

# TWO-DIMENSIONAL MODEL FOR HYDRAULIC FRACTURING WITH FOAMS

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**Abstract.** A simple and accurate model of foam hydraulic fracturing is developed with both compressibility and rheology being taken into account. The governing equations for a compressible power-law fracturing fluid are derived for the classical PKN fracture geometry. Numerical simulations reveal an influence of compressibility and rheology on the temporal evolution of the fracture opening.

**Keywords:** hydraulic fracturing, compressibility, rheology, foam, numerical solution

## 1. Introduction

Foam fracturing modern technique of the hydraulic fracturing looks promising due to several advantages: foam minimizes usage of liquid, limits fluid retention in the formation and has better proppant transport properties due to its high viscosity. However, a modeling of foam faces serious difficulties. Apart from the foam saturation and stability, it is necessary to take into account compressibility and non-Newtonian character of the foam viscosity. Moreover, foam contains both liquid and gas parts. Taking into account all these factors is a difficult task. The compressibility of foam is usually introduced through its quality or the importance of the gas part [1 – 4]. Modeling of viscosity is simpler for incompressible fluids [4 – 9], and for compressible foam experimental data is usually used to fit power-law model [10, 11].

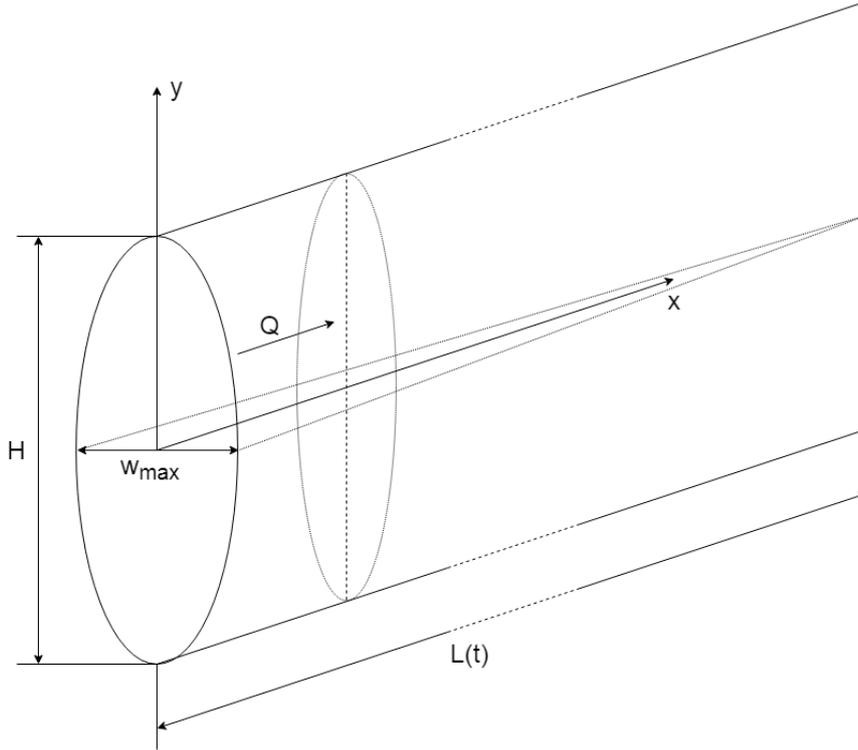
Existing models of foam fracturing could be divided into two groups. The first group uses simpler considerations and models foam as a single-phase fluid with effective properties depending on the foam quality and pressure. Gu's and Mohanty's model [12] is neglecting changing density of the foam and models its properties by relations for the power-law parameters depending on the fluid pressure in the fracture and given constant quality of the foam. For the two-dimensional model, it results in a difference in geometries of the fracture with the same fracturing conditions but different chosen foam qualities, while the total volume of the fracture remains constant. Another model, Park's approach [13], accounts for compressibility of foam by adding formation volume factor to the time-dependent term in the continuity equation. Wang's et al. model [3] allows investigating influence of compressibility of fracturing fluid using linear density-pressure relation.

The second group of models is considering foam as a multi-phase fluid under different assumptions (e.g. dimensions, pressure, temperature, solubility etc.) [10, 11, 14]. The systems of governing equations for these models became more accurate, but at the same time they became noticeably more complex, requiring more setting parameters and increasing time of calculations, especially for three-dimensional cases.

