

OVERALL STABILITY OF STEEL WEB-TAPERED MEMBERS

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Abstract. Efficient analytical-numerical method for the calculation ultimate capacity of thin-walled steel web-tapered members under biaxial loading is presented. It allows to determine ultimate loads and displacements faster than using FEM. Results made by presented method for three loading schemes are demonstrated.

1. Introduction

The use of web-tapered members in frames can reduce the consumption of steel up to 30% in comparison with prismatic members. However, frames composed of web-tapered members haven't obtained wide distribution in Russia because of the lack of information about stability in the national design codes. Therefore, an analytical-numerical method for solving stability problems is presented in the paper. This method allows carrying out the member's overall stability analysis in the elasto-plastic range rapidly and precisely enough.

2. Computational model

Member's boundary conditions at both ends: bending about the major and minor-axis is free, the torsion is prevented but the warping is unrestrained. Member's extraction from the plane frame is conducting by its out-of-plane effective length. To carry out this extraction it is needed to make in-plane non-linear analysis of the frame (without taking torsion into consideration). The aim of this analysis is to find second-order bending moments M_{y0} , M_{y1} and compressive force N at the ends of the member at any loading level.

Figure 1 represents loading case for bar element: compressive force is applied with eccentricities about both axes $e_{y0} = M_{y0} / N$, $e_{y1} = M_{y1} / N$, $e_{x0} = e_{x1} = l_{ef,y} / 750 + i_y^{mid} / 20$. The latter imply all the imperfections which could occur during fabrication and erection processes.

3. Solution method

The solution of the problem is based on the V.Z. Vlasov second-order theory of thin-walled members [1], extended by B.M. Broude [2] and E.A. Beilin [3] to the case when it is necessary to take into account the difference of the fibers' curvatures and the slopes associated with the torsion. In view of this fact the three differential equation system of equilibrium [3] (after the pre-integration of the first two) will be:

$$\begin{cases} EJ_x^* v'' + N^0 v - M_y^0 \theta + M_z^0 u' = -M_x^0 \\ EJ_y^* u'' + N^0 u + M_x^0 \theta - M_z^0 v' = -M_y^0 \\ \left[\frac{EJ_\omega^*}{(h_\omega^*)^2} (\theta h_\omega^*)'' \right] h_\omega^* - [GJ_k^* \theta']' - M_y^0 v'' + M_x^0 u'' + [i_p^{*2} N^0 \theta']' = 0, \end{cases} \quad (1)$$

where $J_x^*, J_y^*, J_\omega^*, J_k^*$ – bending, sectorial and torsional moments of inertia; h_ω^* – the distance between flanges' centroidal axes; parameters marked “*” are variable through the length; v, u, θ – displacements (see Fig. 1); E, G – Young's and shear moduli; i_p – polar radius of inertia. Forces marked “0” are the results of the linear analysis of the “extracted” member.

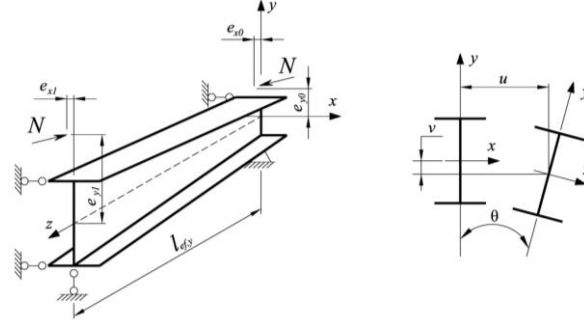


Fig. 1. Loading case and spatial cross-sectional displacements.

To solve the system (1) we use an analytical-numerical method by which the overall solution of the problem in elastic range is a sum of terms which can be obtained by executing separate analyses or calculations (so-called “Member-algorithm”) [4]:

$$v = v_l + v_b + v_a; u = u_l + u_b + u_a; \theta = \theta_l + \theta_b + \theta_a. \quad (2)$$

The first terms in (2) are the results of the linear analysis of the member when only “the active forces” [5] are applied to the member:

$$\begin{aligned} EJ_x^* v'' = -M_x^0, EJ_y^* u'' = -M_y^0, \\ \left[\frac{EJ_\omega^*}{(h_\omega^*)^2} (\theta h_\omega^*)'' \right] h_\omega^* - [GJ_k^* \theta'] = 0, (B_\omega \neq 0). \end{aligned} \quad (3)$$

The solution of the equations (3) can be presented as:

$$v_l = V_l \psi_l(z); u_l = U_l \varphi_l(z); \theta_l = \Theta_l v_l(z), \quad (4)$$

where ψ_l, φ_l, v_l – shapes of deflected bar; V_l, U_l, Θ_l – displacements' amplitudes which are linearly dependent of “the active forces”.

