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THE GENERALIZATION OF THE FLEXIBILITY METHOD FOR ELASTOPLASTIC COMPUTATION OF ROD SYSTEMS

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Abstract. On the basis of generalization of the flexibility method mathematical models are developed describing the elastoplastic deformation of the rods. The algorithm of calculation of the tangential rigidities is presented.

1. Introduction

Now process of elastoplastic deformation of designs can be calculated only using potent computer programs which are based on application of FEA. In this work the generalized method of forces was developed for elasto-plastic calculation of rod systems. This generalization assumes use of a computing circuit, apparent on time, and definitions on each step of the tangential rigidities of system.

At the solution of elastoplastic tasks on the basis of FEA on each temporary step it is necessary to solve system of the algebraic equations which number is proportional to number of finite elements. These are thousands or tens of thousands of the equations. At application of the generalized flexibility method the number of preparatory operations increases, however the number of the algebraic equations on each step is equal only to number of static indefinability of rod system. This circumstance defines effectiveness of a method.

2. The main ratios for definition of the tangential rigidities

Let us consider rigidly closed up console core of AB. Speed of the center of section of the free end will be the fact that kinematic parameters [1, 2]:

$$\boldsymbol{V}_{AB} = \int_{AB} \boldsymbol{r}(\boldsymbol{s}) \times \frac{\partial' \boldsymbol{\chi}^1(s)}{\partial \boldsymbol{t}} ds + \int_{AB} \boldsymbol{p}(\boldsymbol{s}) ds, \tag{1}$$

$$\frac{\partial' \boldsymbol{\chi}^1}{\partial t} = \begin{Bmatrix} \dot{\chi}_1^{(1)} \\ \dot{\chi}_2^{(1)} \\ \dot{\chi}_3^{(1)} \end{Bmatrix}, \qquad \boldsymbol{p} = \begin{Bmatrix} \frac{\partial w}{\partial s} \\ \dot{\gamma}_3 \\ \dot{\gamma}_2 \end{Bmatrix}, \tag{2}$$

where $\boldsymbol{r}$(s) – the radius-vectors (with pole in B) points on the axis of the rod; $\frac{\partial' \boldsymbol{\chi}^1}{\partial t}$ – the vector of changes in time of the generalized curvatures; $\dot{\gamma}_2, \dot{\gamma}_3$ – the shear velocity cross-section; $\frac{\partial w(s)}{\partial s}$ – the speed in the axial direction.

The following geometrical ratios [3] connect kinematic parameters with speeds of deformations of points of a core:

$$\begin{aligned} \dot{\varepsilon}_{\zeta\zeta} &= \frac{1}{a^2+b^2}\left(a\left(\dot{w} - \dot{\chi}_2^{(1)}\xi + \dot{\chi}_3^{(1)}\eta\right) + b\dot{\chi}_1^{(1)}\sqrt{\xi^2+\eta^2}\right), \\ \dot{\gamma}_{\eta\xi} &= \frac{1}{a}\left(\dot{\gamma}_2 + \dot{\chi}_1^{(1)}\xi\right), \\ \dot{\gamma}_{\xi\zeta} &= \frac{1}{a}\left(\dot{\gamma}_3 - \dot{\chi}_1^{(1)}\eta\right), \end{aligned} \tag{3}$$

where ζ, η, ξ - coordinates of points of a core in system of orthogonal axes, bound to core section (the axis ζ is directed perpendicular to section).

For rods with large (compared to the size of the cross section) radius of curvature, accepted:

$a = 1, b = 0.$ (4)

Assumptions (4) match rods with not natural twisting and large (compared to the cross-section dimensions) radius of curvature.

Coupling equations between tension and deformations in a point of a core represent the generalized Hooke law with pseudo-resilient coefficients.

