$$

Here the operator T defined by

$$(T\mu)(z) = 2\int_S \mu(\zeta)(\partial G/\partial n_z)(z,\zeta) \, \mathrm{d}s_\zeta$$

is not a compact operator in $C_\kappa(S)$. However, if κ satisfies the inequality

$$\kappa < \min_{\pm}\left[1 + \left|1 - \frac{2\beta_\pm}{\pi}\right|\right]^{-1}, \tag{3.5}$$

then the Fredholm theorems are true for $I - T$ in $C_\kappa(S)$ (I is the identity operator). This follows from the inequality

$$|T| < \max_{\pm} \frac{\sin\kappa|\pi - 2\beta_\pm|}{\sin\kappa\pi}, \tag{3.6}$$

where $|T|$ is the essential norm of T, i.e. $\inf\|T-K\|$ with K going through the set of linear compact operators in $C_\kappa(S)$. The last estimate combined with (3.5) implies that $|T| < 1$, which is sufficient for validity of the Fredholm theorems (see, for example, Maz'ya (1991), ch. 4).

The inequality (3.6) is due to Carleman (1916, ch. 1, § 2), but, of course, it was not expressed in terms of functional analysis in his celebrated original paper.

From (3.1) and (3.4) we get an integral equation with additional algebraic terms:

$$-\mu(z) + (T\mu)(z) + 2\sum_{\pm} \mu_\pm(\partial G/\partial n_z)(z, \pm a) = 2f(z), \quad z \in \operatorname{int} S. \tag{3.7}$$

Using (2.11) we complement this equation by algebraic system for $\mu_\pm$ containing integral functionals of μ. Really, according to (A.5) the wave term in the asymptotics of (3.2) as $|z| \to \infty$ has the form:

$$-2\mathrm{e}^{\nu y}\left(\int_S \mu(\zeta)\mathrm{e}^{\nu\eta}\sin\nu(x-\xi)\,\mathrm{d}s_\zeta + \sum_\pm \mu_\pm \sin\nu(x\mp a)\right).$$

Comparing this with (2.7) we obtain:

$$\begin{aligned}(\mu_+ + \mu_-)\cos\nu a + \int_S \mu(\zeta)\mathrm{e}^{\nu\eta}\cos\nu\xi\,\mathrm{d}s = 0,\\ (\mu_+ - \mu_-)\sin\nu a + \int_S \mu(\zeta)\mathrm{e}^{\nu\eta}\sin\nu\xi\,\mathrm{d}s = 0.\end{aligned} \tag{3.8}$$

Here (2.11) is taken into account. The equations (3.7) and (3.8) constitute an integro-algebraic system for the unknown vector $X = (\mu, \mu_+, \mu_-)^t$.

To prove the solvability theorem for the system (3.7), (3.8) (see Theorem 3.3 below) it is necessary to show that the Fredholm alternative holds for it. The suitable Banach space is $C_\kappa(S) \times \mathbb{R}^2$ supplied with norm $\max\{\|\mu\|_\kappa, |\mu_+|, |\mu_-|\}$, where the system takes the form:

$$(-\mathcal{I} + \mathcal{N})\,X = V. \tag{3.9}$$

Here $V = (2f,\ 0,\ 0)^t$, $\mathcal{I}$ is the identity matrix operator, and $\mathcal{N}$ is the operator

$$\begin{bmatrix} T & N_+ & N_- \\ T_+ & a_{22} & a_{23} \\ T_- & a_{32} & a_{33} \end{bmatrix}, \tag{3.10}$$

where $a_{2,(5\pm1)/2} = \frac{1}{2} \mp \frac{1}{2} + \cos\nu a$, $a_{3,(5\pm1)/2} = \frac{1}{2} \pm \frac{1}{2} \mp \sin\nu a$, $N_\pm$ is the operator of multiplication by $2(\partial G/\partial n_z)(z, \pm a)$, and the formula

$$T_\pm\mu = \int_S \mu(\zeta)\mathrm{e}^{\nu\eta}\cos\left(\nu\xi - \frac{\pi}{4} \pm \frac{\pi}{4}\right)\,\mathrm{d}s$$

defines the functional $T_\pm$.

THEOREM 3.1. If κ satisfies (3.5), then the Fredholm alternative holds for the equation (3.9) in the space $C_\kappa(S) \times \mathbb{R}^2$.

Proof. Let us write the matrix (3.10) as a sum

$$\begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & N_+ & N_- \\ T_+ & a_{22} & a_{23} \\ T_- & a_{32} & a_{33} \end{bmatrix}.$$

In the second matrix the functionals $T_\pm$ are continuous in $C_\kappa(S)$, and $(\partial G/\partial n_z)(z, \pm a)$ belongs to $C_\kappa(S)$ (see Appendix A). Therefore, this matrix defines a finite-dimensional operator in $C_\kappa(S) \times \mathbb{R}^2$. Then $\mathcal{N}$ and the

first matrix have the same essential norm in $C_\kappa(S)\times\mathbb{R}^2$, which is equal to $|T|$ in $C_\kappa(S)$. The latter norm is strictly less than 1 by (3.5), and hence, $|\mathcal{N}|<1$. As it was mentioned above, this implies that the Fredholm alternative is valid for (3.9). The theorem is proved.

