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EXTENSIONAL WAVES IN A TRANSVERSELY ISOTROPIC SOLID BAR IMMERSED IN AN INVISCID FLUID CALCULATED USING CHEBYSHEV POLYNOMIALS

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Abstract. The extensional vibration in a homogeneous transversely isotropic solid bar immersed in an inviscid fluid is studied using the linearized, three-dimensional theory of elasticity. The equations of motion of solid bar and fluid are respectively formulated using the constitutive equations of a transversely isotropic cylinder and the constitutive equations of an inviscid fluid. The solution of the frequency equations are obtained by Chebyshev polynomial series using the geometric boundary conditions. The computed non-dimensional frequencies are presented in the form of dispersion curves for the material Zinc. To compare the model with exiting literature, the longitudinal vibration of cylindrical bar without fluid are obtained and they show good agreement.

1. Introduction

In many structural applications the extensional loadings has taking interest because of high tensile strength and high corrosion resistance properties. The extensional modes often used to evaluate the material properties of thin metal wires, reinforcement filament in ultrasonic transducers and resonators. Applying Chebyshev polynomial series as the admissible function for each displacement component has distinct advantages like rapid convergence and better numerical stability in computation than other algebraic polynomial series.

The most general form of harmonic waves in a hollow cylinder of circular cross section of infinite length has been analyzed by Gazis [1]. Mirsky [2] investigated the wave propagation in transversely isotropic circular cylinders of infinite length and presented the frequency equation in Part I and numerical results in Part II. A method, for solving wave propagation in arbitrary cross-sectional cylinders and plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional and flexural, by constructing frequency equations was devised by Nagaya [3-5]. He formulated the Fourier expansion collocation method for this purpose. Following Nagaya, Paul and Venkatesan [6] studied the wave propagation in an infinite piezoelectric solid cylinder of arbitrary cross section using Fourier expansion collocation method. The longitudinal waves inhomogeneous anisotropic cylindrical bars immersed in a fluid is studied by Dayal [7]. Guided waves in a transversely isotropic cylinder immersed in a fluid are analyzed by Ahmad [8]. Following Ahmad, Nagay [9] have studied the longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid, later, Nagy with Nayfeh [10] discussed the viscosity-induced attenuation of longitudinal guided waves in fluid-loaded rods. Easwaran and Munjal [11] reported a note on the effect of wall compliance on lowest-order mode propagation in fluidfilled/submerged impedance tubes. Sinha et al. [12] have discussed the axisymmetric wave propagation in circular cylindrical shell immersed in fluid, in two parts. In Part I, the theoretical analysis of the propagating modes are discussed and in Part II, the axisymmetric modes excluding torsional modes are obtained theoretically and experimentally and are compared. Berlinear and Solecki [13] have studied the wave propagation in fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results. Ponnusamy and Selvamani [14] have studied the dispersion analysis of generalized magneto-thermo elastic waves in a transversely isotropic cylindrical panel using the wave propagation approach. Later, Selvamani [15] obtained mathematical modeling and analysis for damping of generalized thermoelastic waves in a homogeneous isotropic plate. Venkatesan and Ponnusamy [16] have obtained the frequency equation of the free vibration of a solid cylinder of arbitrary cross section immersed in a fluid using Fourier expansion collocation method. Gladwell and Tahbildar [17] and Buchanan and Yii [18] used the finite element method to study the 3D vibration of cylinders while Wang and Williams [19] compared their finite element result with the experimental data. Singal and Williams [20] used simple algebraic polynomials as admissible functions to study the 3D vibration of completely free hollow cylinders and the theoretical results agreed well with the experimental results. The same problem was also studied by So and Leissa [21] using simple algebraic polynomials as admissible functions, and high accurate result were given. Later, Zhou [22] studied the three dimensional vibration analyses of solid and hollow circular cylinders using Chebyshev-Ritz method.