Our goal is to develop quite simple and fast but accurate model of foam fracturing, which takes into account fluid compressibility and non-Newtonian character of its viscosity by introducing both density-quality and rheology-quality relations. The governing equations are derived for the classical PKN [15] fracture geometry in order to develop two-dimensional model and qualitatively investigate it using numerical simulation.

## 2. Governing equations

We consider a problem of a hydraulic fracture propagation driven by injection of compressible non-Newtonian fluid in an infinite homogenous linear elastic medium. The geometry of the problem presented in Fig. 1:



**Fig. 1.** PKN fracture geometry

This is a classical PKN [15] fracture geometry approach: the length of the fracture  $L$  is much greater than its constant height  $H$ , and height is much greater than fracture opening  $w$ . Then the approximate plane strain condition can be assumed in every plane orthogonal to the direction of propagation. Assuming that net pressure is independent of  $y$ -axis leads to elliptical fracture cross section. The fracture width along  $z$ -axis is then given by [15]:

$$w(z, x, t) = p_{net}(x, t) \frac{1}{E'} \sqrt{H^2 - 4z^2}, \quad (1)$$

where  $E'$  is defined as

$$E' = \frac{G}{1-\nu},$$

$p_{net} = p - \sigma_{min}$ ,  $p$  is fluid pressure,  $\sigma_{min}$  is minimum horizontal stress,  $\nu$  is Poisson's ratio,  $G$  is the shear modulus.

It is assumed that density and pressure of foam are constant in the each fracture cross section:  $\rho = \rho(x, t)$ ,  $p = p(x, t)$ . Under these assumptions and according to the fracture geometry the continuity equation is given by:

$$\frac{\pi H}{4} \frac{\partial}{\partial t} (\rho w_{max}) + \frac{\partial}{\partial x} (\rho q) = 0, \quad (2)$$

where  $q$  is the volumetric flow rate,  $w_{max} = w(0, x, t)$ .

Equations of motions for single-phase fluid with one-dimensional flow in an elliptical fracture are reduced to the simple relation between volumetric flow rate and pressure gradient, neglecting inertial terms for Newtonian fluid:

$$q = -\frac{\pi w_{max}^3 H}{64\mu} \frac{\partial p}{\partial x}, \quad (3)$$

where  $\mu$  is Newtonian viscosity. However, most investigations [16 – 19] are showing that foams rheology is well approximated by power-law models. That is why it can also be used another known relation for power law fluid motion in an elliptical fracture [20]:

$$q = -\frac{\varphi(n)H(\pi w_{max})^{2n+1/n}}{(2K)^{1/n}} \left(\frac{\partial p}{\partial x}\right)^{1/n} \text{sign}\left(\frac{\partial p}{\partial x}\right), \quad (4)$$

where  $\text{sign}(x)$  is a signum function,  $K$  is flow consistency index and  $n$  is flow behavior index. Term  $\varphi(n)$  depends on the fracture's cross section geometry and is derived as follows:

$$\varphi(n) = \frac{n}{2(2n+1)H} \int_{-H/2}^{H/2} \left(\frac{w}{w_{max}}\right)^{\frac{2n+1}{n}} dy. \quad (5)$$

For the elliptic cross section, the solution is known as [21]:

$$\varphi(n) = \frac{n}{2(2n+1)} \frac{\Gamma\left(\frac{4n+1}{2n}\right)}{\Gamma\left(\frac{5n+1}{2n}\right)} \left(\frac{2}{\sqrt{\pi}}\right)^{\frac{3n+2}{n}}, \quad (6)$$

where  $\Gamma(x)$  is gamma-function.

### 3. Foam compressibility model

The quality of the foam  $\Gamma$  is introduced as [1, 2]

$$\Gamma = \frac{V_g}{V_g + V_l}, \quad (7)$$

where  $V_g$ ,  $V_l$  are the volumes of the gas and liquid phases of the foam respectively. The presence of a gas phase makes foams highly compressible. Indeed, for isothermal processes the Boyle's law is

$$p_0 V_{0g} = p V_g, \quad (8)$$

where  $p_0$  is the injection pressure in our case,  $V_{0g}$  is initial volume of gas phase. It follows from Eqs. (7), (8) that

$$\frac{p}{p_0} = \frac{\Gamma_0(1-\Gamma)V_{0l}}{(1-\Gamma_0)\Gamma V_l}, \quad (9)$$

where  $\Gamma_0$  is the foam injection quality,  $V_{0l}$  is initial volume of liquid phase. The density of the foam  $\rho$  is defined as

$$\rho = \frac{\rho_l V_l + \rho_g V_g}{V_l + V_g} = (1 - \Gamma)\rho_l + \Gamma\rho_g. \quad (10)$$