For small $\nu>0$ the system (3.7), (3.8) can be investigated in more detail.

Lemma 3.1. For any $a>0$ and sufficiently small values $\nu>0$ the equations (3.8) are solvable with respect to $\mu_\pm$.

This is obvious because, the determinant Δ of (3.8) is equal to $-\sin 2\nu a$ and does not vanish for sufficiently small values of $\nu>0$. Moreover, we have

$$\mu_\pm=\frac{1}{\Delta}\int_S \mu(\zeta)\Delta_\pm(\zeta)\,\mathrm{d}s_\zeta,$$

where $\Delta_\pm=\mathrm{e}^{\nu\eta}\sin\nu(a\pm\xi)$. Thus, we arrive at

Lemma 3.2. For sufficiently small $\nu>0$ the integro-algebraic system (3.7), (3.8) is equivalent to the integral equation

$$-\mu(z)+(T_\nu\mu)(z)=2f,\ \ z\in\operatorname{int}S. \tag{3.11}$$

Here the operator T_ν is defined by

$$(T_\nu\mu)(z)=(T\mu)(z)+\frac{2}{\Delta}\int_S \mu(\zeta)\sum_\pm \Delta_\pm(\zeta)\frac{\partial G}{\partial n_z}(z,\pm a)\,\mathrm{d}s_\zeta.$$

Remark 3.1. In view of (A.2) the difference $T_\nu-T$ is a finite-dimensional operator in the space $C_\kappa(S)$. Hence the Fredholm alternative holds for (3.11), if κ satisfies (3.5).

Theorem 3.2. If κ satisfies (3.5), then for sufficiently small $\nu>0$ the equation (3.11) is uniquely solvable in the space $C_\kappa(S)$.

Proof. Let us consider the equation

$$-\mu(z)+(T_0\mu)(z)=2f,\ \ z\in\operatorname{int}S, \tag{3.12}$$

where

$$(T_0\mu)(z)=\frac{1}{\pi}\int_S \mu(\zeta)\frac{\partial}{\partial n_z}(\log|z-\overline{\zeta}|-\log|z-\zeta|)\,\mathrm{d}s_\zeta.$$

We extend the functions μ and f to $S'=\{z\in\overline{\mathbb{R}^2_+}:\overline{z}\in S\}$ as odd functions of y. Then (3.12) coincides with the integral equation for the Neumann problem in the domain exterior to the closed contour $S\cup S'$. Therefore, (3.12) is uniquely solvable in $C_\kappa(S)$ with κ satisfying (3.5) (see Carleman 1916). Thus, it is sufficient to show that the norm of $T_\nu-T_0$ is small in this space for positive ν close to zero.

Due to (A.2) and Lemma 3.2 the kernel of this operator has the form

$$-\frac{1}{\pi}\left[\frac{\partial g}{\partial n_z}(z,\zeta)+\frac{1}{\Delta}\sum_{\pm}\Delta_{\pm}(\zeta)\frac{\partial g}{\partial n_z}(z,\pm a)\right]. \tag{3.13}$$

According to (A.2) and (A.3) for all $\zeta \in S$

$$(\partial g/\partial n_z)(z,\zeta) = -2\nu\log\nu\cos(n_z,y)+O(\nu) \quad \text{as } \nu\to 0.$$

Then (3.13) can be written for small ν as follows:

$$[2\pi^{-1}\cos(n_z,y)\nu\log\nu+O(\nu)][1+\Delta^{-1}\sum_{\pm}\Delta_{\pm}(\zeta)]. \tag{3.14}$$

One easily finds that

$$\Delta^{-1}\sum_{\pm}\Delta_{\pm}(\zeta) = -1+O(\nu).$$

Hence, (3.13) tends to zero as $O(\nu^2\log\nu)$ as $\nu\to 0$. This completes the proof.

By Lemma 3.2 the system (3.7), (3.8) is equivalent to (3.11) for small ν. Hence Theorem 3.2 implies the following corollary.

Corollary 3.1. If κ satisfies (3.5), then for sufficiently small $\nu > 0$ the system (3.7), (3.8) is uniquely solvable in the space $C_\kappa(S)\times\mathbb{R}^2$.

Now, the general result on solvability of the integro-algebraic system can be proved.

Theorem 3.3. For all $\nu > 0$, except possibly for a sequence tending to infinity, (3.9) is uniquely solvable in the space $C_\kappa(S)\times\mathbb{R}^2$, where κ satisfies (3.5).

Proof. By (A.1) and (A.2) the operator $\mathcal{N}$ depends analytically on ν in a vicinity of the ray $\{\operatorname{Re}\nu > 0,\ \operatorname{Im}\nu = 0\}$. By Corollary 3.1 the operator $\mathcal{I}-\mathcal{N}$ is invertible for sufficiently small $\nu > 0$. Besides, the Fredholm alternative holds for (3.9). Then the assertion follows from the theorem on invertability of an operator-function analytically depending on a parameter (see Trofimov 1968).