These equations in relation to stressed state conditions in a core

$\sigma_{\eta\eta} = \sigma_{\xi\xi} = \tau_{\xi\eta} = 0$, will take a form [4]:

$$\dot{\sigma}_j = \sum_{k=1}^{3} a_{jk}\dot{\varepsilon}_k\,, j = 1,2,3, \quad (6)$$

where

$$\dot{\sigma}_1 = \dot{\sigma}_{\zeta\zeta}, \dot{\sigma}_2 = \dot{\tau}_{\eta\zeta}, \dot{\sigma}_3 = \dot{\tau}_{\xi\zeta}, \quad (7)$$

$$\dot{\varepsilon}_1 = \dot{\varepsilon}_{\zeta\zeta}, \dot{\varepsilon}_2 = \dot{\gamma}_{\eta\zeta}, \dot{\varepsilon}_3 = \dot{\gamma}_{\xi\zeta}\,. \quad (8)$$

Pseudo-resilient coefficients of a_{jk} are calculated on formulas:

$$a_{ii} = 2(1+\nu)G(1 - 2(1+\nu)\alpha\langle\frac{\sigma_1^2}{\overline{\sigma}^2}\rangle);$$

$$a_{jj} = G(1 - 9\alpha\langle\frac{\sigma_j^2}{\overline{\sigma}^2}\rangle), j = 2, 3; \quad (9)$$

$$a_{ij} = a_{ji} = -2(1+\nu)G\cdot 3\alpha\langle\frac{\sigma_i\sigma_j}{\overline{\sigma}^2}\rangle\,, j = 2,3;$$

$$a_{2,3} = a_{3,2} = -G9\alpha\langle\frac{\sigma_2\sigma_3}{\overline{\sigma}^2}\rangle;$$

$$\alpha = \frac{G}{3G+\lambda}\cdot\frac{1}{1-(1-2\nu)\frac{G}{3G+\lambda}\cdot\frac{\sigma_1^2}{\overline{\sigma}^2}};$$

$$\overline{\sigma}^2 = \sigma_1^2 + 3\sigma_2^2 + 3\sigma_3^2, \quad (10)$$

where G – the shear modulus, ν - Poisson's ratio, $\lambda = \frac{E\cdot E_{pl}}{E-E_{pl}}$, $E = 2(1+\nu)G$ – the modulus of elasticity, E_{pl} – tangent plastic modulus. The sense $\langle\rangle$ means the following: $\langle\phi\rangle = 0$ in the conditions of elastic deformation, $\langle\phi\rangle = \phi$ in the conditions of a plasto-elastic deformation.

Transitions from an elasto-plastic loading to resilient and back are defined by logical conditions:

a) resilient loading $\overline{\sigma}^2 \le \sigma_n^2$,

b) elasto-plastic loading

$$\overline{\sigma}^2 > \sigma_n^2, \frac{d(\overline{\sigma}^2)}{dt} > 0. \quad (11)$$

In (11) through $\overline{\sigma}_n$ is designated $\overline{\sigma}$ at the time of the beginning of n of a half-cycle of a resilient loading (if by $n = 0$, $\overline{\sigma}_0 = \sigma_s$).

Let us consider matrixes columns:

$$\{\dot{\sigma}\}=\{\dot{\sigma}_1, \dot{\sigma}_2, \dot{\sigma}_3\};\ \{\dot{\varepsilon}\}=\{\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3\};\ \{\dot{\psi}\}=\{\dot{\chi}_1^{(1)}, \dot{\chi}_1^{(1)}, \dot{\chi}_1^{(1)}, \frac{\partial\omega}{\partial s}, \dot{\gamma}_3, -\dot{\gamma}_2\}. \quad (12)$$

Taking into (12) account a ratio (3) and (6) it is possible to write down in a matrix form:

$$\dot{\sigma} = A\dot{\varepsilon},\ \dot{\varepsilon} = S\dot{\psi}. \quad (13)$$

Components of matrixes of A, S naturally are defined by equalities (3), (6)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad (14)$$

$$S = \begin{bmatrix} 0 & -\xi & \eta & 1 & 0 & 0 \\ \xi & 0 & 0 & 0 & 0 & 1 \\ -\eta & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \quad (15)$$

