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3D FLUID STRUCTURE INTERACTION PROBLEM SOLVING METHOD IN EULER VARIABLES BASED ON THE MODIFIED GODUNOV SCHEME

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Abstract. We present an explicit numerical method for three-dimensional modeling of fast processes of fluid structure interaction problems in Euler variables. The method does not require complex spatial grid generation. To set the initial geometry and follow the deformation of the calculating domains in the process of interaction it is enough to take into account the interacting surfaces constituted by a set of triangles created by CAD systems. The method is based on the modified Godunov scheme with the increased accuracy and uniform for solving equations of fluid dynamics and elastic-plastic flows. Fixed Cartesian grid and local mobile grids associated with each triangle of the surface are used. The flow parameters are interpolated from the Cartesian grid to the local grids and vice versa. At the fluid structure interaction boundary the exact Riemann solver is used.

1. Introduction

The solution of many problems of the interaction of gases, liquids, and groundwater environments with the elastic-plastic structures (Fluid Structure Interaction - FSI) is associated with the significant deforming of the contacting domains and requires the allocation and maintenance of the contact and free boundaries. In some cases, we also have to treat other difficulties like shock waves, phase transitions and so on. The most effective method for solving of such kind of two-dimensional planar and axisymmetric problems is the Godunov scheme and its modifications [1-3]. They use the Lagrangian and Eulerian variables with the continuous curvilinear grid fitted with the body interface (Arbitrary Lagrange Euler - ALE). This approach proved to be practically useless for solving the spatial problems of FSI. In the 3D case in the ALE technology, there are fundamental difficulties associated with the accuracy of calculation of the integrated areas and volumes, which leads to significant errors, especially for the problems of the dynamics of deformable solids (Computational Structure Dynamics - CSD). For the 3D FSI problems the approaches [4, 5] are more perspective, where a fixed Eulerian grid is used to resolve the contact discontinuities within computational cells. It is the extension of Immersed Boundary Method (IBM) in Computational Fluid Dynamics (CFD). Godunov scheme of high accuracy in a compact $3\times3\times3$ stencil [2] and uniform for CFD and CSD, with the exact uniform Riemann solver (Riemann's Problem - RP) of a discontinuity of elastic medium-elastic medium, gas-gas, gas-elastic medium, permits to extend the IBM technology on the 3D FSI problem.

In this paper, the IBM approach based on the use of the three types of grid is presented. The first kind of grid is in the form of a set of triangles (STL files) that define the surface of the interacting domains, second - fixed Cartesian grid and third - local mobile grids that are coupled with the surface of each triangle of the first grid.

2. Governing system of equations for Cartesian coordinates and the solution method

A closed system of equations describing the deformation of a continuous medium approximation model of a compressible elastic-plastic body in a Cartesian coordinate system is as follows [2]:

$$\rho_{,t} + \left(\rho u_i\right)_{,x_i} = 0 \tag{1}$$

$$\left(\rho u_i\right)_{,t} + \left(\rho u_i u_j - \sigma_{ij}\right)_{,x_j} = 0 \tag{2}$$