Corollary 3.2. For all values $\nu > 0$, except possibly for a sequence tending to infinity, Problem (L) has a solution for any $f \in C^{0,\alpha}(S)$.

Proof. By Theorem 3.3 (3.9) is solvable for all $\nu > 0$ except for a discrete sequence of values. Substituting the solution $(\mu,\mu_+,\mu_-)^t$ of this equation into (3.2) we obtain the function u which meets (due to the properties of the Green function) all the conditions of Problem (L) except for (3.1) and (2.11). The conditions (2.11) are satisfied provided the equations (3.8) hold.

Now we have to verify that $\mu \in C^{0,\alpha}(\operatorname{int} S)$, because in this case the potential $\mathcal{U}\mu$ has the normal derivative on $\operatorname{int} S$ and (3.1) follows from the equation (3.7). According to (A.3) the function $(\partial G/\partial n_z)(z,\pm a)$ belongs

to $C^{0,\alpha}(\operatorname{int} S)$. Writing (3.7) in the form

$$-\mu(z) + (T\mu)(z) = 2[f(z) - \sum_{\pm} \mu_{\pm}(\partial G/\partial n_z)(z, \pm a)],$$

we see that the right-hand side belongs to $C^{0,\alpha}(\operatorname{int} S)$ for any $\mu_{\pm}$. Then μ is in the same class due to the well-known property of T (see, for example, Colton & Kress 1983).

4. On the uniqueness of the waveless potential

Following the scheme proposed in Kuznetsov & Maz'ya (1989) we want to prove that Problem (L) has no more than one solution for all $\nu > 0$ except for a certain discrete sequence of values. The method applied in Kuznetsov & Maz'ya (1989) is based on the following general theorem (see Hille & Phillips (1957), § 2.12).

An operator equation in a Banach space has no more than one solution when the equation with adjoint operator is solvable for an arbitrary right-hand term.

The supplementary conditions used by Kuznetsov & Maz'ya (1989) are well suited to define a uniquely solvable "adjoint" problem by virtue of Green's identity. These conditions vanish all out of integral terms in this identity, and this is the crucial point in their proof. Since the supplementary conditions (2.11) are poorly adapted to vanishing the out of integral terms, we impose a geometrical restriction when applying the same method to Problem (L). Namely, we assume that the contour S is symmetric with respect to the y-axis. This allows to introduce symmetric and antisymmetric solutions and to consider them separately. In either case it is possible to define an appropriate "adjoint" Problem $(\mathrm{L}_1)/(\mathrm{L}_2)$, which is coupled with symmetric/antisymmetric solution by Green's formula containing only integral. In Appendix B the exact definition and investigation of Problems (L_1) and (L_2) are given.

Let u be a solution to the homogeneous problem (2.1)–(2.6) and (2.11). In view of symmetry of W it is possible to represent u as a sum of the even and odd functions with respect to x:

$$u(x, y) = u^{(s)}(x, y) + u^{(a)}(x, y),$$

$$2u^{(s)}(x, y) = u(x, y) + u(-x, y), \qquad 2u^{(a)}(x, y) = u(x, y) - u(-x, y).$$

By Theorem 4.1 $u^{(s)}$ and $u^{(a)}$ satisfy the homogeneous Problem (L). Furthermore, the equalities

$$u_x^{(s)}(P_+) + u_x^{(s)}(P_-) = 0, \qquad u_x^{(a)}(P_+) - u_x^{(a)}(P_-) = 0 \tag{4.1}$$

hold for these functions. At last, according to (2.7) and (2.11) we have as $|z| \to \infty$:

$$u^{(s)}(x, y) = C + Q \log(\nu|z|) + \psi^{(s)}(x, y), \tag{4.2}$$

$$u^{(a)}(x, y) = \psi^{(a)}(x, y). \tag{4.3}$$

Here $\psi^{(s)}, \psi^{(a)} = O(|z|^{-1})$ and $|\nabla\psi^{(s)}|, |\nabla\psi^{(a)}| = O(|z|^{-2})$; C is an arbitrary constant and $\pi\nu Q = 2u^{(s)}(P_-)$ on account of (2.8) and (4.1).

THEOREM 4.1. The problem (2.1)–(2.6) and (2.11) for a symmetric body has at most one solution (up to a constant term) for all $\nu > 0$ except possibly a discrete sequence of values.

Proof. We have to show that both functions $u^{(s)}$ and $u^{(a)}$ introduced above are constants (it is obvious that $u^{(a)} = 0$, when it is a constant). According to Theorem B.1 (see Appendix B) Problem $(\mathrm{L}_1)/(\mathrm{L}_2)$ is solvable for all $\nu > 0$, except possibly a sequence tending to infinity. Let ν be such a value of the parameter that both these problems have solutions $u^{(1)}$ and $u^{(2)}$ for arbitrary Neumann data. We suppose the latter to have the zero mean value on S.