In this paper, the extensional vibration in a finite, homogeneous transversely isotropic solid bar immersed in an inviscid fluid is studied using the linearized three-dimensional theory of elasticity**.** Two displacement potential functions are introduced to uncouple the equations of motion and the solutions are obtained by Chebyshev method. The computed non-dimensional frequencies are presented in the form of dispersion curves for the material Zinc.

2. Formulation of the problem

We consider a transversely isotropic cylindrical bar of length L and radius a immersed in inviscid fluid. The system is assumed to be linear so that the linearized three-dimensional stress equations of motion are used for both the bar and the fluid. The system displacements and stresses are defined by the cylindrical coordinates r, θ and z. In cylindrical coordinates, the three-dimensional stress equations of motion and strain-displacement relations in the absence of body are given as

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt}\,, \tag{1a}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}\,, \tag{1b}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho w_{,tt}\,, \tag{1c}$$

where

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz}\,, \tag{2a}$$

$$\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz}\,, \tag{2b}$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz}\,, \tag{2c}$$

$$\sigma_{r\theta} = 2c_{66}e_{r\theta}\,,\; \sigma_{\theta z} = 2c_{44}e_{\theta z}\,,\; \sigma_{rz} = 2c_{44}e_{rz}\,, \tag{2d}$$

where $\sigma_{rr,}\sigma_{zz,}\sigma_{rz}$ are the stress components, $e_{rr,}\ e_{\theta\theta,}\ e_{zz,}\ e_{r\theta,}\ e_{\theta z,}\ e_{rz}$ are the strain components, $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66} = (c_{11} - c_{12})/2$ are the five independent elastic constants, ρ is the mass density of the material.
The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{,r}\,, \quad e_{\theta\theta} = r^{-1}\left(u + v_{,\theta}\right),\; e_{zz} = w_{,z}\,, \tag{3a}$$

$$2e_{r\theta} = v_{,r} - r^{-1}\left(v - u_{,\theta}\right), \qquad 2e_{zr} = \left(u_{,z} + w_{,r}\right), \qquad 2e_{\theta z} = \left(v_{,z} + r^{-1}w_{,\theta}\right), \tag{3b}$$

in which u, and w are the displacement components along radial, axial directions respectively. The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (3) and (2) in the Eq. (1), results in the following three-dimensional displacement equations of motion:

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) - r^{-2}\left(c_{11} + c_{66}\right)v_{,\theta} + r^{-2}c_{66}u_{,\theta\theta} +$$

$$+ c_{44}u_{,zz} + \left(c_{44} + c_{13}\right)w_{,rz} + r^{-1}\left(c_{66} + c_{12}\right)v_{,r\theta} = \rho u_{,tt}\,, \tag{4a}$$

$$r^{-1}\left(c_{12} + c_{66}\right)u_{,r\theta} + r^{-2}\left(c_{66} + c_{11}\right)u_{,\theta} + c_{66}\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) +$$

$$+ r^{-2}c_{11}v_{,\theta\theta} + c_{44}v_{,zz} + r^{-1}\left(c_{44} + c_{13}\right)w_{,\theta z} = \rho v_{,tt}\,, \tag{4b}$$

$$c_{44}\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{,z} + v_{,\theta z}\right) +$$

$$+ \left(c_{44} + c_{13}\right)u_{,rz} + c_{33}w_{,zz} = \rho w_{,tt}\,. \tag{4c}$$

For extensional wave ,it is assumed that the displacement along the hoop direction, v is zero and Eqs. (4) reduce to

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + c_{44}u_{,zz} + \left(c_{44} + c_{13}\right)w_{,rz} = \rho u_{,tt}\,, \tag{5a}$$

$$c_{44}\left(w_{,rr} + r^{-1}w_{,r}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{,z}\right) + \left(c_{44} + c_{13}\right)u_{,rz} + c_{33}w_{,zz} = \rho w_{,tt}\,. \tag{5b}$$

