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ENERGY ESTIMATIONS OF PHASE TRANSFORMATIONS UNDER THE ACTION OF A SPHERICALLY CONVERGING COMPRESSION WAVE

I.A. Brigadnov, A. B. Freidin, D.A. Indeitzev, N.F. Morozov and Yu.V. Petrov

Institute of Problems of Mechanical Engineering, Russian Academy of Sciences,
Bolshoj 61, Vas. Ostrov, St.Petersburg, 199178, Russia

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Abstract. A model for description of the deformation processes initiated by phase transformations in a ball subjected to the action of a spherically converging compression wave of high power is proposed. Explanation for the effect of cavity origination in the center of a ball is given.

1. FORMULATION OF THE PROBLEM AND BASIC HYPOTHESES

A continuous homogeneous ball of radius R made of isotropic material is subjected to the spherically symmetric radial-compressing action applied to the surface [1].

The equilibrium equation of continuous medium is of the form: $\rho\ddot{\boldsymbol{u}} - \nabla\cdot\boldsymbol{\sigma} = 0$, where ∇ is the differential Hamilton's operator, $\boldsymbol{\sigma}$ is the Cauchy (true) stress tensor; ρ is the density of material, and $\boldsymbol{u}(t,\boldsymbol{r})$ is the displacement vector, $\boldsymbol{r}$ is the point of a ball, the upper dots denote the partial derivative with respect to time t.

At the initial moment the ball is at rest, i.e. $\boldsymbol{u}(0,\boldsymbol{r}) = \dot{\boldsymbol{u}}(0,\boldsymbol{r}) = 0$. At $t>0$ the ball surface is subjected to the normal spherically symmetric action $\sigma_r|_{r=R} = F(t)$, where r is the radial coordinate, and $F(t)$ is the law of transformation of impact pulse in time. The displacement in the ball center is left bounded. Under the high power impact action and with allowance for phase transitions, the material is described by an unknown tensor constitutive relation, $\boldsymbol{\sigma} = \boldsymbol{S}\{\nabla\boldsymbol{u}\}$, where braces point to the operational dependence which has to include a possibility of phase transformations. We do not concretize the dependence and further we are going to develope an approximate energetic approach. The approach is based on the approximation of the true wave process by basic functions, which are defined from the solution of the preliminary simplified initial boundary-value problem by the following hypotheses:

1) All the values are solely radius dependent: i.e. the displacement is radial and up to the initiation of the discontinuity the Cauchy stress tensor is spherical: $\boldsymbol{\sigma} = -p\,\boldsymbol{I}$, where p is the hydrostatic pressure and $\boldsymbol{I}$ is the unit tensor.
2) By virtue of a weak compressibility of iron, the ball deformation is described by the linear strain tensore $\boldsymbol{\varepsilon}$.
3) Material is described by the Hooke linear model, $p = -k_0\mathrm{tr}(\varepsilon)$, where k_0 is the bulk modulus.

As a result, the basic functions are found from the solution of the classical problem on the acoustical wave propagation in a ball. Depending on the retention of a variety of terms in the expansion of wave solution, different models of real physical processes can be constituted, such as: 1) the retention of the first term (the compression wave) only permits to elucidate the occurrence of a cavity in the ball center due to the formation of a gaseous nucleus surrounded with a liquid layer and a solid crust thereupon, 2) the retention of the second term (the rarefaction wave) allows to explain the origination of the cavity due to the cavitation in the center of a liquid nucleus formed at melting under the action of

Corresponding author: A.B. Freidin, e-mail: freidin@abf.ipme.ru

the first compression wave, 3) the retention of the following terms is worth-while for sufficiently continuous pulse loading. The adequacy of different models can be confirmed from experiment.

2. ESTIMATION OF THE LIQUID LAYER SIZES AND SUBLIMATION NUCLEUS

Within the framework of the first model, the processes arisen in the course of the first compression wave running are considered. In this case

$$p(t,r) = \frac{R}{r} F\left(t - \frac{R-r}{c}\right), \quad t \in \left(0, \frac{R}{c}\right),$$

where c is the rate of front motion [2]. At this moment within the central domain of the ball a transition of the potential strain energy into the internal energy is assumed to take place and as a result melting and sublimation can arise.

For the processes of melting and sublimation of mass m, the energies $Q^{(i)}=mc_p^{(i)}$ (i=1,2) are needed, where $c_p^{(1)}$ and $c_p^{(2)}$ are the specific energies of melting and sublimation, respectively. Therefore, within the framework of accepted hypotheses, the radius of the sublimation nucleus, $r_a = aR$, and outer radius of the liquid layer, $r_b = bR$ are found by solution of the following nonlinear equations

$$\int_0^a F^2\left(\frac{Rx}{c}\right)dx = \frac{3}{2}\rho k_0 c_p^{(2)} a^3,$$
$$\int_a^b F^2\left(\frac{Rx}{c}\right)dx = \frac{3}{2}\rho k_0 c_p^{(1)} (b^3 - a^3). \qquad (1)$$