Equation (9) allows us to express quality through the pressure  $p$ . Substitution of the resulting expression into Eq. (10) gives rise to the connection between density and pressure,

$$\rho = \frac{\Gamma_0 p_0}{\Gamma_0 p_0 + (1-\Gamma_0)p} \rho_g + \frac{(1-\Gamma_0)p}{\Gamma_0 p_0 + (1-\Gamma_0)p} \rho_l. \quad (11)$$

The second term in Eq. (11) is negligibly small for low quality foams due to  $\frac{\rho_g}{\rho_l} \ll 1$ .

Then the compressibility relationship is

$$\rho = \frac{(1-\Gamma_0)p}{\Gamma_0 p_0 + (1-\Gamma_0)p} \rho_l. \quad (12)$$

However, for the high quality foam the density of gas becomes comparable with the density of the liquid phase, so at least an estimation for  $\rho_g$  is needed. We assume that the value of  $\rho_g$  can be simply estimated as the density of an ideal gas at the defined values of pressure and temperature  $T$ :

$$\rho_g = \frac{Mp}{RT}, \quad (13)$$

where  $M = 28$  g/mol for  $N_2$  and  $M = 44$  g/mol for  $CO_2$ ,  $R$  is ideal gas constant. As we consider isothermal case, the averaged temperature of the reservoir is assumed to be known and used in Eq. (13).

#### 4. Foam rheology model

We consider foams as non-Newtonian fluids by using known models that predict power law parameters of the foam as functions of quality. In presented work we are using known empirical correlations [22] for  $K(\Gamma)$  and  $n(\Gamma)$ .

The correlation for 20-lbm/Mgal foam is:

$$n = n_0(1 - 2.1006\Gamma^{7.3003}), \quad K = K_0 \exp(-1.9913\Gamma + 8.9722\Gamma^2) \quad (14)$$

The correlation for 30-lbm/Mgal foam is:

$$n = n_0(1 - 0.1535\Gamma^{6.5152}), \quad K = K_0 \exp(-2.3761\Gamma + 8.8830\Gamma^2) \quad (15)$$

The correlation for 40-lbm/Mgal foam is:

$$n = n_0(1 - 0.6633\Gamma^{5.1680}), \quad K = K_0 \exp(-0.4891\Gamma + 5.6203\Gamma^2) \quad (16)$$

$K_0$  and  $n_0$  are constant power law parameters of the base fluid which are assumed to be known. It should be noted that any other known correlations (e.g., Ref. [16 – 19]) for  $K(\Gamma)$ ,  $n(\Gamma)$  may be used instead of Eqs. (14), (15) and (16). It is also possible to use Newtonian  $\mu(\Gamma)$  correlations (e.g., Ref. [5, 23]) with Eq. (3) in order to simplify the model.

#### 5. Numerical simulation

Equations (1), (2), (4), (6), (11), (13) and relations (14), (15) or (16) together form a closed system of equations and can be modified to a single pressure or a fracture width equation. It can be solved implicitly using FDM with inner iterations. The boundary conditions are:

$$w_{max}(x, t)|_{x=L} = 0,$$

$$\rho q|_{x=0} = Q_{in},$$

where  $Q_{in}$  is an inlet mass flow rate. Fracture length  $L(t)$  is evaluated as follows:

$$L(t) = \min(x|_{w_{max}(x)=0}). \quad (17)$$

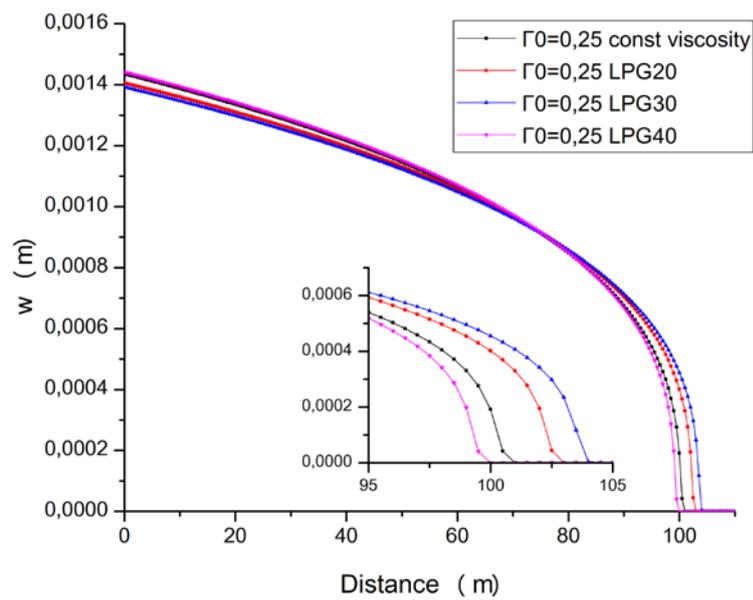
The initial condition is a closed fracture:

$$w_{max}(x, 0) = 0.$$

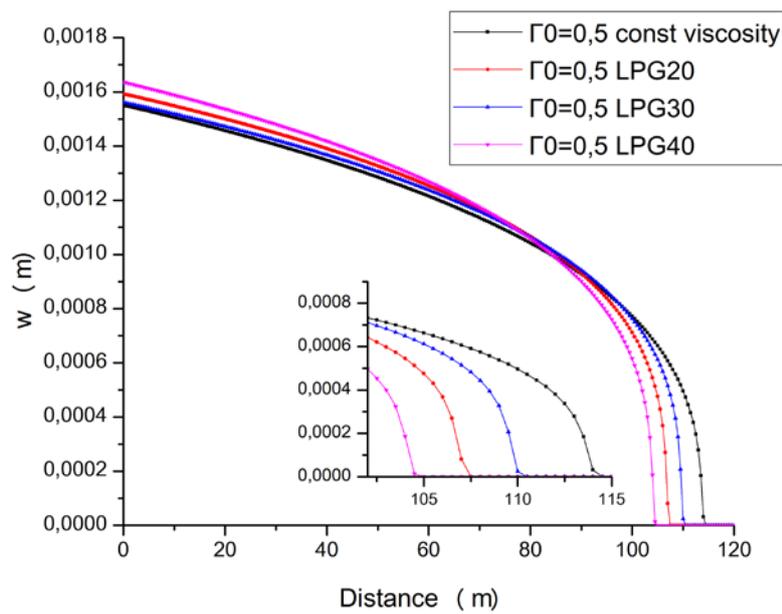
Following hydraulic fracturing parameters are chosen to be fixed during the numerical simulations in order to investigate the dependence of the initial foam quality and its rheology on the fracture geometry:

$$\begin{aligned} Q_{in} &= 1.5 \text{ kg/s}, \quad P_0 = 1 \text{ MPa}, \\ \rho_l &= 1000 \text{ kg/m}^3, \quad K_0 = 0.01 \text{ Pa} \cdot \text{s}, \quad n_0 = 1, \\ E &= 25 \text{ GPa}, \quad H = 20 \text{ m}, \quad \sigma = 1 \text{ MPa} \end{aligned}$$

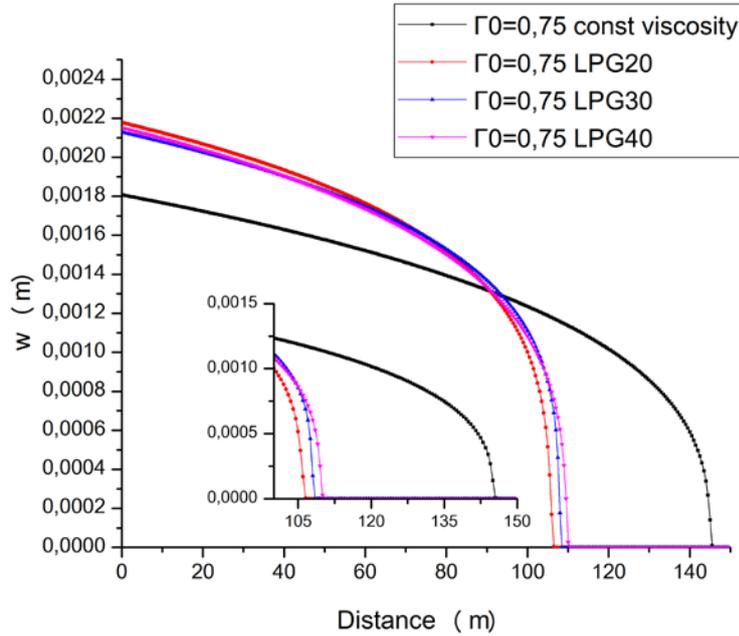
Firstly, we are studying the influence of rheology relations defined by Eqs. (14), (15) and (16) for the different initial foam qualities by comparing obtained numerical results with the constant viscosity case ( $K(\Gamma) = K_0 = \text{const}$ ,  $n(\Gamma) = n_0 = \text{const}$ ). The fracture width distributions along  $L$  at  $t = 1000$  s are shown for  $\Gamma_0 = 0.25$ ,  $\Gamma_0 = 0.5$  and  $\Gamma_0 = 0.75$  in Fig. 2, Fig. 3, Fig. 4 respectively. For this set of initial parameters non-Newtonian character of the foam viscosity starts to influence fracture geometry at  $\Gamma_0 = 0.5$ , and its influence become essential for the case  $\Gamma_0 = 0.75$ : fracture width is increasing while the fracture length decreases. In addition, it is shown that for different foams (20, 30 and 40lbm/Mgal) fracture geometry is changing not significantly.