$$e_{,t} + \left(e u_j - u_i \sigma_{ij}\right)_{,x_j} = 0 \tag{3}$$

$$\mathrm{DS_{ij}/Dt} + \lambda_t \mathrm{S_{ij}} = 2\mu e_{ij} \tag{4}$$

$$\varepsilon = \varepsilon(\mathrm{p}, \rho), \tag{5}$$

where t - time, $x_{i\ (i=1,2,3)}$ - space coordinates, u_i - velocity component for x_i coordinate, ρ - density, $e = \rho(\varepsilon + u_i u_i / 2)$ - energy per unit of volume, ε -specific energy per unit of mass from state eq. (2.5), σ_{ij} - stress tensor, which is divided into pressure $p = -1/3\sigma_{ii}$ and deviatoric stress parts $\sigma_{ij} = -p\delta_{ij} + S_{ij}$, e_{ij}- deviator of the tensor of the velocity of deformation tensor, $e_{ij} = \varepsilon_{ij} - 1/3\varepsilon_{kk}\delta_{ij}$, where $\varepsilon_{ij} = 1/2(u_{i,j} + u_{j,i})$. Sign D/Dt, used for the Jaumann derivative, allows for the rotation of stress tensor in Euler variables. $DS_{ij}/Dt = S_{ij,t} + u_k \partial S_{ij}/\partial x_k - S_{ik}\omega_{jk} - S_{jk}\omega_{ik}$ $\omega_{ij} = 1/2(u_{i,j} - u_{j,i})$, μ is the shear modulus. To describe the behavior of the plastic material the splitting method proposed by Kukudzhanov [6] is used. In the absence of shear stress system (1)-(5) obviously goes to the Euler equations for the motion of a compressible fluid (gas). The system of equations (1)-(5) is closed by equation of state with the appropriate parameters. A modified Godunov method [2], which provides a second order accuracy approximation on a compact $3\times3\times3$ stencil for CFD and CSD, with the exact RP solver for FSI, is used to solve (1)-(5) numerically.

The method presented is based on the solution of the following model problem. Let us consider the local cubic stencil $3\times3\times3$, adjacent to the boundary of the domain from the inside. Let us build it out of the domain, if there is a contact with the other domain up to $3\times3\times6$ stencil (see Fig. 1 left), a bold line denotes the contact boundary, consider it is flat. This stencil is enough for integrating the four central cells (marked with dots in Fig. 1) with the second order accuracy with this modified Godunov scheme or only for the two center cells adjacent to the boundary in the absence of contact (only boundary conditions). Let us solve the RP on the contact boundary, then move the interface with the normal velocity for only two integrating cells, get new mesh (Fig. 1 right). Then let us integrate four (two if boundary) cells for the new mesh. It is the ALE integration.

Method of calculation of the 3D FSI problem based on this model consists of a sequence of the following steps:

1. The calculating domains are defined as the sets of spatial surfaces of the triangles with the required accuracy (STL files). Figure 2a shows an example of such surface.

2. Each computational domain with curved boundaries is bordered by appropriate parallelepiped and covered with a regular Cartesian grid. Only the Cartesian grid cell size is specified. For example in Fig. 2b the cylinder defined by a set of triangles (STL file format) is inside the appropriate box. Four types of cubic cells for the computational domain are obtained, 1 - cells, cut by triangles of the domain’s surface (boundary cells), 2 - cell outside surface, 3 - cells inside the surface with the stencil which is enough for integrating them with

a predetermined approximation error, 4 - cells inside the surface without enough stencil to integrate them. The cells of the third type are integrated on the fixed Cartesian grid.

3. On the surface of each triangle the local Cartesian grid of $3\times3\times3$ inside this surface parallel to this triangle is constructed, the cell size of this local 3D grid is taken close to the Cartesian one. In the case of the contact of this triangle with another triangle of any domain this local grid is extended outside (into the contacting domain) on additional $3\times3\times3$ cells (Fig. 1). From the main grid cell parameters are interpolated to these local grids. For each local grid, the model problem is solved and a new time step parameters near the boundaries are obtained (boundary layer).

4. Using the velocity at the center of each triangle derived from the RP in step 3 calculate the velocity at the vertices of triangles STL file and move the vertices to the new time step position. Obtain the position of the surface (STL file) at the new time step.

5. On the new position of the domain surface the cells of type 4 are obtained. The parameters in these cells are obtained by interpolating appropriate parameters from the third type cells (obtained on the step 2) and from the boundary layer cells (obtained on the step 3).

6. Next, make a basic restructuring of the parallelepiped in accordance with the new surface of the domain (add or minus of the necessary number of cells, forcing the domain to be bounded by this new parallelepiped), thus completing the integrating step.