According to [1], function $F(t)$ is a fast decreasing function. Therefore, it is obviously that the system of equations (1) has always a real pair of roots $0 \le a \le b$. The following cases are realized in accordance with the location of roots, namely: 1) $a=b=0$, *i.e.* the whole ball remains solid, 2) $0=a<b<1$, *i.e.* a liquid nucleus surrounded with a solid crust is formed, 3) $0<a<b<1$, *i.e.* a gaseous nucleus surrounded with a liquid layer and a solid crust thereupon is formed, 4) $0<a<1\le b$, *i.e.* a gaseous nucleus surrounded with a liquid layer is formed, 5) $a\ge 1$, *i.e.* the ball completely transforms into gaseous state, *i.e.* it is evaporated. Note that the system (1) has always the trivial solution $a=b=0$, and the two last variants lead to the complete destruction of the ball.

3. PROCESS IN THE SOLID CRUST AFTER PHASE TRANSITIONS.

The sublimate pressure p_a is assumed to be equalized along the radius practically momentary. The pressure can be estimated by the condition of equality for the sublimation energy, $Q^{(2)}=\rho V_a c_p^{(2)}$, and the potential energy of compressed gas $p_a V_a$, where V_a is the volume of a sublimate nucleus. The pressure at the inner surface of solid crust is given by the law of hydraulics. Consequently, the following estimations are true

$$p_a = \rho c_p^{(2)},$$
$$p_b = \eta^2 c_p^{(2)} \quad (\eta = a/b). \qquad (2)$$

The solid crust starts to blow up under the action of the applied pressure. In this case the stress tensor is not spherical, which can lead to an irreversible shear deformation. Simultaneously, within the ball the processes of sublimate condensation on the inner surface of liquid layer and growing the solid crust due to the recrystallization of liquid layer are carried out. As a result, the pressure under the growing crust drops and the deformation is stopped. After cooling, the outer radius of the ball grows up to $R_* = R + u_r(R)$.

Irreversible deformation of the solid crust is estimated by the solution of the problem on blowing up of the spherical layer under the internal pressure for material relation of the rigid-plastic type [3]

$$\boldsymbol{\varepsilon}^D = \frac{\sigma_s}{2\mu\alpha} H\left(\frac{|\sigma^D|}{\sigma_s} - 1\right)\left(\frac{|\sigma^D|}{\sigma_s} - 1\right)\frac{\boldsymbol{\sigma}^D}{|\sigma^D|}, \qquad (3)$$

where $\boldsymbol{\varepsilon}^D$ and $\boldsymbol{\sigma}^D$ are the deviatoric parts of the strain and stress tensors, respectively, μ is the shear modulus; $\alpha > 0$ and $\sigma_s > 0$ are the dynamic parameters of strain hardening and yield stress, respectively, $H(x)$ is the Heaviside function.

From the plastic incompressibility condition, $\mathrm{tr}(\boldsymbol{\varepsilon}) = u_r' + 2u_r/r = 0$, the radial displacement $u_r(r) = C/r^2$ is derived, where the constant C is determined from the boundary condition on the internal surface of the crust $\sigma_{rr}(r_b) = -p_b$.

Thus, for a relative increase of the outer solid crust radius the following estimation holds true

$$\delta_R = \frac{u_r(R)}{R} = \frac{b^3\sigma_s}{2\sqrt{6}\mu a} H(s-1)(s-1), \qquad (4)$$

where

$$s = \max\left\{\frac{|\sigma^D|}{\sigma_s}\right\} = \left(\frac{3}{2}\right)^{1/2} \eta^2 \frac{p_a}{\sigma_s}$$

