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NUMERICAL METHODS FOR CALCULATING THE STRENGTH AND STABILITY OF STIFFENED ORTHOTROPIC SHELLS

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Abstract. The article discusses coatings of building structures in the form of shells, made of advanced composite materials. A mathematical shell model takes into account the geometric nonlinearity, orthotropy of material, transverse shifts, presence of reinforcement ribs. The study algorithm of this model is based on the minimization of the functional of the total potential energy of shell deformation and the linearization of the problem through best parameter continuation. Based on the software product developed, a comprehensive study of the strength and stability of various shell structures has been conducted. The efficiency of the use of orthotropic composite materials compared to traditional isotropic ones is shown.

1. Introduction

To cover large-span building structures, different types of shells of revolution are used. In the area of building construction, the composite materials have not yet received proper application, although they are promising [1]. One reason for this is insufficient study of the strength and stability of constructions of such materials. Since the reinforcement elements in the material are often placed along the coordinate lines of the shell, such structures can be considered as orthotropic.

All works relating to the stability of shells mostly consider the shells of isotropic materials [2], while the postbuckling behavior of shells and the relationship between stability and strength are not investigated. In addition, there are no publications about ribbed shells made of orthotropic materials.

2. Mathematical model

In this paper, we used a mathematical model of deformation of shell structures, that holistically takes into account the geometric nonlinearity, transverse shifts, orthotropy of material, inclusion of ribs by the method of structural anisotropy with their shear and torsional rigidity [3].

A mathematical model represents a functional of total potential energy of deformation of the shell, which can be written in the dimensionless parameters as follows [4]:

$$\overline{E}_p = \int_{\overline{a}_1}^{1}\int_0^1 \Big\{ (1+\overline{F}_x)\overline{\varepsilon}_x^2 + \overline{G}_2(1+\overline{F}_y)\overline{\varepsilon}_y^2 + \mu_{21}(2+\overline{F}_x+\overline{F}_y)\overline{\varepsilon}_x\overline{\varepsilon}_y + \frac{1}{2}\overline{G}_{12}(2+\overline{F}_x+\overline{F}_y)\overline{\lambda}^2\overline{\gamma}_{xy}^2 +$$

$$+\overline{G}_{13}k\overline{A}^2(1+\overline{F}_x)(\overline{\Psi}_x-\overline{\theta}_1)^2 + \overline{G}_{23}k\overline{A}^2\overline{\lambda}^2(1+\overline{F}_y)(\overline{\Psi}_y-\overline{\theta}_2)^2 + 2\overline{S}_x\overline{\varepsilon}_x\overline{\chi}_1 + \mu_{21}(\overline{S}_x+\overline{S}_y)\overline{\varepsilon}_x\overline{\chi}_2 +$$

$$+\mu_{21}(\overline{S}_x+\overline{S}_y)\overline{\varepsilon}_y\overline{\chi}_1 + 2\overline{G}_2\overline{S}_y\overline{\varepsilon}_y\overline{\chi}_2 + 2\overline{G}_{12}(\overline{S}_x+\overline{S}_y)\overline{\lambda}^2\overline{\gamma}_{xy}\overline{\chi}_{12} + \left(\frac{1}{12}+\overline{J}_x\right)\overline{\chi}_1^2 +$$

$$+\left(\frac{1}{12}+\bar{J}_x\right)\bar{\chi}_1^2+\bar{G}_2\left(\frac{1}{12}+\bar{J}_y\right)\bar{\chi}_2^2+\mu_{21}\left(\frac{1}{6}+\bar{J}_x+\bar{J}_y\right)\bar{\chi}_1\bar{\chi}_2+2\bar{G}_{12}\left(\frac{1}{6}+\bar{J}_x+\bar{J}_y\right)\bar{\lambda}^2\bar{\chi}_{12}^2-$$

$$\left.-2(1-\mu_{12}\mu_{21})\bar{P}\bar{W}\right\}\bar{A}\bar{B}d\xi d\eta. \qquad (1)$$

Here $\bar{\varepsilon}_x, \bar{\varepsilon}_y, \bar{\gamma}_{xy}$ are the dimensionless expressions of strain, taking into account the geometric nonlinearity; $\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_{12}$ are the dimensionless functions of curvature change and torsion change; $\bar{F}_x, \bar{S}_x, \bar{J}_x, \bar{F}_y, \bar{S}_y, \bar{J}_y$ are the dimensionless values of stiffness properties of the ribs.