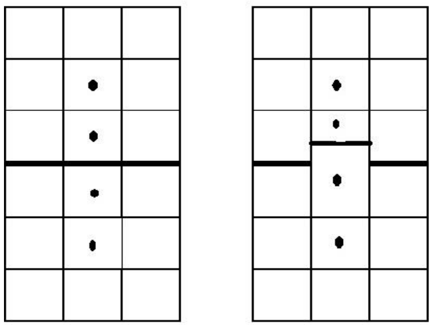

Fig. 1. Local mesh before and after moving boundary.

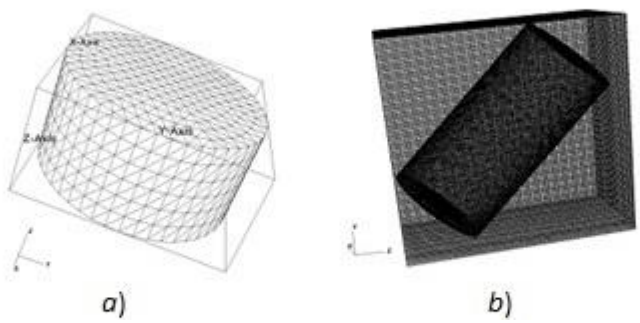

Fig. 2. Calculation domain defined by STL file and by appropriate box of a regular Cartesian grid.

3. Results of calculations

The results of calculation of standard and applied problems demonstrate the advantages of the proposed method. In Fig. 3 the results of 3D modeling of explosive loading of a closed rigid cylindrical shell with the 2D codes results are compared (axisymmetric formulation of the 3D problem).

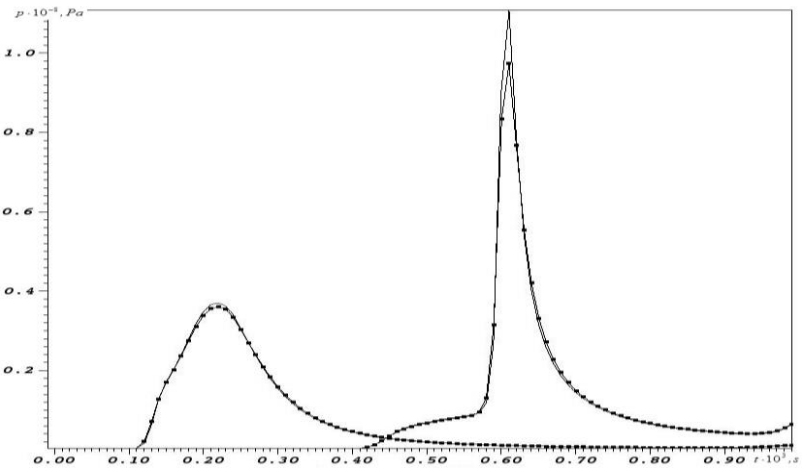

Fig. 3. Comparison of the pressure in time on the wall for 2D and 3D calculations.

The curves with dependence of pressure from time on the cylinder wall in the plane of symmetry (left side of Fig. 3) and on the top of the cylinder in the axis of symmetry (right) are presented. The results are similar and confirm the accuracy of the 3D calculations. The surface of the cylinder shell was approximated by STL file consisting of a set of 10^5 triangles with sides of 0.02 to 0.04 m. The main grid consists of 432,000 ($60 \times 60 \times 120$) cubic cells with side of 0.03 m. The diameter of a spherical charge is 0.2 m. The charge center was in the plane of symmetry of the cylinder on the axis of symmetry (axisymmetric problem). The initial values in the detonation products are: pressure - 10^4 MPa, the density - 1600 kg /m^3, the velocity is 0, the effective adiabatic coefficient 3.0; in ambient air: pressure - 0.1 MPa, the density - 1.28 kg /m^3, the adiabatic coefficient is taken as 3.0.

To demonstrate the capabilities of the methodology the following model problem also was considered. In the cylindrical envelope of diameter D and height H = 2D, filled with air, comprising two fixed rigid body of cubic shape, two identical cylindrical solid explosive charge diameter of D/8, and a height of D/4, the axes of which lie in one plane with the cylinder axis are initiated simultaneously. In Fig. 4a sectional view of the cylinder and the charges in plane Y = 0 are presented.