Due to (4.2) and (4.3) Lemmas B.1 and B.2 can be used. By Lemma B.1 we get

$$\int_S u^{(s)}(\partial u^{(1)}/\partial n)\,\mathrm{d}s = 0.$$

Here (4.1) and (B.1) vanish the right-hand side term in (B.5). Since the second factor in the last integral is an arbitrary function orthogonal to constant, $u^{(s)} = \text{const}$ on S. Now the uniqueness theorem for the Cauchy problem for Laplace's equation yields that $u^{(s)} = \text{const}$ on W.

Similar consideration based on Lemma B.2 leads the conclusion that $u^{(a)} = 0$ on W, that completes the proof.

5. The waveless potential and resistance to the forward motion

In the present section we are concerned with the horizontal component of force ("resistance") acting on a cylinder under assumption that the velocity field in the fluid is described by the waveless potential. According to Kuznetsov (1990) the total resistance R to the forward motion of a 2D surface-piercing body can be expressed as follows:

$$R = -\frac{\rho\nu}{4}(\mathcal{A}^2 + \mathcal{B}^2) - \frac{\rho}{2\nu}[u_x^2(a,0) - u_x^2(-a,0)], \tag{5.1}$$

where ρ is the fluid density and u satisfies (2.1)–(2.6). Here the first term can be naturally identified with the wave resistance. Since the second term is connected with the constant Q, which is proportional to the additional flux of fluid due to body's presence (see Remark 2.1), this term should be associated with the so-called spray resistance. By (2.11) the wave resistance vanishes if u is the waveless potential. The spray resistance reveals some interesting properties in this case. In particular, the following theorem means that the waveless statement of the Neumann–Kelvin problem is, in fact, resistanceless for any symmetric contour.

THEOREM 5.1. Let S be symmetric about the y-axis, and let u be the unique (up to constant term) waveless potential in W. Then $R = 0$.

First we prove an auxiliary result which seems to be of its own interest. Let $\widehat{W} = \{(x, y) : (-x, y) \in W\}$ be the domain symmetric to W with respect to the y-axis. By $\hat{S}$ we denote the arc symmetric to S. Let us put $\hat{u}(x, y) = -u(-x, y)$. This function is defined on $\widehat{W}$. If u is the waveless potential in W, then $\lim_{x\to+\infty} |\nabla\hat{u}| = 0$. Conversely, the condition $\lim_{x\to+\infty} |\nabla u| = 0$ means that there are no wave terms in the asymptotics (2.7) for $\hat{u}$. It is easy to verify by direct calculation that the Neumann condition (2.3) is equivalent to

$$\partial\hat{u}/\partial n = U\cos(n, x) \quad \text{on} \quad \operatorname{int}\hat{S}.$$

Since all other relations in the definition of waveless potential are obviously true for $\hat{u}$, we get

THEOREM 5.2. Let u be a solution to (2.1)–(2.6) and (2.11) in W. Then the function $\hat{u}$ is a waveless potential in $\widehat{W}$.

Proof of Theorem 5.1. Since W is symmetric about the y-axis, Theorem 5.2 yields that $\hat{u}$ satisfies Problem (L) along with u. From the assumption about uniqueness of the waveless potential it follows that

$$u(x, y) + u(-x, y) = \text{const}.$$

This immediately yields that $u_x(a, 0) = u_x(-a, 0)$. Substituting the last equality and (2.11) into (5.1) proves the theorem.

For cylinders with asymmetric geometry the behaviour of spray resistance can be investigated only numerically when u satisfies the waveless statement of the problem. As an example we take a family of Pascal's snails given by the following parametric equation:

$$x(t) = b\cos^2 t + a\cos t - b, \quad y(t) = -\sin t(b\cos t + a), \quad t \in [0, \pi].$$

On fig. 2(a) two patterns of this curve are given. We see on fig. 2(b,c) that the spray resistance is not a monotonic function of b/a and there exist geometries with the towing force instead of the resistance.

Fig. 2(b) demonstrates that the horizontal force takes the opposite values for two snails which are symmetric to each other with respect to the y-axis. The proof of this assertion for an arbitrary pair of symmetric contours follows from Theorem 5.2.

6. Conclusion

The existence and uniqueness of the solution to a new statement of the two-dimensional Neumann–Kelvin problem for a surface-piercing body has been considered. It is non-local unlike the investigated earlier statements, and prescribes the coefficients of wave terms in the asymptotics of velocity potential at infinity downstream to vanish. The considered problem can be perceived as a paradoxical one from physical point of view, but it proves to be well-posed mathematically. The solvability theorem has been proved for an arbitrary con-