In the inviscid fluid-solid interface, the perfect-slip boundary condition allows discontinuity in planar displacement components. That is, the radial component of displacement of the fluid and solid must be equal and the extensional components are discontinuous at the interface. The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a$

$$\left(\sigma_{rr} + p^{f}\right) = \left(\sigma_{rz}\right) = \left(u - u^{f}\right) = 0\,. \tag{6}$$

3. Solution to solid medium

The Eq. (5) is coupled partial differential equations of the three displacement components. This system of equations can be uncoupled by eliminating two of the three displacement components through two of the three equations, but this results in partial differential equations of fourth order as defined in Eq.(10). To uncouple the Eq. (5), we follow Mirsky [2] and assume the solution of Eq. (5) as follows:

$$u(r,z,t)=\left[\left(\phi\left(\bar{r},\bar{z}\right)_{,r}\right)\right]e^{i(\omega t)}, \tag{7a}$$

$$w(r,z,t)=\left(i/a\right)\left[W\left(\bar{r},\bar{z}\right)\right]e^{i(\omega t)}, \tag{7b}$$

where $i=\sqrt{-1}$, ω is the angular frequency, $\phi\left(\bar{r},\bar{z}\right)$, $W\left(\bar{r},\bar{z}\right)$, are the displacement potentials and a is the geometrical parameter of the cylindrical bar.

By introducing the dimensionless quantities such as, $\Omega^2=\rho\omega^2a^2/c_{44}$, $\bar{c}_{11}=c_{11}/c_{44}$, $\bar{r}=2r/a$, $\bar{z}=2z/H-1$, $\bar{c}_{13}=c_{13}/c_{44}$, $v_2=c_{44}/\rho$, $\bar{c}_{33}=c_{33}/c_{44}$, $T=t\sqrt{c_{44}/\rho/a}$ and $x=r/a$; and substituting Eqs. (7) in Eqs. (5), we obtain

$$\left(\bar{c}_{11}\nabla^2+\Omega^2\right)\phi-\left(1+\bar{c}_{13}\right)W=0, \tag{8a}$$

$$\left(1+c_{13}\right)\phi+\left(\nabla^2+\Omega^2\right)W=0, \tag{8b}$$

where $\nabla^2\equiv\partial^2/\partial x^2+x^{-1}\,\partial/\partial x$.

A non-trivial solution of the algebraic systems (7) exists only when the determinant of Eqs. (7) is equal to zero.

$$\begin{vmatrix}\left(\overline{c_{11}}\nabla^2+\left(\Omega^2\right)\right) & -\left(1+\overline{c_{13}}\right)\\ \left(1+\overline{c_{13}}\right) & \left(\nabla^2+\left(\Omega^2\right)\right)\end{vmatrix}\left(\phi,W\right)=0. \tag{9}$$

Eq. (9), on simplification reduces to the following differential equation:

$$\left(A\nabla^4+B\nabla^2+C\right)\phi=0, \tag{10}$$

where

$$A=\overline{c_{11}},\qquad B=\left[\left(1+\overline{c_{11}}\right)\Omega^2-\varsigma^2\left(1+\overline{c_{11}}\,\overline{c_{33}}\right)\right],\ C=\left(\Omega^2-\varsigma^2\right)\left(\Omega^2-\overline{c_{33}}\,\varsigma^2\right). \tag{11}$$

Solving the Eq. (10), the displacement amplitude functions $\phi\left(\bar{r},\bar{z}\right)$ and $w\left(\bar{r},\bar{z}\right)$ are obtained by the orthogonal series of Chebyshev polynomials multiplied by a geometry boundary function for the extensional mode as follows:

$$\phi\left(\bar{r},\bar{z}\right)=F_u\left(\bar{z}\right)\sum_{i=1}^{2}\sum_{j=1}^{2}A_{ij}P_i\left(\bar{r}\right)P_i\left(\bar{z}\right), \tag{12a}$$

$$W\left(\bar{r},\bar{z}\right)=F_w\left(\bar{z}\right)\sum_{i=1}^{2}\sum_{j=1}^{2}d_{ij}A_{ij}P_i\left(\bar{r}\right)P_j\left(\bar{z}\right). \tag{12b}$$