**Fig. 2.** Fracture width distribution for initial foam quality 0.25

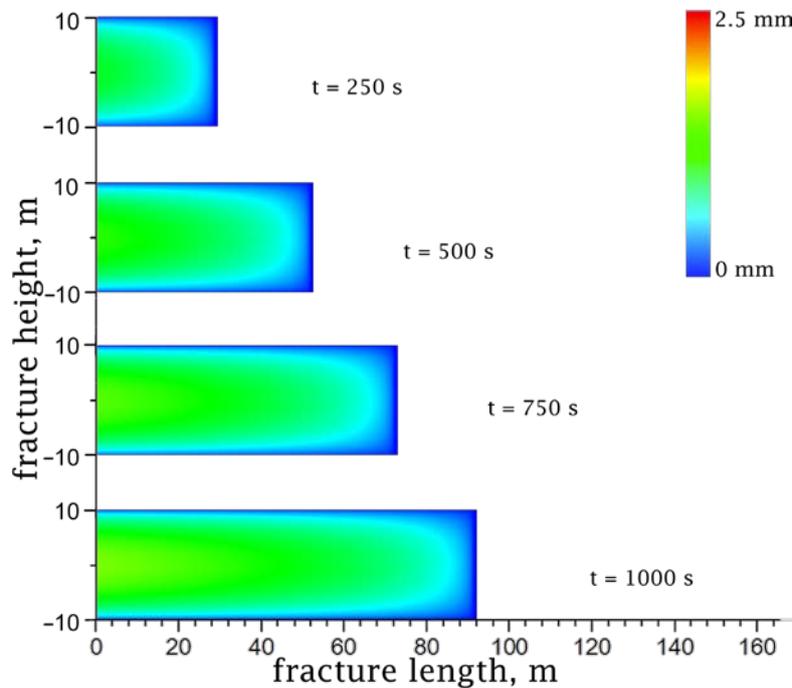


**Fig. 3.** Fracture width distribution for initial foam quality 0.5

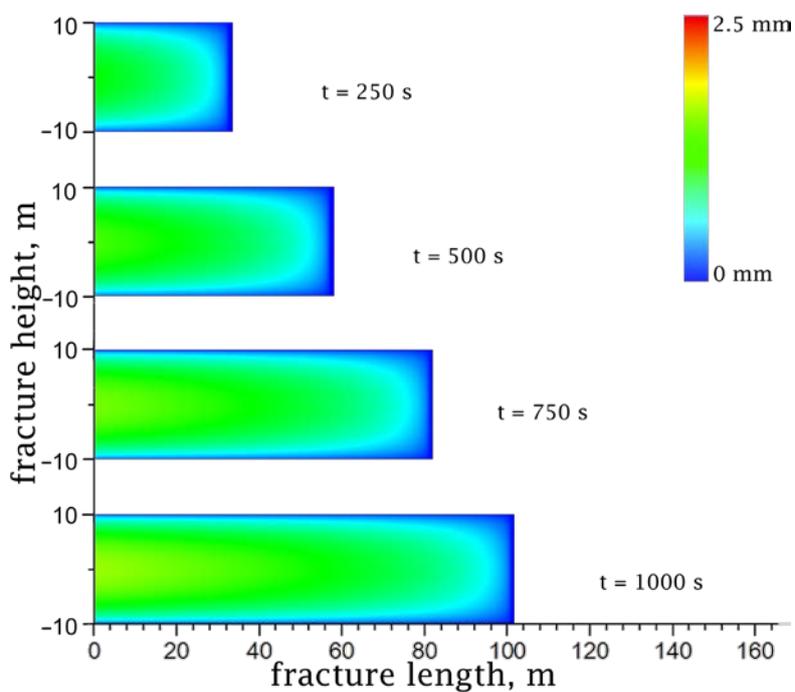


**Fig. 4.** Fracture width distribution for initial foam quality 0.75

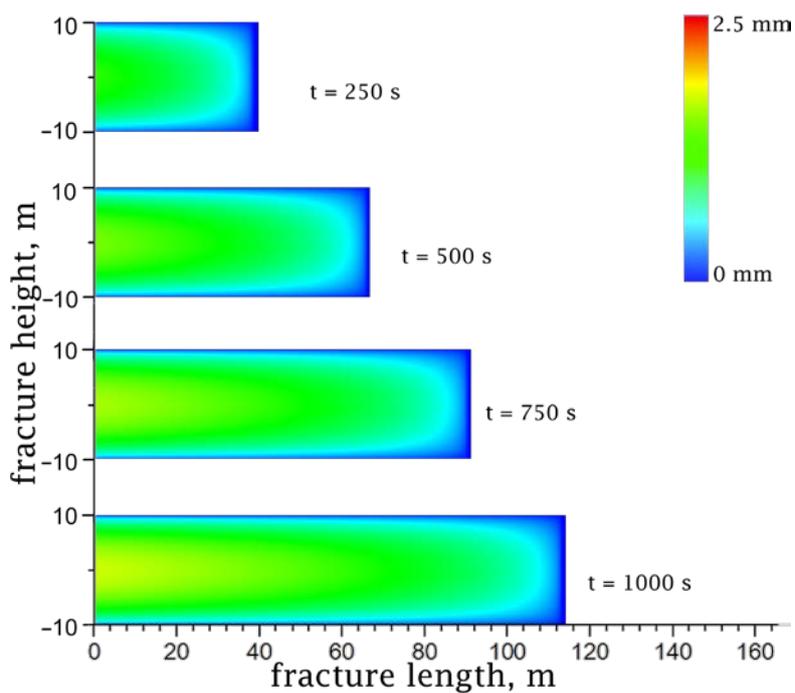
Secondly, we investigate the compressibility factor influence on the fracture geometry. Considering constant viscosity  $K(\Gamma) = K_0 = \text{const}$ ,  $n(\Gamma) = n_0 = \text{const}$  for all the simulations we compare fracture propagation process for incompressible fluid (Fig. 