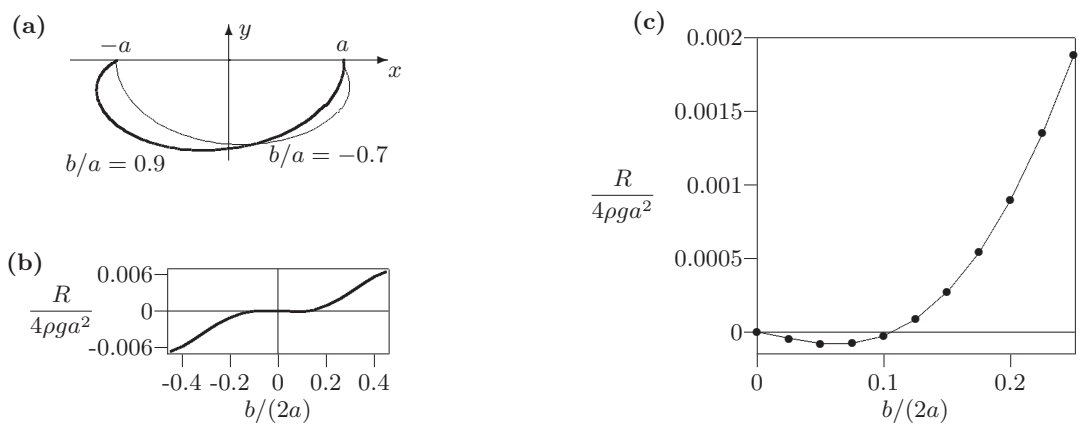

Figure 2: (a) the lower half of Pascal's snail for different values of the ratio b/a; (b,c) the dependence of the spray resistance on b/a for Pascal's snail when the velocity potential is waveless (the Froude number $\mathrm{Fr} = (2a\nu)^{-1/2} = 0.8$).

tour intersecting the free surface transversally. According to this theorem the waveless potential does exist for all values U of the forward velocity except possibly a discrete sequence. For any contour, which is symmetric about a vertical axis, the waveless potential has been demonstrated to be unique under the same restriction on U.

It is clear that the wave resistance vanishes if the flow about a body is described by waveless potential. The other component of the resistance, i.e. the so-called spray resistance, has been investigated both numerically and mathematically. It has been proved that this component also vanishes for symmetric contours, for which uniqueness of the solution holds. For two contours, which are not symmetric themselves but are symmetric with respect to each other, the spray resistance has been shown to have opposite values. This resistance has been calculated numerically for a family of Pascal's snails depending on a shape parameter. It reveals complicated non-monotonic behaviour.

Appendix A. The Green function of the Neumann-Kelvin problem

Here we summarize some known properties of the Green function (the 2D Kelvin source). We begin with the boundary value problem for $G(x, y; \xi, \eta)$ $(= G(z, \zeta))$:

$$\nabla^2_{x,y} G = -\delta(|z - \zeta|) \text{ for } y < 0,$$

$$G_{xx} + \nu G_y = 0 \text{ for } y = 0,$$

$$\limsup_{x\to\infty} |\nabla_{x,y} G| < \infty, \quad \lim_{x\to+\infty} |\nabla_{x,y} G| = 0.$$

We suppose that $\eta < 0$ in this boundary value problem. The function $G(x, y; \xi, 0)$ is a harmonic function in the half-plane $\{-\infty < x < +\infty,\ y < 0\}$, and it satisfies the boundary condition

$$G_{xx} + \nu G_y = \nu\delta(|x - \xi|) \text{ for } y = 0.$$

Wehausen & Laitone (1960) give the following formula for Green's function:

$$\begin{aligned} G(z, \zeta) = &- (2\pi)^{-1}\big[\log(\nu|z - \zeta|) + \log(\nu|z - \overline{\zeta}|) \\ &+ 2\int_0^\infty \frac{\cos\mu(x - \xi)}{\mu - \nu} \mathrm{e}^{\mu(y+\eta)}\, \mathrm{d}\mu + 2\pi \mathrm{e}^{\nu(y+\eta)} \sin\nu(x - \xi)\big], \end{aligned}$$

where the integral is understood in the sense of the Cauchy principal value. Another representation can be found in Kochin (1937):

$$G(z, \zeta) = -(2\pi)^{-1}\{\log|z - \zeta| - \log|z - \overline{\zeta}| + g(z, \zeta)\}, \tag{A.1}$$

where

$$\begin{aligned} g(z, \zeta) = &- 2\,\mathrm{Re}\Big\{\log(\nu|z - \overline{\zeta}|) \sum_{m=1}^{\infty} \frac{[-i\nu(z - \overline{\zeta})]^m}{m!} \\ &+ \exp\{-i\nu(z - \overline{\zeta})\}\Big(\gamma - \frac{\pi}{2}i + \sum_{m=1}^{\infty} \frac{[i\nu(z - \overline{\zeta})]^m}{m!m}\Big)\Big\}. \end{aligned} \tag{A.2}$$

Here $\gamma = 0,5772\ldots$ is Euler's constant. From here it follows (cf Ursell 1981):

$$\begin{aligned} &G_x(x, y; \xi, 0) = \\ &\frac{\nu}{\pi}\sum_{m=0}^{\infty} \frac{(-\nu r)^m}{m!} \left\{\left(\varphi - \frac{\pi}{2}\right)\cos m\left(\varphi + \frac{\pi}{2}\right) + \left[\log(\nu r) - \frac{\Gamma'(m+1)}{\Gamma(m+1)}\right]\sin m\left(\varphi + \frac{\pi}{2}\right)\right\}, \\ &G_y(x, y; \xi, 0) = \\ &\frac{\nu}{\pi}\sum_{m=0}^{\infty} \frac{(-\nu r)^m}{m!} \left\{\left(\frac{\pi}{2} - \varphi\right)\sin m\left(\varphi + \frac{\pi}{2}\right) + \left[\log(\nu r) - \frac{\Gamma'(m+1)}{\Gamma(m+1)}\right]\cos m\left(\varphi + \frac{\pi}{2}\right)\right\}. \end{aligned} \tag{A.3}$$