3. Algorithm of research and software implementation

The algorithm of study of this mathematical model is based on the application of several numerical methods, i.e. the Ritz method for the conversion of a variational problem to systems of nonlinear algebraic equations; method of the best parameter continuation, when the continuation of arc length of the curve of equilibrium states is taken as a parameter, which allows to convert the solution of a system of nonlinear algebraic equations to the successive solution of systems of linear algebraic equations [5]; the Gauss method for solution of systems of linear equations; the Simpson method for definite integration.

Ritz method. In order to minimize the functional of the total potential energy of shell deformation, the Ritz method is used. To solve the problem in the dimensionless parameters, the required displacement functions and functions of rotation angles of the normal can be represented as follows:

$$\bar{U}(\xi,\eta)=\sum_{I=1}^{N}\bar{U}(I)\bar{Z}1(I);\ \bar{V}(\xi,\eta)=\sum_{I=1}^{N}\bar{V}(I)\bar{Z}2(I);\ \bar{W}(\xi,\eta)=\sum_{I=1}^{N}\bar{W}(I)\bar{Z}3(I); \qquad (2)$$

$$\bar{\Psi}_x(\xi,\eta)=\sum_{I=1}^{N}\overline{PS}(I)\bar{Z}4(I);\ \bar{\Psi}_y(\xi,\eta)=\sum_{I=1}^{N}\overline{PN}(I)\bar{Z}5(I),$$

here $\bar{U}(I), \bar{V}(I), \bar{W}(I), \overline{PS}(I), \overline{PN}(I)$ are unknown dimensionless numerical coefficients, $\bar{Z}1(I)-\bar{Z}5(I)$ are known approximating functions of arguments ξ and η, satisfying the given boundary conditions on the shell contour, N is the number of expansion terms.

Substituting the expansions of required functions (2) in the functional (1), and conducting the procedure of the Ritz method, we obtain a system of nonlinear algebraic equations, which can be written briefly in vector form $F(X,\bar{P})=\mathbf{0}$, where $X=\left(\bar{U}(I),\bar{V}(I),\bar{W}(I),\overline{PS}(I),\overline{PN}(I)\right)^T, I=1..N$ is the vector of unknown numerical parameters, $\bar{P}$ is the load.

The resulting system will be solved through the method of the continuation by best parameter [5].

Method of the best parameter continuation. As the best parameter for continuation of the solution, it is proposed to take the length of the arc of load trajectory λ in the state space. Unlike the conventional method of parameter continuation, here load parameter $\bar{P}$ is equivalent to the other unknown parameters, and it is convenient to add it to the other parameters:

$$\tilde{X}=\left(X,\bar{P}\right)^T=\left(\bar{U}(I),\bar{V}(I),\bar{W}(I),\overline{PS}(I),\overline{PN}(I),\bar{P}\right)^T, I=1..N.$$

Parameter λ does not appear explicitly in the equations and is associated with the variables of the vector $\tilde{X}$ problem as follows:

$$(d\lambda)^2 = \sum_{I=1}^{N}\left[\left(d\overline{U}(I)\right)^2 + \left(d\overline{V}(I)\right)^2 + \left(d\overline{W}(I)\right)^2 + \left(d\overline{PS}(I)\right)^2 + \left(d\overline{PN}(I)\right)^2\right] + \left(d\overline{P}\right)^2.$$

Differentiating the system of equations by parameter λ, and assuming that the variables of the problem depend on it, we obtain the system of differential equations $\bar{J}\,\frac{d\tilde{X}}{d\lambda} = \mathbf{0}$, with initial condition $\tilde{X}(\lambda_0) = \mathbf{0}$, $\lambda_0 = 0$. Here $\bar{J} = \frac{\partial F(\tilde{X})}{\partial \tilde{X}}$ is the extended Jacobi matrix having $5N$ rows and $(5N+1)$ columns.