The second terms in (2) are the results of the buckling analysis when only “the parametric forces” [5] are applied to the member. Equations for buckling analysis can be extracted from the system (1):

$$EJ_x^* v'' + N^0 v_y = 0; \quad (5)$$

$$\begin{cases} EJ_y^* u'' + N^0 u_y + M_x^0 \theta = 0, \\ \left[\frac{EJ_\omega^*}{(h_\omega^*)^2} (\theta h_\omega^*)'' \right] h_\omega^* - [GJ_k^* \theta'] + M_x^0 u'' + [i_p^{2*} N^0 \theta'] = 0. \end{cases} \quad (6)$$

As one can be seen from (5) and (6), the system (1) has been divided into the buckling analysis of the centrally loaded bar with the results represented in the similar way:

$$v_b = V_b \psi_b(z), \quad (7)$$

and the two equation system (6) of the lateral-torsional buckling of thin-walled member. After the execution of the latter, we can obtain the lateral-torsional buckled shape functions

$$u_b = U_b \varphi_b(z); \theta_b = \Theta_b v_b(z). \quad (8)$$

In (7) and (8) V_b, U_b, Θ_b – some unknown constants having dimensions corresponding to v, u, θ (buckling analyses are executing with precision up to these constants); ψ_b, φ_b, v_b – buckled shapes.

The third terms in (2) are the displacements about both axes and the angle of torsion caused by spread of plastic deformations, possible local buckling and damages reducing cross-section. As well, initial geometric imperfections can be taken into account by using these terms. They can be calculated in the process of step-by-step load increase by determination the equilibrium state for any cross section on each step. It can be carried out using so-called “Section-algorithm” [4]. Following the method [4] substitute (2) in the main system (1) then obtain:

$$\begin{aligned} L_v &= EJ_x^* v_y'' + N^0 v - M_y^0 \theta + M_z^0 u' = 0, \\ L_u &= EJ_y^* u_y'' + N^0 u + M_x^0 \theta - M_z^0 v' = 0, \\ L_\theta &= \left[\frac{EJ_\omega^*}{(h_\omega^*)^2} (\theta_y h_\omega^*)'' \right] h_\omega^* - [GJ_k^* \theta_y']' - M_y^0 v'' + M_x^0 u'' + [i_p^{2*} N^0 \theta']' = 0. \end{aligned} \quad (9)$$

From (9) one can see that they formally represent equations of the equilibrium for web-tapered member with initial geometric imperfections $v_l + v_b, u_l + u_b, \theta_l + \theta_b$ and which has received additional displacements from the action of “the parametric forces” [5].

Use Galerkin method in order to solve the system (9):

$$\int_0^L L_v \cdot \psi_y''(z) dz = 0; \int_0^L L_u \cdot \varphi_y''(z) dz = 0; \int_0^L L_\theta \cdot v_y(z) dz = 0. \quad (10)$$

As a result, we get the system of three algebraic equations for the unknown constants of the buckled shape functions. After the solution of algebraic system we will have all terms in (2) (v_a, u_a, θ_a – assumed to be known and are determined using “Section-algorithm”). By using (2) we can obtain second-order internal forces:

$$\begin{aligned} M_x &= M_x^0 + N^0 v - M_y^0 \theta, \\ M_y &= M_y^0 + N^0 u + M_x^0 \theta, \\ B_\omega &= -(EJ_\omega^* / h_\omega^*) (\theta h_\omega^*)'', \end{aligned} \quad (11)$$

and stresses at any cross-sectional point. At the same time, the ultimate load carrying capacity of the “extracted” member is determined by the disruption of the stable deformation state.

Returning to the general solution of (2), it should be noted that the difficult non-linear analysis was reduced to buckling analyses. The latter were carried out numerically for the each loading case.