The constants d_{ij} defined in the Eq. (12b) can be calculated from the equation

$$d_{ij}=\left(1+\bar{c}_{13}\right)\left(\alpha_i a\right)^2\Big/\left(\left(\alpha_i a\right)^2+\Omega^2-\bar{c}_{33}^{-2}\varsigma^2\right),\ \ i=1,2, \tag{13}$$

where the boundary functions for the stress free boundary condition is taken $F_u\left(\bar{z}\right)=F_u^{-1}\left(\bar{z}\right)F_u^{1}\left(\bar{z}\right)$=1 unity, and *i, j* are the order of Chebyshev polynomial series, A_{ij} and B_{kl} are the coefficients of the polynomial. $P_s\left(\psi\right)\left(s=1,2,3,4,5;\psi=\bar{r},\bar{z}\right)$ is the Chebyshev polynomial which can be written in terms of cosine function as follows:

$$P_s\left(\psi\right)=\cos\left[\left(s-1\right)arc\cos\left(\psi\right)\right],\ s=1,2,3...,\quad \psi=\bar{r},\bar{z}. \tag{14}$$

The advantage of using Chebyshev polynomials over using other polynomial functions as trial function has been shown due to its simple and unified expression with cosine function and excellent mathematical properties in numerical approximation containing a set of orthogonal and complete series.

4. Solution of fluid medium

In cylindrical polar coordinates r, θ and z, the acoustic pressure and radial displacement equations of motion for an in viscid fluid are of the form Berliner [16]

$$p^f=-B^f\left(u_{,r}^f+r^{-1}(u^f)+w_{,z}^f\right), \tag{15}$$

and

$$c_f^{-2}u_{,tt}^f=\Delta,_r\ , \tag{16}$$

respectively, where B^f, is the adiabatic bulk modulus, ρ^f is the density, $c^f=\sqrt{B^f/\rho^f}$ is the acoustic phase velocity in the fluid, and $\left(u^f,w^f\right)$ is the displacement vector.

$$\Delta=\left(u_{,r}^f+r^{-1}(u^f)+w_{,z}^f\right). \tag{17}$$

Substituting

$$u^f=\phi_{,r}^f\ \text{ and }\ w^f=\phi_{,z}^f, \tag{18}$$

and seeking the solution of (15) in the form

$$\phi^f(r,\theta,z,t)=\left[\phi^f\left(r\right)\right]e^{i\omega t}. \tag{19}$$

The fluid represents the oscillatory waves propagating away is given by

$$\phi^f = A_{33} H_n(\delta a x), \tag{20}$$

where $(\delta a)^2 = \Omega^2 / \overline{\rho}^f \overline{B}^f - \varsigma^2$, in which $\overline{\rho}^f = \rho / \rho^f$, $\overline{B}^f = B^f / c_{44}$, H_n is the Hankel function of the first kind. If $(\delta_2 a)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n, where K_n is the modified Bessel function of the second kind. The velocity and density ration between the cylinder and fluid is defined by $c_0 = v_2 / c^f$ and $\rho_0 = \rho / \rho^f$, respectively. By substituting Eq. (18) in (15) along with (19) and (20), the acoustic pressure for the fluid can be expressed as

$$p^f = A_{33} \Omega^2 \overline{\rho}\, H_n(\delta a x) e^{i(\Omega T_a)}. \tag{21}$$

5. Frequency equations

In this section we shall derive the frequency equation for the three dimensional vibration of the solid cylindrical bar immersed in fluid subjected to perfect slip boundary conditions at $r = a$. Substituting the expressions in Eqs. (1)- (4) into Eqs. (6), we can get the frequency equation for free vibration as follows:

$$\left|A_{ij}\right| = 0, \qquad i, j = 1, 2, 3, \tag{22}$$

$$A_{11} = 2\overline{c_{66}}\left(F_u(z) \frac{d}{dr}\left(p_1(\bar{r})\right) p_1(\bar{z}) - F_u(z)\, p_1(\bar{r})\, p_1(\bar{z}) + d_{11} F_w(z)\, p_1(\bar{r}) \frac{d}{dz}\left(p_1(\bar{z})\right)\right),$$