From (13) we receive

$$\dot{\sigma} = AS\dot{\psi}. \quad (16)$$

Let us increase right and left-hand (16) on a matrix:

$$L = \begin{bmatrix} \eta & 0 & 0 \\ -\xi & 0 & 0 \\ 0 & \xi & -\eta \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (17)$$

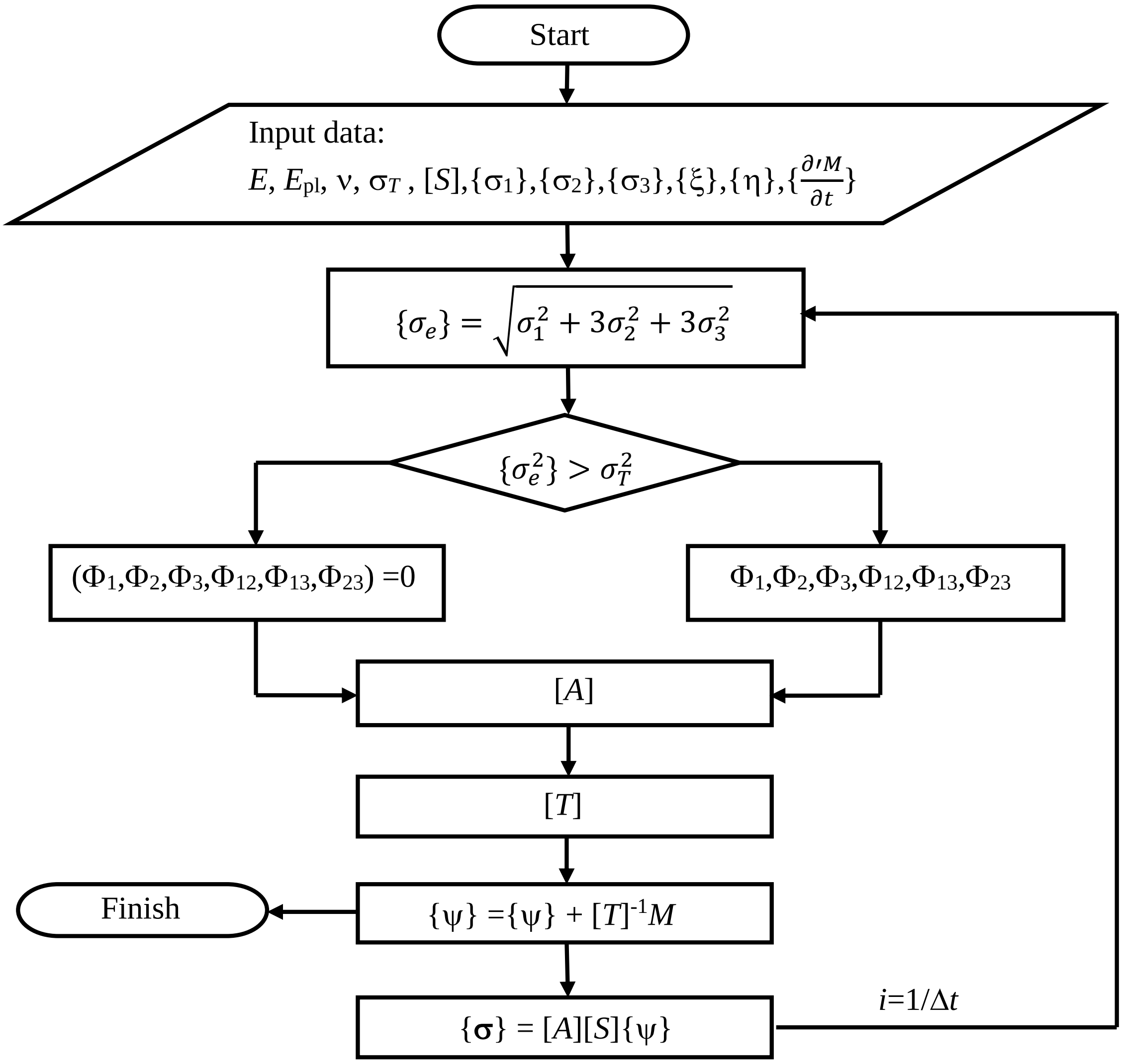

Fig. 1. Algorithm of calculation of the tangential rigidities.