5), compressible  $CO_2$ -foam with  $\Gamma_0 = 0.25$  (Fig. 6),  $\Gamma_0 = 0.5$  (Fig. 7) and  $\Gamma_0 = 0.75$  (Fig. 8) at 50K-temperature cases. The numerical results show that increasing of initial foam quality leads to an increase in the final volume of the fracture for similar mass flow rate. For  $\Gamma_0 = 0.25$  overall calculated mass fraction of pumped gas in foam is equal to  $\sim 0.54\%$ , for  $\Gamma_0 = 0.5$  this value reaches  $\sim 1.54\%$ , and for  $\Gamma_0 = 0.75$  is  $\sim 4.69\%$ .



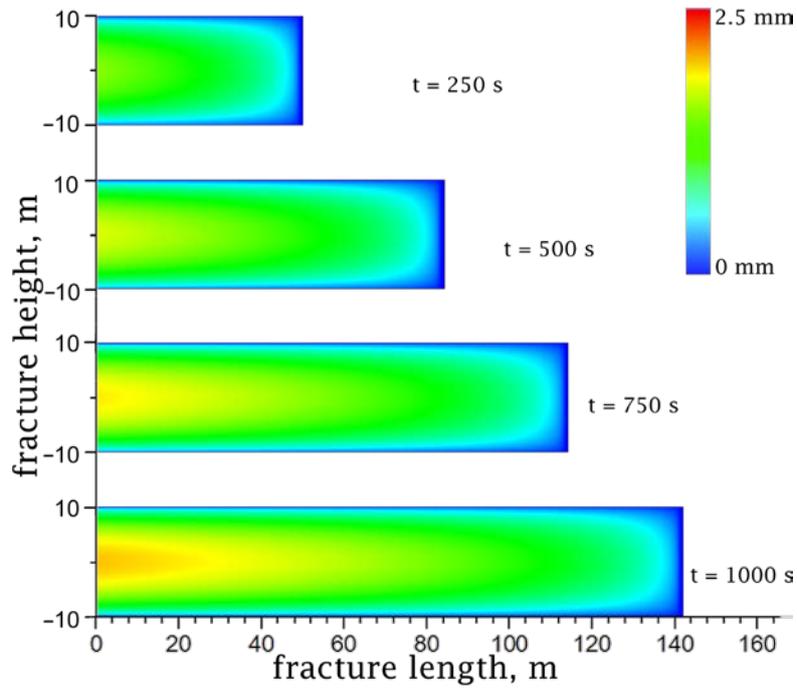
**Fig. 5.** Fracture growth for incompressible fluid