$$A_{12} = 2\overline{c_{66}}\left(F_u(z) \frac{d}{dr}\left(p_1(\bar{r})\right) p_2(\bar{z}) - F_u(z)\, p_1(\bar{r})\, p_2(\bar{z}) + d_{12} F_w(z)\, p_1(\bar{r}) \frac{d}{dz}\left(p_2(\bar{z})\right)\right),$$

$$A_{13} = \Omega^2 \overline{\rho} (ax)^2 \left(n H_n(\delta a x) - (\delta a x) H_{n+1}(\delta a x)\right),$$

$$E_{21} = 2\mu\left(F_w(\bar{z})\, d_{21} \frac{d}{dr}\left(p_2(\bar{r})\right) p_1(\bar{z}) + F_u(\bar{z})\, p_2(\bar{r}) \frac{d}{dr}\left(p_1(\bar{z})\right)\right),$$

$$E_{22} = 2\mu\left(F_w(\bar{z})\, d_{22} \frac{d}{dr}\left(p_2(\bar{r})\right) p_2(\bar{z}) + F_u(\bar{z})\, p_2(\bar{r}) \frac{d}{dr}\left(p_2(\bar{z})\right)\right),$$

$$E_{23} = 0,$$

$$E_{31} = n J_n(\alpha_1 a x) - (\alpha_1 a x) J_{n+1}(\alpha_1 a x),$$

$$E_{32} = n J_n(\alpha_2 a x) - (\alpha_2 a x) J_{n+1}(\alpha_2 a x),$$

$$E_{33} = n H_n(\delta a x) - (\delta a x) H_{n+1}(\delta a x).$$

6. Numerical results and discussion

The coupled free wave propagation in a homogenous transversely isotropic solid cylindrical bar immersed in water is numerically solved for the Zinc material. The material properties of Zinc are given as follows and for the purpose of numerical computation the liquid is taken as water.

For the solid the elastic constants are $c_{11} = 1.628\times10^{11}\,Nm^{-2}$, $c_{12} = 0.362\times10^{11}\,Nm^{-2}$, $c_{13} = 0.508\times10^{11}\,Nm^{-2}$, $c_{33} = 0.627\times10^{11}\,Nm^{-2}$, $c_{44} = 0.385\times10^{11}\,Nm^{-2}$ and density $\rho = 7.14\times10^{3}\,kg\,m^{-3}$ and for the fluid the density $\rho^{f} = 1000\,kg\,m^{-3}$ and phase velocity $c^{f} = 1500\,ms^{-1}$.

A comparison is made for the non-dimensional frequencies in case of free and clamped edges with respect to the velocity ratio for the symmetric and anti symmetric modes of the solid bar immersed in fluid in Tables 1 and 2. From these tables, it is clear that as the sequential number of the velocity ratio increases, the non dimensional frequencies also increases for both the free and clamped solid bar. The present solutions for frequency are compared with those of Leissa and So [21] and Zhou [22] for the solid bar without fluid interaction in Table 3 with *L/a* = 2.The first five frequency parameters for the extensional vibration are considered. A good agreement has been achieved. When a solid medium such as the solid bar is surrounded by fluid medium, guided waves are transmitted across the interface. Thus bulk waves are excited in the embedding medium, radiating away from the solid medium.

Table 1. The non-dimensional frequencies for first three symmetric extensional modes of the free and clamped edges of solid bar with velocity ratio.

Velocity ratio (c_0)	Free edge						Clamped edge				
	S_1		S_2		S_3		S_1		S_2		S_3
0.1	0.0635		0.1236		0.1681		0.0259		0.1039		0.1284
0.2	0.1801		0.2636		0.2747		0.1702		0.1213		0.2530
0.4	0.2063		0.3928		0.4492		0.1950		0.3563		0.4220
0.6	0.3967		0.4727		0.5391		0.2837		0.4702		0.5295
0.8	0.5010		0.6036		0.7853		0.5029		0.5031		0.6752

Table 2. The non-dimensional frequencies for first three anti symmetric extensional modes of the free and clamped edges of solid bar with velocity ratio.