and we will integrate the received expressions on core section. As a result we have

$$\int_F L\dot{\sigma}\, dF = \int_F LAS\dot{\Psi}\, dF = \int_F LAS\, dF \cdot \dot{\Psi} = T\dot{\Psi}, \quad (18)$$

where

$$T = \int_F LAS\, dF \quad (19)$$

tangential rigidities of the cross section in terms of elastic-plastic deformation.

Let us enter a matrix column

$$\frac{\partial' M}{\partial t} = \left\{\frac{\partial' M_1}{\partial t} \quad \frac{\partial' M_2}{\partial t} \quad \frac{\partial' M_3}{\partial t} \quad \frac{\partial' Q_1}{\partial t} \quad \frac{\partial' Q_2}{\partial t} \quad \frac{\partial' Q_3}{\partial t}\right\}, \quad (20)$$

where M_1, M_2, M_3, Q_1, Q_2, Q_3 – components of vectors of $\boldsymbol{M}$ and $\boldsymbol{Q}$ in a frame ζ, η, ξ.

The left part in (18) is a matrix-column derivative with time of the internal forces in the section:

$$\int_F L\dot{\sigma}\, dF = \frac{\partial' M}{\partial t}. \quad (21)$$

Thus, we obtain the generalized formula of Mora:

$$\frac{\partial' M}{\partial t} = T\dot{\Psi} \text{ or } \dot{\Psi} = T^{-\mathbf{1}}\frac{\partial' M}{\partial t}. \quad (22)$$

At creation of computing algorithms at each time step, the above formulas are written in the form of incremental ratios. Based on (22) the equations of flexibility method to elastoplastic deformation conditions are also written in incremental form. The number of these equations equals the degree of static indetermination of the system.

On the developed mathematical model, the algorithm of calculation of the tangential rigidities in core section was made (Fig. 1).

Using (22) it is easy to write the generalized formula of Mohr for the determination of the influence coefficients δ_{ij} and free terms Δ_i geometrically linear rod system. Then the equations of the generalized method would be:

$$\sum_{i=1}^{k}\delta_{ij}\dot{R}_i + \dot{\Delta}_i = 0,\ j = 1, 2, \ldots, k. \quad (23)$$

At creation of computing algorithms at each time step, the above formulas are written in the form of incremental ratios. Based on (22) the equations of the generalized flexibility method to elastoplastic deformation conditions are also written in incremental form. The number of these equations equals the degree of static indetermination of the system.

3. Conclusions

For the first time the equations of a flexibility method are generalized on a case of the solution of elastoplastic tasks. Effectiveness of use of the generalized flexibility method (in comparison with FEA) consists in sharp increase in quick action, decrease of the used computer resources and simplicity of physical interpretation of results.

References

[1] U.L. Rutman, In: *Load and reliability of mechanical systems* (Naukova Dumka, Kiev, 1987), p. 83.

[2] V.A. Svetlitskiy, *Mechanics of flexible rods and filaments* (Mashinostroenie, Moscow, 1978). (In Russian).

[3] A.P. Filin, O.D. Tananaiko, I.M. Cherneva, M.A. Schwartz, *Algorithms for construction of resolving equations of the mechanics of rod systems* (Stroiizdat, Leningrad, 1983). (In Russian).

[4] J. Argyris, D. Sharpf, In: *Mechanics: collection of translations of foreign articles*, ed. by A.U. Ishlinskiy (Mir, Moscow, 1972), №134, p. 107. (In Russian).