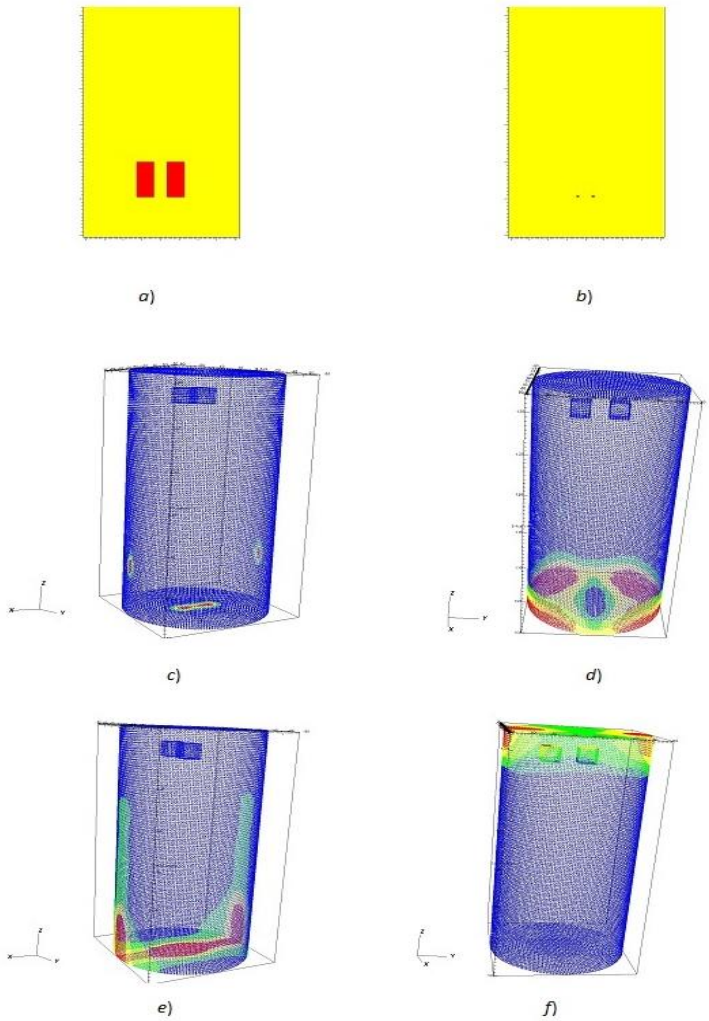

Fig. 4. Detonation of two cylindrical solid charges in the rigid cylinder with the two solid cubical bodies inside. Evolution of the pressure wave in time on the surfaces.

The area occupied by the charges BB is marked in red. In Fig. 4b the points of detonation initiation are indicated in red. The charge density is 1600 kg/ m^3, the detonation velocity 7800 m/c, pressure, and density of the ambient air, respectively 0.1 MPa and $1.28\ \mathrm{kg/m^3}$, the adiabatic coefficient for the product of the detonation and air $\chi = 1{,}25 + 1{,}13 \cdot (\rho / \rho_{BB})$, where ρ_{BB} - the density of the explosive. The size of triangles of the STL surfaces and basic Cartesian grid is about 0.01D. In Figs. 4c,d,e,f are respectively the reflected shock wave from the bottom and side of the walls of the shell, spread of the pressure wave along the cylinder, the interaction of the shock waves with the top of the cylinder and rigid cubic bodies.

4. Conclusions

The numerical studies demonstrate the applicability of the proposed method for the calculating the problems of interaction of gases and liquids with elastoplastic medium. The main advantage of this method for calculations of three-dimensional FSI problems is that it does not need complex 3D mesh generators. Actually, the construction of the spatial grid is excluded not only of the preparation phase of the initial data for the numerical solution, but also from the process of meshing the deformable domains.

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