Velocity ratio (c_0)	Free edge						Clamped edge				
	S_1		S_2		S_3		S_1		S_2		S_3
0.1	0.0249		0.0865		0.1572		0.0215		0.0342		0.1408
0.2	0.0651		0.2219		0.2345		0.0554		0.1969		0.2205
0.4	0.1774		0.4977		0.5337		0.1444		0.3888		0.4473
0.6	0.1994		0.5385		0.7292		0.2487		0.5023		0.5941
0.8	0.3964		0.6952		0.9408		0.5504		0.6050		0.7303

Dispersion curves. The results of extensional (symmetric and anti symmetric) modes of vibrations are plotted in the following figures with respect to the parameters aspect ratio a/b and L/a. The notations ES1, ES2 and EAS1, EAS2 denote the extensional symmetric and anti-symmetric mode respectively, “1” and “2” refer to the first and the second modes.

Table 3. Comparison of the non-dimensional frequencies for extensional modes of a solid bar with those of Leissa and So [21] and Zhou [22] with $L/a = 2$.

Mode	Ref. [21]		Ref. [22]		Present
1	1.2860		1.2851		1.2790
2	2.9601		2.9600		2.9734
3	3.1686		3.1675		3.1671
4	4.1815		4.1817		4.1814
5	4.2970		4.2947		4.2958

The variation of frequencies with the thickness to length ratio (h/L) is discussed in Fig. 1 and Fig. 2 for both symmetric and anti symmetric modes of the solid bar immersed in fluid for first two modes of vibration. In Fig.1 the frequency is increases monotonically in all the ranges of thickness to length ratio for symmetric modes of vibration. For the case of anti symmetric modes there is a small deviation on the frequency pattern in Fig. 2 due to the damping effect of fluid medium and the mechanical properties of the material. The energy transmission occurs only on the surface of the solid bar because the bar acts as the semi-infinite medium. In general, from the Figs. 1 and 2 it is observed that the non-dimensional frequency of the fundamental mode is non dispersive and increases rapidly in the presence of liquid with increasing thickness to length ratio.

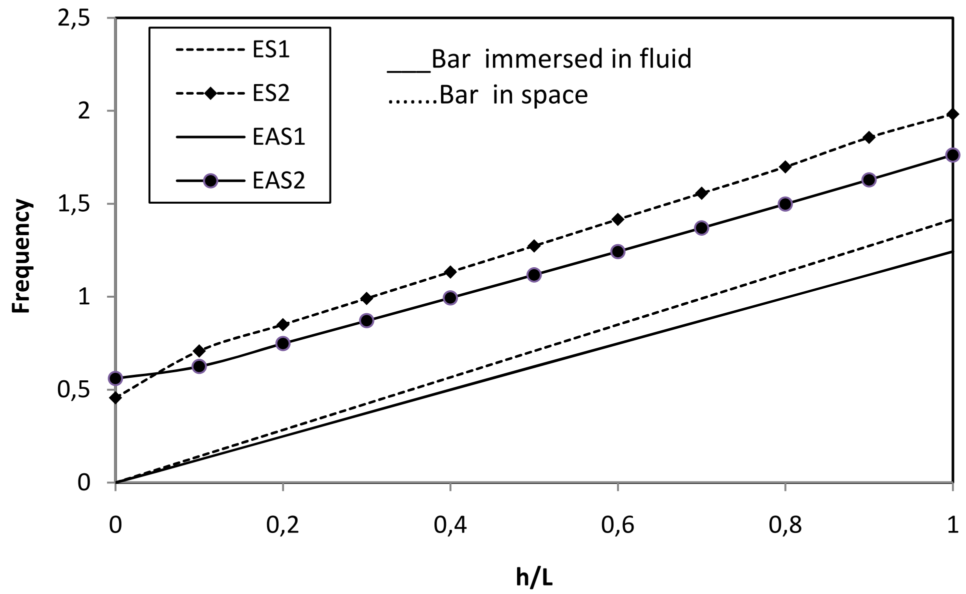

Fig. 1. Variation of frequency with *h/L* for symmetric mode of Zinc solid bar.