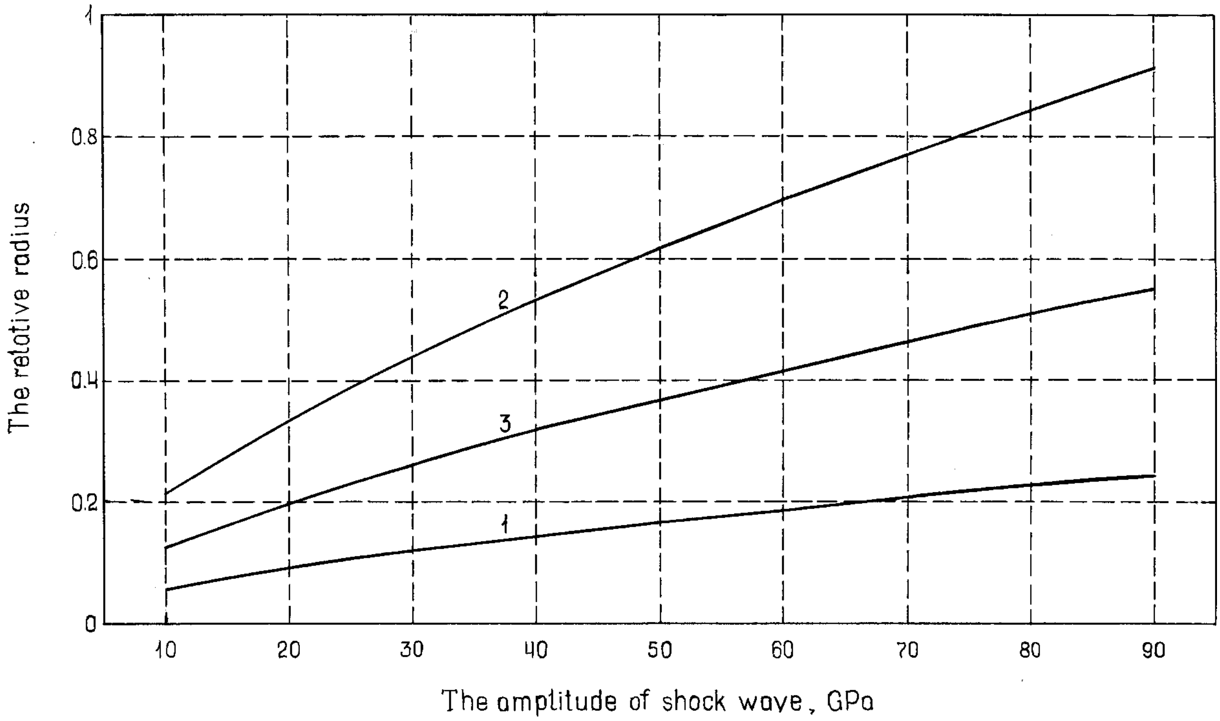

Fig.1. The relative values of the sublimation nucleus radius *a* (1), the outer liquid layer radius *b* (2) and the spherical cavity radius δ_* under the assumption that $\rho_* = \rho$ (3) *vs* shock wave amplitudes F_0.

is the relative intensity of tangential stresses on the inner surface of the solid crust. For $s \le 1$, the outer ball radius does not grow, since δ_R=0. It is easily to verify that within the framework of the model considered a cavity can not be formed at $\eta \le \eta_* = (2/3)^{1/4} (\sigma_s/p_a)^{1/2}$. In this case, it is necessary to examine the second model.

The relative radius of a spherical cavity is found using the law of mass conservation

$$\delta_* = \frac{r_*}{R} = \left[(1+\delta_R)^3 - 1 - \left(\frac{\rho}{\rho_*} - 1 \right) b^3 \right]^{1/3}, \qquad (5)$$

where ρ_* is the density of the recrystallization layer.

4. NUMERICAL RESULTS.

An example of analysis of the derived relations was performed for the iron ball of radius R = 0.05 m with the following parameters [4]: $k_0 \approx 108$, $\mu \approx 84$ and $\sigma_s \approx 1$ GPa; $\alpha \approx 0.1$ relative units, $\rho \approx 7.6\cdot10^3$ kg/m^3, $c_p^{(1)} \approx 1.3\cdot10^4$ and $c_p^{(2)} \approx 3.5\cdot10^5$ J/mol, molar weight of iron $m_0 \approx 6\cdot10^{-2}$ kg/mol.

The simplest approximation of the impact effect $F(t) = F_0 e^{-t/\lambda}$ with the parameter λ=0.15 μs was considered [1].

Relations of the relative values of the sublimation nucleus radius *a* (Curve 1), the outer liquid layer radius *b* (Curve 2) and the spherical cavity radius δ_* under the assumption that $\rho_* = \rho$ (Curve 3), for different amplitudes F_0, are presented in Fig. 1.

For the typical amplitude F_0=40 GPa [1], the relative values of the sublimation nucleus and inner liquid layer radius $a \approx 0.14$ and $b \approx 0.53$ correspond to the third from the above mentioned states of the ball. In this case the sublimate pressure $p_a \approx 44.3$ GPa and the relative increase in the ball outer radius $\delta_R \approx 0.01$, which, with the assumption that $\rho_* = \rho$, give rise to the following estimate value of the relative radius of the spherical cavity: $\delta_* \approx 0.32$. This value is followed, that is in a good agreement with experimental data [1].

Recrystallization of the liquid phase proceeds at the pressure p_a and temperature T_a which can be estimated by the law for ideal gas:

$${}_a = p_a \frac{3m_0}{2\rho R_0} = \frac{3c_p^{(2)}}{2R_0} \approx 3.75\cdot10^3\,K,$$

where R_0 = 8.31 J/(degree·mol) is the universal gas constant.

The presented model can be refined due to: 1) a more accurate estimation of the potential compression energy at the moment of focusing, 2) taking into account the temperature variation during the adiabatic compression and also the temperature and pressure influences upon the specific melting and sublimation heats, 3) a more accurate estimation of the sublimate pressure, 4) taking into account the sublimate condensation and melt recrystallization in the course of blowing up of the solid crust that is resolved into solution of the connected Stephan's problem and visco-plastic flow; 5) the consideration of the phase composition of recrystallization layer.

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