**Fig. 6.** Fracture growth for 0.25-quality foam (no rheology)



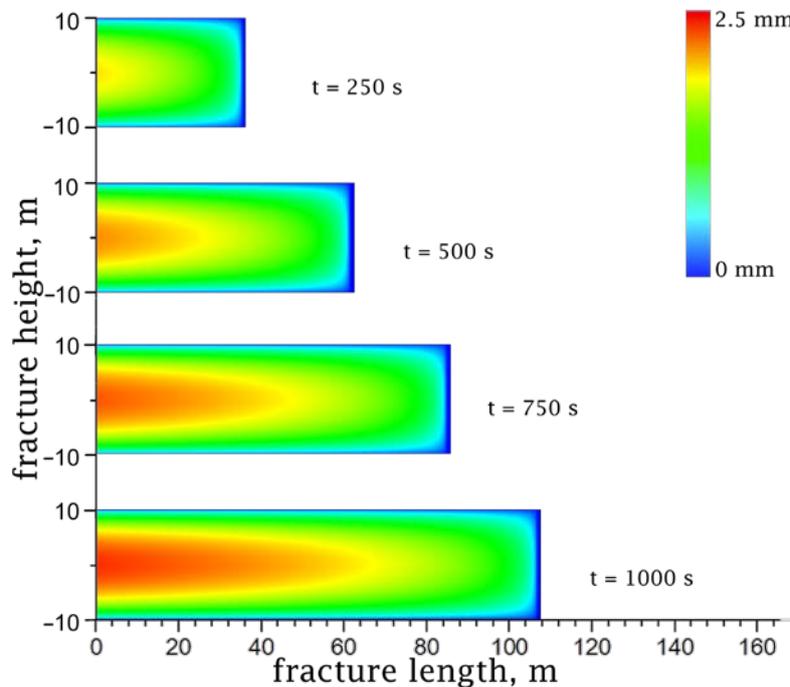
**Fig. 7.** Fracture growth for 0.5-quality foam (no rheology)



**Fig. 8.** Fracture growth for 0.75-quality foam (no rheology)

Also it should be noted that assuming  $\Gamma_0 = 0$  and  $n(\Gamma) = 1$  (the case corresponding to Fig. 5) the model reduces to classical PKN model for an incompressible fracturing fluid without leak-off.

Finally we present the numerical results (Fig. 9) for 0.75-quality 30-lbm/Mgal foam (rheology is defined by Eq. 15). Comparing to the no-rheology case (Fig. 8) the modeling results show that the length of the fracture is decreasing while the fracture width increases.



**Fig. 9.** Fracture growth for 0.75-quality 30-lbm/Mgal foam

## 6. Conclusion

The two-dimensional model of foam fracturing is presented. It takes into account fluid compressibility and non-Newtonian character of its viscosity by introducing density-quality relation based on the Boyle's law and rheology-quality correlations. This model allows to calculate fracture geometry for the foams with different quality at given pressure. Both rheology and compressibility properties have been studied independently and together. The developed model shows that for higher quality foam it is expected to produce a fracture with higher opening value and lower length due to rheology properties, and with greater volume for the same mass of pumped foam due to compressibility. Due to the simplicity of the introduced system of equations, it is expected that one may develop fast and accurate pseudo3D foam fracturing model based on the presented model.

However, the presented model does not take into account very important aspects of hydraulic fracturing, such as leak-off and proppant transport. Future generalization of the problem implies not only conversion to pseudo3D-geometry, but also consideration of proppant transport, its influence on the foam rheology and adding multi-phase leak-off terms to the governing equations.

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