In Fig. 3, the variation of frequency with respect to the length to radius ratio L/a of the solid bar immersed in fluid is presented for symmetric modes of vibration. The magnitude of the frequency decreases monotonically in the range $0 \le L/a \le 3$ for first two modes of the solid cylinder immersed in fluid, become asymptotically linear in the remaining range of L/a ratios. The variation of frequency with respect to the aspect ratio for first two anti symmetric modes of the solid bar is presented in Fig. 4, where the non dimensional frequency slashes down in $0 \le L/a \le 3$ with a small oscillation in the starting L/a ratios, and become linear due to the fluid medium in the rest. From Figs. 3 and 4, it is clear that the effects of length to radius ratio of the solid bar are quite pertinent due to the combined effect of mechanical property and damping effect of the fluid medium. When the ratio of the densities of the fluid and elastic material is small (0.14), then the mode spectrum of fluid loaded bar is slightly different from that of free bar.

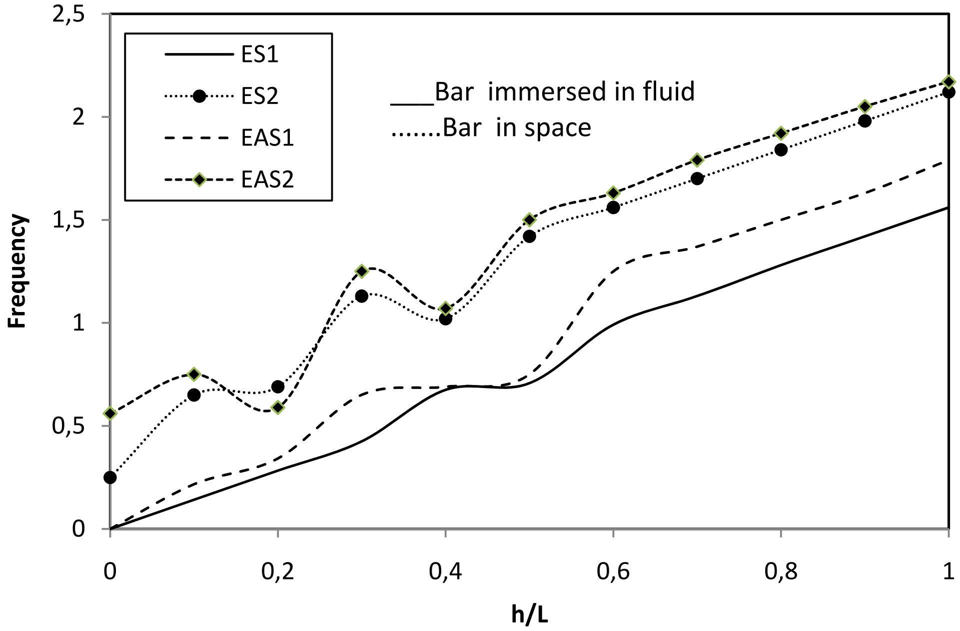

Fig. 2. Variation of frequency with *h/L* for anti symmetric mode of Zinc solid bar.