Thus, instead of solving a system of nonlinear algebraic equations, at each step of the parameter continuation a system of linear algebraic equations is being solved.

At each step of loading, calculation and evaluation of Jacobi matrix determinant $\det(J)$ are performed. Moments of change in the sign of the determinant correspond either to critical loads (upper and lower) or to bifurcation points.

To determine the buckling loads, we obtain the curve "load–deflection" in the characteristic points of the structure. Extrema points on this curve correspond to the upper and lower critical loads, these loads correspond to the transition to a new equilibrium state [6]. Also, there may be a sequence of local stability losses resulting in the formation of dents in various parts of the shell (local and general modes of the shell buckling).

To study the strength of shell structures, multiple strength criteria are used: the criterion of maximum stresses, criterion of Mises–Hill, criterion of Fisher, criterion of Goldenblat–Kopnov and criterion of Pisarenko–Lebedev.

According to the developed algorithm, the software module is made in Maple environment of analytical calculations with the possibility of parallelizing computations and use of graphical interface that allows conducting the comprehensive study of the strength and stability of shell structures and studying postbuckling behavior.

4. Strength and stability study

Table 1 shows the values of critical buckling loads q_{kr} and ultimate loads of strength q_{nlin} found by different criteria.

Table 1. Values of the maximum permissible load for the considered variants of shells, depending on the strength criterion.

Option	q_{kr}, MPa	Maximum allowable load q_{nlin} by criteria, MPa				
		Criterion of maximum stresses	Criterion of Mises–Hill	Criterion of Fisher	Criterion of Goldenblat–Kopnov	Criterion of Pisarenko–Lebedev
1	0.014	0.032, F_{xy}	0.029	0.033	–	–
2	0.034	0.029, F_y^+	0.030	0.030	–	–
3	0.066	0.041, F_y^-	0.042	0.042	0.041	0.027

We considered the following options of shells:

1. Shallow shell, square in plan, with linear dimensions $a = b = 600h$, principal radii of curvature $R_1 = R_2 = 1510h$ and thickness $h = 0.09$ m. It is made of carbon fiber M60J/Epoxy.

2. Cylindrical shell panel with parameters $a = 20$ m, $R = 5.4$ m, $h = 0.01$ m and angle of rotation π radian. This one is made of carbon fiber LU-P / ENFB.

3. The panel, the geometry of which coincides with option 2. This one is made of fiberglass T-10 / UPE22-27.

To all the considered shell structures a uniformly distributed lateral load is applied. Fastening of contour is fixed-hinged. The calculations were performed by holding 16 members in the expansion of the required functions in series ($N = 16$).

Results of the study of the strength and stability of the orthotropic conical shell made of CFRP T300 / 976 showed the effectiveness of using this material compared to traditional isotropic ones. Table 2 shows the results of calculations of the truncated conical panel 20 m long, with cone angle $\theta = 0.78$ radian, angle of rotation π and thickness of 0.01 m.

Table 2. Values obtained for the considered options of panels of conical shells.

Parameter	Panels of conical shells		
	plexiglas	steel	CFRP T300 / 976
Critical load q_{kr} , MPa	0.0152	0.2847	–
Maximum deflection at q_{kr} , m	0.971	0.3515	–
Maximum load q_{nlin}, MPa	0.0113	0.2847	0.023
Gravity load, MPa	0.000118	0.00078	0.00015
Component of maximum stresses	σ_i	σ_i	F_2^+

5. Conclusion

Using of the technique based on the Ritz method and best parameter continuation method, with regard to the adaptive choice of mesh, allows exploring the strength and stability of shells, bypass singular points of "load–deflection", obtain critical load values and examine the postbuckling behavior of a shell.

A joint study of strength and stability of shell structures will allow selecting optimal parameters of the reinforcement ribs and material of shells for safe operation of the structure.

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