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STABILITY OF THE MICROPOLAR THIN ROUND PLATE

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Abstract. In this paper, a thin round plate of isotropic micropolar elastic material is considered, in which the elastic deflections are comparable with their thickness, and small in relation to the basic size, also both the angles of rotation of the normal elements to the middle plane before deformation and their free rotations are small. Thus, the strain tensor and tensor of bending-torsion takes into account not only linear but also the nonlinear terms in the gradients of displacement and rotation. The stability problem is solved in the case when the solid round plate is hinge supported along the contour and is under the action of radial compressive forces. After solving the obtained boundary value problem, the critical value of the external force is determined. The critical force of the micropolar problem is compared with the value of the classical solution. The important properties of micropolar material are established.

Keywords: micropolar, elastic, thin round plate, curvilinear coordinates, geometrically nonlinear, applied model, stability

1. Introduction

As structural elements, thin rods, plates, and shells are widely used, the bearing capacity of which is determined mainly by their stability. The theory of stability of thin rods, plates, and shells in the framework of the classical theory of elasticity is described in monographs [1,2,etc.]. The work [3] is devoted to the stability problem in the framework of the micropolar (momental) theory of elasticity. Review of works on the micropolar theory of thin plates and shells was carried out in work [4]. In works [5-8], on the basis of the hypothesis method (which has an asymptotic substantiation), the linear theory of micropolar elastic thin plates and shells is constructed. In works [9,10], the theory of micropolar elastic flexible plates and shells are constructed. In work [11] the stability problem of micropolar elastic rectangular plates is studied.

In this paper, the stability problem of micropolar elastic round solid plate in an axisymmetric formulation is studied, when on its contour uniformly distributed radial load is applied. To obtain stability equations, disturbance is given to the initial state of the plate, equations of micropolar flexible plates are used, performing linearization. The boundary-value problem of stability of micropolar elastic round plate is solved exactly using Bessel functions; as a result, the critical value of the load is determined. After comparison with the classical case, effective manifestations of the micropolarity of the material are established.

2. Geometrically nonlinear mathematical model of micropolar elastic thin plates in curvilinear coordinates with independent fields of displacements and rotations

We consider a plate of constant thickness $2h$ as a three-dimensional elastic isotropic body. We assign the plate to the curvilinear coordinate system α_1, α_2, z. The coordinate plane

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α_1, α_2 will be combined with the median plane of the plate. The axis Oz is directed along the normal to the median plane.

The basic equations and the natural boundary conditions of micropolar elastic geometrically nonlinear thin plates with independent fields of displacements and rotations in curvilinear coordinates have the form [10]:

Balance Equations

$$\frac{1}{A_i}\frac{\partial T_{ii}}{\partial \alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_j}{\partial \alpha_i}(T_{ii}-T_{jj})+\frac{1}{A_j}\frac{\partial S_{ji}}{\partial \alpha_j}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}(S_{ji}+S_{ij})=-(p_i^+-p_i^-),$$

$$\frac{1}{A_i}\frac{\partial M_{ii}}{\partial \alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_j}{\partial \alpha_i}(M_{ii}-M_{jj})+\frac{1}{A_j}\frac{\partial M_{ji}}{\partial \alpha_j}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}(M_{ji}+M_{ij})-N_{3i}=-h(p_i^++p_i^-),$$

$$\frac{1}{A_1A_2}\left\{\left[\frac{\partial(A_2N_{13})}{\partial \alpha_1}+\frac{\partial(A_1N_{23})}{\partial \alpha_2}\right]+\frac{A_2}{A_1}\frac{\partial^2 w}{\partial \alpha_1^2}T_{11}+\frac{A_1}{A_2}\frac{\partial^2 w}{\partial \alpha_2^2}T_{22}+\frac{\partial^2 w}{\partial \alpha_1\partial \alpha_2}\left(S_{12}+S_{21}\right)+\right.$$

$$+\frac{\partial w}{\partial \alpha_1}\left[\frac{A_2}{A_1}\frac{\partial T_{11}}{\partial \alpha_1}-\frac{A_2}{A_1^2}\frac{\partial A_1}{\partial \alpha_1}T_{11}+\frac{1}{A_1}\frac{\partial A_2}{\partial \alpha_1}T_{11}+\frac{1}{2}\frac{\partial\left(S_{12}+S_{21}\right)}{\partial \alpha_2}\right]+$$

$$\left.+\frac{\partial w}{\partial \alpha_2}\left[\frac{A_1}{A_2}\frac{\partial T_{22}}{\partial \alpha_2}-\frac{A_1}{A_2^2}\frac{\partial A_2}{\partial \alpha_2}T_{22}+\frac{1}{A_2}\frac{\partial A_1}{\partial \alpha_2}T_{22}+\frac{1}{2}\frac{\partial\left(S_{12}+S_{21}\right)}{\partial \alpha_1}\right]\right\}=-(p_3^+-p_3^-),$$