The first of these formulae yields that

$$\lim_{x\to\xi\pm 0} G_x(x, 0; \xi, 0) = \nu(-1 \pm 1/2). \tag{A.4}$$

The derivation of the following asymptotic formula can be found in Vainberg & Maz'ya (1973):

$$G(z, \zeta) = -\pi^{-1}\log(\nu|z|) - H(-x)2\mathrm{e}^{\nu(y+\eta)}\sin\nu(x - \xi) + \varphi_(x, y), \tag{A.5}$$

where $|\zeta| < C < \infty$ and $\varphi = O(|z|^{-1}),\ |\nabla\varphi| = O(|z|^{-2})$ as $|z| \to \infty$.

Appendix B. Auxiliary Problems (L_1) and (L_2)

The proof of uniqueness theorem in § 4, based on the method proposed in Kuznetsov & Maz'ya (1989), needs two auxiliary problems. They are introduced and investigated in the present Appendix.

Definition B.1. We say that $u^{(i)}$ $(i = 1, 2)$ is a solution to Problem (L_i), if it satisfies (2.1), (2.2), (3.1), (2.4)–(2.6), and the following supplementary conditions

$$u_x^{(i)}(P_+) + (-1)^i u_x^{(i)}(P_-) = 0, \qquad u^{(i)}(P_+) - (-1)^i u^{(i)}(P_-) = 0 \tag{B.1}$$

hold.

Remark B.1. Problem (L_2) is very close to Problem I.2 in Kuznetsov & Maz'ya (1989). However, Problem (L_1) differs from other consistent statements of the Neumann–Kelvin problem because the second condition in (B.1) does not allow to add an arbitrary constant to $u^{(1)}$.

The scheme applied in § 3 to demonstrate the solvability of Problem (L) can be also used for Problems (L_1) and (L_2). Let us seek a solution to Problem (L_i) $(i = 1, 2)$ in the form (3.2). Then, from (B.1) we obtain algebraic equations coupled with (3.7):

$$\sum_{\pm} \mu_{\pm}^{(i)} [G_x(a, \pm a) + (-1)^i G_x(-a, \pm a)] + \int_S \mu^{(i)}(\zeta)[G_x(a, \zeta) + (-1)^i G_x(-a, \zeta)]\, \mathrm{d}s = 0,$$

$$\sum_{\pm} \mu_{\pm}^{(i)} [G(a, \pm a) - (-1)^i G(-a, \pm a)] + \int_S \mu^{(i)}(\zeta)[G(a, \zeta) - (-1)^i G(-a, \zeta)]\, \mathrm{d}s = 0, \tag{B.2}$$

where $G_x(a, a)$ and $G_x(-a, -a)$ should be calculated with the help of (A.4). We denote by $(\mu^{(i)}, \mu_+^{(i)}, \mu_-^{(i)})^t$ the unknown vector in the representation (3.2) for $u^{(i)}$.

Similarly to Lemma 3.1 one proves that the system (B.2) is solvable for any a and sufficiently small $\nu > 0$. Really, using (A.3) and (A.4) one obtains that the determinant $\Delta^{(i)}$ of this system has the following behaviour as $\nu \to 0$:

$$\Delta^{(i)} = \nu^2 \big[(-1)^i 4a + O(\nu \log \nu)\big], \quad i = 1, 2.$$

Hence, it does not vanish for small $\nu > 0$. Substituting the solution of (B.2)

$$\mu_{\pm}^{(i)} = \frac{1}{\Delta^{(i)}} \int_S \mu^{(i)}(\zeta) \Delta_{\pm}^{(i)}(\zeta)\, \mathrm{d}s_\zeta$$

into the integral equation (3.7) one arrives at

$$-\mu^{(i)}(z) + (T_\nu^{(i)} \mu^{(i)})(z) = 2f(z), \ z \in \operatorname{int} S.$$

Here the operator $T_\nu^{(i)}$ is defined by

$$(T_\nu^{(i)}\mu^{(i)})(z) = (T\mu)(z) + \frac{2}{\Delta^{(i)}}\int_S \mu^{(i)}(\zeta)\sum_{\pm}\Delta_\pm^{(i)}(\zeta)\frac{\partial G}{\partial n_z}(z,\pm a)\,\mathrm{d}s_\zeta,$$

where $\Delta_\pm^{(i)}(\zeta)$ are the determinants arising when solving the system (B.2).