The procedure of determining spatial displacements and the overall stability in elasto-plastic range using “Member-” and “Section-algorithm” is well described for prismatic members (see, for example, [6]) and here it was used the same.

4. Results

For the purposes of practical application the results of the study are presented in the form of spatial stability coefficients $\varphi_{exy} = N^{ult} / R_y A_{mid}$ depending on the angle of taper α , loading case, member's slenderness $\lambda_y = l_{ef,y} / i_y^{mid}$ and non-dimensional eccentricity of compressive force m_{x1} . Then the spatial stability check can be made in accordance with traditional formulae existed in the steel design code

$$N / (\varphi_{exy} \gamma_c R_y A_{mid}) \leq 1, \quad (12)$$

where A_{mid}, i_y^{mid} – middle cross section area and radius of inertia, correspondingly.

The graphs in Fig. 2 for the three loading cases shows how φ_{exy} (when $\alpha = 4^\circ$ and 8°) depends on the member's slenderness λ_y and non-dimensional eccentricity m_{x1} , when $R_y = 24.5 \text{ kN/cm}^2$. The solid lines correspond to the one-sided end eccentricities (Scheme 1),

dashed – when there is $m_{x0} = 0$ at the smaller end (Scheme 2), dash-dot correspond to the case when the eccentricities are opposite in direction at the ends (Scheme 3).

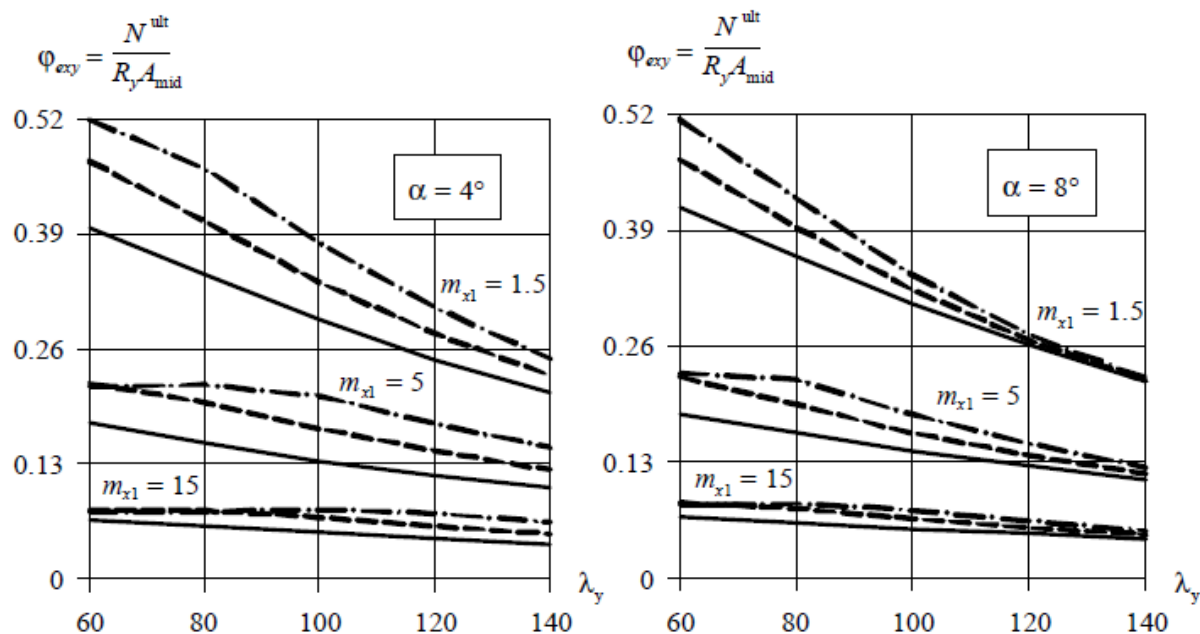


Fig. 2. The results of the study for the three loading cases.

The graphs in Fig. 2 reveal that when the non-dimensional eccentricities are equal, the load carrying capacity is considerably higher for the schemes 2 and 3 than for the scheme 1. However, with the growth of slenderness these differences become insignificant.

5. Conclusions

The use of analytical-numerical method when the buckled shapes are determined numerically allows to study the overall stability of the web-tapered members quite fast and precisely enough. Thus, the results of this investigation can be widely applied in structural steel design.

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