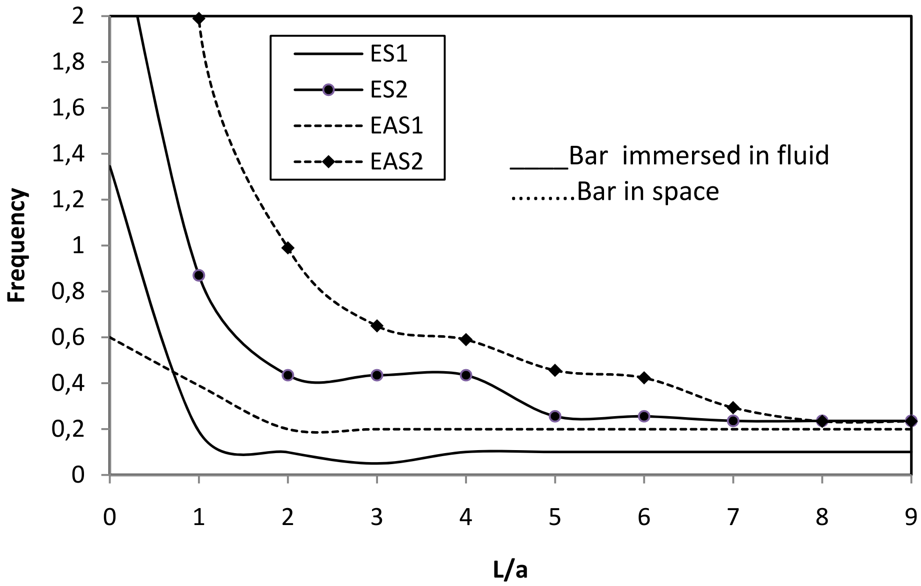

Fig. 3. Variation of frequency with *L/a* for symmetric mode of Zinc solid bar.

7. Conclusions

In this paper, the extensional vibration in a finite, homogeneous transversely isotropic solid bar immersed in a in viscid fluid is studied using the linearized, three-dimensional theory of elasticity**.** Two displacement potential functions are introduced to uncouple the equations of motion. In the present analysis, a set of Chebyshev polynomials multiplied by a boundary function which satisfies the geometric boundary conditions of the cylinder are taken as the trial functions. The computed non-dimensional frequencies are presented in the form of dispersion curves for the material Zinc. In addition, a comparative study is made to prove the feasibility of the model with exiting literature and they show good agreement.

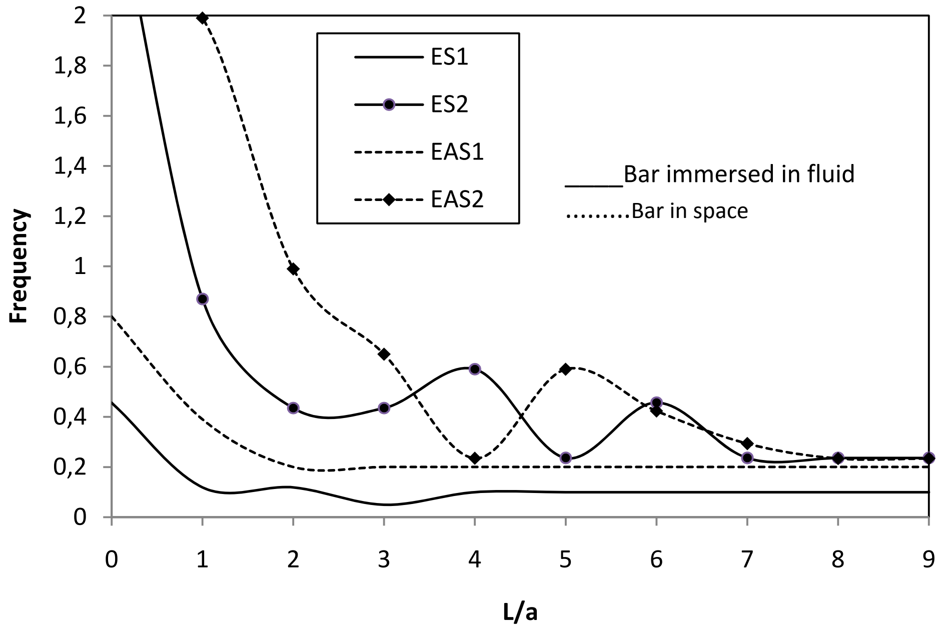

Fig. 4. Variation of frequency with *L/a* for anti symmetric mode of Zinc solid bar.

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