As in § 3 the crucial point of the proof that Problems (L_1) and (L_2) are solvable is to show that the norm of $T_\nu^{(i)} - T_0$ is small when ν is close to zero. Similarly to (3.13) and (3.14) this norm is small if

$$\frac{1}{\Delta^{(i)}}\sum_{\pm}\Delta_\pm^{(i)}(\zeta) \tag{B.3}$$

is bounded as $\nu \to 0$. Direct calculation based on (A.3) shows that the following formulae hold as $\nu \to 0$:

$$\sum_{\pm}\Delta_\pm^{(1)}(\zeta) = \nu^2\big[4a\pi^{-1}\{\varphi_+(\zeta) - \varphi_-(\zeta)\} + O(\nu\log\nu)\big],$$
$$\sum_{\pm}\Delta_\pm^{(2)} = \nu^2\big[2a\pi^{-1}(3\pi + \varphi_+(\zeta) + \varphi_-(\zeta)) + O(\nu\log\nu)\big],$$

where $\varphi_\pm(\zeta) = \arg(\zeta \mp a) \in [-\pi, 0]$. These formulae and the asymptotics for $\Delta^{(i)}$ demonstrate that (B.3) is bounded for small ν. Following the considerations in the end of § 3 one proves

Theorem B.1. For all values $\nu > 0$, except possibly for a sequence tending to infinity, Problems (L_1) and (L_2) have solutions for any $f \in C^{0,\alpha}(S)$.

Now we prove two lemmas used in § 4.

Lemma B.1. Let u be a waveless potential such that

$$u(x,y) = C + Q\log(\nu|z|) + \psi(x,y) \quad \text{as } |z| \to \infty, \tag{B.4}$$

where ψ decays at infinity like the remainder term in (2.7). If $u^{(1)}$ has the form (3.2) and satisfies Problem (L_1) with f having zero mean value over S, then

$$\int_S \left(u\frac{\partial u^{(1)}}{\partial n} - u^{(1)}\frac{\partial u}{\partial n}\right)\mathrm{d}s = \nu^{-1}\left[u(x,0)u_x^{(1)}(x,0) - u^{(1)}(x,0)u_x(x,0)\right]_{x=-a}^{x=a}. \tag{B.5}$$

Proof. According to the assumptions made we have that

$$u^{(1)}(x,y) = H(-x)\mathrm{e}^{\nu y}(\mathcal{A}^{(1)}\sin\nu x + \mathcal{B}^{(1)}\cos\nu x) + \psi^{(1)}(x,y) \quad \text{as } |z| \to \infty, \tag{B.6}$$

where $\psi^{(1)}$ has the same behaviour at infinity as ψ. Really, by (3.2) and (A.5) there is no constant term in (B.6). Furthermore, by (2.8) we get that $Q = 0$ in the same asymptotics, because of (B.1) and the orthogonality of f to constant.

Let $R_d = \{|x| < d, -d < y < 0\}$ be a rectangle containing D. By p_0, p_d and $q_{\pm d}$ we denote the upper, lower,

right and left sides of R_d respectively, $W_d = R_d \setminus \overline{D}$. By Green's formula

$$0 = \int_{W_d} (u^{(1)}\nabla^2 u - u\nabla^2 u^{(1)})\mathrm{d}x\mathrm{d}y = \int_{\partial W_d} \left(u\frac{\partial u^{(1)}}{\partial n} - u^{(1)}\frac{\partial u}{\partial n} \right) \mathrm{d}s,$$

where n is directed into W_d. Thus,

$$\int_S \left(u\frac{\partial u^{(1)}}{\partial n} - u^{(1)}\frac{\partial u}{\partial n} \right) \mathrm{d}s = \int_{\partial W_d \setminus \overline{D}} \left(u^{(1)}\frac{\partial u}{\partial n} - u\frac{\partial u^{(1)}}{\partial n} \right) \mathrm{d}s,$$

and we have to consider the integrals along straight segments whose sum gives the latter integral.

Substituting (B.4) and (B.6) into the integral over p_d gives after simple calculation that this integral is $O(d^{-1}\log d)$ as $d \to \infty$. Similarly, we get the same estimate for the integral over q_d and show that the integral over q_{-d} is equal to

$$-[C + Q\log(\nu d)](\mathcal{A}^{(1)}\cos\nu d + \mathcal{B}^{(1)}\sin\nu d) + O(d^{-1}\log d).$$

By the boundary condition (2.2) we obtain

$$\int_{p_0 \cap \partial W_d} \left(u^{(1)}\frac{\partial u}{\partial n} - u\frac{\partial u^{(1)}}{\partial n} \right) \mathrm{d}s = \nu^{-1} \left(\int_{-d}^{-a} + \int_a^d \right) \left[u^{(1)}u_{xx} - uu^{(1)}_{xx} \right]_{y=0} \mathrm{d}x.$$

Integrating by parts we find this integral:

$$\nu^{-1}\Big\{ \Big[u^{(1)}(x,0)u_x(x,0) - u(x,0)u^{(1)}_x(x,0) \Big]_{x=-d}^{x=d} \\ - \Big[u^{(1)}(x,0)u_x(x,0) - u(x,0)u^{(1)}_x(x,0) \Big]_{x=-a}^{x=a} \Big\}.$$

Taking into account the asymptotics of u and $u^{(1)}$ at infinity we get that the first term in braces is equal to

$$[C + Q\log(\nu d)](\mathcal{A}^{(1)}\cos\nu d + \mathcal{B}^{(1)}\sin\nu d) + O(d^{-1}\log d).$$

Summing up the asymptotics of the segment integrals and tending d to infinity we obtain (B.5).

Lemma B.2. Let u be a waveless potential decaying at infinity like the remainder term in (2.7). If $u^{(2)}$ satisfies Problem (L_2) and has the form (3.2), then (B.5) holds for u and $u^{(2)}$ replacing $u^{(1)}$.

The proof of this lemma literally repeats that of Lemma B.1.

REFERENCES

Bessho, M. 1976 Line integral, uniqueness and diffraction of wave in the linearized theory. In: *International Seminar on Wave Resistance*. Tokyo, 45–55.

Bessho, M. 1994 On a consistent linearized theory of the wave-making resistance of ships. *J. Ship Res.* **38**, 83–96.

Carleman, T. 1916 *Über das Neumann-Poincarésche Problem für ein Gebiet mit Ecken.* Uppsala: Ålmqvist & Wiksell.

Colton, D. & Kress, R. 1983 *Integral equation methods in scattering theory.* New York et al.: Wiley.

Eggers, K. 1976 Discussion to Bessho (1976). In: *International Seminar on Wave Resistance.* Tokyo, 401–404.

Hille, E. & Phillips, R.S. 1968 *Functional Analysis and Semi-Groups.* Providence: AMS.

Kantorovich, L.V. & Akilov, G.P. 1982 *Functional analysis*. Oxford: Pergamon.

Kochin, N.E. 1937 On the wave resistance and lift of bodies submerged in a fluid. In: *Trudy Konferentsii po Teorii Volnovogo Soprotivleniya, Moscow*: TsAGI, 65–134 (in Russian), English translation in: 1951 *SNAME Tech. & Res. Bull.* **1-8**.

Kuznetsov, N.G. 1990 Wave resistance of a cylinder partially immersed in the infinite depth fluid. In: *High Speed Hydrodynamics*. Cheboksary: Chuvashian University, 53–60 (in Russian).

Kuznetsov, N.G. 1992 Waves due to the forward motion of a surface-piercing cylinder in a fluid of infinite depth. *Modelling in Mechanics* **6**(4), 70–84 (in Russian).

Kuznetsov, N. 1995 On uniqueness and solvability in the linearized two-dimensional problem of a supercritical stream about a surface-piercing body. *Proceedings of the Royal Society of London A* **450**, 233–253.

Kuznetsov, N.G. & Maz'ya, V.G. 1989 On unique solvability of the plane Neumann-Kelvin problem. *Mathematics USSR Sbornik*. **63** 425–446.

Kuznetsov, N.G. & Motygin, O.V. 1995 On the statement of the two-dimensional Neumann–Kelvin problem describing the resistanceless forward motion of a surface-piercing tandem. *A.M. Basin Memorial Conference on Ship Hydrodynamics.* St. Petersburg, 154–162 (in Russian).

Lenoir, M. 1982 Methodes de couplage en hydrodynamique navale et application a la resistance de vagues bidimensionnelle. *Rapport de Recherche ENSTA* **164**.

Maz'ya, V.G. 1991 Boundary integral equations. In: *Encyclopaedia of Mathematical Sciences.* **27**, (*Analysis IV*), 127-222. Berlin et al.: Springer-Verlag.

McIver, M. 1996 An example of non-uniqueness in the two-dimensional linear water wave problem. *Journal of Fluid Mechanichs* **315**, 257–266.

Motygin, O. & Kuznetsov, N. 1995 The 2D Neumann–Kelvin problem for a surface-piercing tandem. *Proceedings of the 10th International Workshop on Water Waves and Floating Bodies.* University of Oxford: Department of Engineering Science, 181–185.

Suzuki, K. 1982 Numerical studies of the Neumann-Kelvin problem for a two-dimensional semisubmerged body. In: *Proceedings of the 3d International Conference on Numerical Ship Hydrodynamics*. Paris: Bassin d'Essais des Carènes, 83–95.

Trofimov, V.P. 1968 The root subspaces of operators that depend analytically on a parameter. *Matematicheskie Issledovaniya* **3**(9), 117–125 (in Russian).

Ursell, F. 1981 Mathematical notes on the two-dimensional Kelvin-Neumann problem. In: *Proceedings of the 13th Symposium on Naval Hydrodynamics*. Tokyo: Shipbuilding Research Association of Japan, 245–251.

Vainberg, B.R. & Maz'ya, V.G. 1973 On the plane problem of the motion of a body immersed in a fluid. *Transactions of Moscow Mathematical Society*. **28**, 33-55.

Wehausen, J.V. & Laiton, E.V. 1960 Surface waves. *Handbuch der Physik*. **9**. Berlin: Springer-Verlag, 446-778.

Author's typeset of the paper published in IMA Jl of Applied Mathematics 1997, No.59: 25–42.