$$\frac{1}{A_i}\frac{\partial L_{ii}}{\partial \alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_j}{\partial \alpha_i}(L_{ii}-L_{jj})+\frac{1}{A_j}\frac{\partial L_{ji}}{\partial \alpha_j}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}(L_{ji}+L_{ij})+$$

$$+(-1)^j(N_{j3}-N_{3j})=-(m_i^+-m_i^-),$$

$$\frac{1}{A_1A_2}\left[\frac{\partial(A_2L_{13})}{\partial \alpha_1}+\frac{\partial(A_1L_{23})}{\partial \alpha_2}\right]+(S_{12}-S_{21})=-(m_3^+-m_3^-),$$

$$L_{33}-\frac{1}{A_1A_2}\left[\frac{\partial(A_2\Lambda_{13})}{\partial \alpha_1}+\frac{\partial(A_1\Lambda_{23})}{\partial \alpha_2}\right]-(M_{12}-M_{21})=h(m_3^++m_3^-). \tag{1}$$

Elasticity Relations

$$T_{ii}=\frac{2Eh}{1-\nu^2}[\Gamma_{ii}+\nu\Gamma_{jj}],\quad S_{ij}=2h[(\mu+\alpha)\Gamma_{ij}+(\mu-\alpha)\Gamma_{ji}],$$

$$M_{ii}=\frac{2Eh^3}{3(1-\nu^2)}[K_{ii}+\nu K_{jj}],\quad M_{ij}=\frac{2h^3}{3}[(\mu+\alpha)K_{ij}+(\mu-\alpha)K_{ji}],$$

$$N_{i3}=2h[(\mu+\alpha)\Gamma_{i3}+(\mu-\alpha)\Gamma_{3i}],\quad N_{3i}=2h[(\mu+\alpha)\Gamma_{3i}+(\mu-\alpha)\Gamma_{i3}],$$

$$L_{ii}=2h[(\beta+2\gamma)\kappa_{ii}+\beta(\kappa_{jj}+\iota)],\ L_{33}=2h[(\beta+2\gamma)\iota+\beta(\kappa_{11}+\kappa_{22})],$$

$$L_{ij}=2h[(\gamma+\varepsilon)\kappa_{ij}+(\gamma-\varepsilon)\kappa_{ji}],\ L_{i3}=2h\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\kappa_{i3},\ \Lambda_{i3}=\frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon}l_{i3}. \tag{2}$$

Geometrically Relations

$$\Gamma_{ii}=\frac{1}{A_i}\frac{\partial u_i}{\partial \alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}u_j+\frac{1}{2}\left(\frac{1}{A_i}\frac{\partial w}{\partial \alpha_i}\right)^2,\ \Gamma_{i3}=\frac{1}{A_i}\frac{\partial w}{\partial \alpha_i}+\left(-1\right)^j\Omega_j,$$

$$\Gamma_{ij}=\frac{1}{A_i}\frac{\partial u_j}{\partial \alpha_i}-\frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}u_i-\left(-1\right)^j\Omega_3+\frac{1}{2}\frac{1}{A_1A_2}\frac{\partial w}{\partial \alpha_1}\frac{\partial w}{\partial \alpha_2},\ \Gamma_{3i}=\psi_i-\left(-1\right)^j\Omega_j,$$

$$K_{ii}=\frac{1}{A_i}\frac{\partial\psi_i}{\partial\alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\psi_j,\ K_{ij}=\frac{1}{A_i}\frac{\partial\psi_j}{\partial\alpha_i}-\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\psi_i-(-1)^j\iota,$$

$$\kappa_{ii}=\frac{1}{A_i}\frac{\partial\Omega_i}{\partial\alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\Omega_j,\quad \kappa_{33}=\iota,\quad \kappa_{ij}=\frac{1}{A_i}\frac{\partial\Omega_j}{\partial\alpha_i}-\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\Omega_i,$$

$$\kappa_{i3}=\frac{1}{A_i}\frac{\partial\Omega_3}{\partial\alpha_i},\quad l_{i3}=\frac{1}{A_i}\frac{\partial\iota}{\partial\alpha_i}. \tag{3}$$

Boundary conditions

$$T_{11}=T_{11}^0,\ S_{12}=S_{12}^0\ ,\ M_{11}=M_{11}^0, M_{12}=M_{12}^0,$$

$$T_{11}\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1}+\frac{S_{12}+S_{21}}{2}\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2}+N_{13}=N_{13}^0, L_{11}=L_{11}^0,\ \ L_{12}=L_{12}^0,\ \Lambda_{13}=\Lambda_{13}^0. \tag{4}$$

Here u_1,u_2 – are displacements of the points of the median plane of the plate around the axes α_1,α_2; w – is displacement of the points of the median plane of the plate in the direction of the axis z; ψ_1,ψ_2 – are complete rotation angles; $\Omega_1,\Omega_2,\Omega_3$ – are certain free rotations of the initially normal elements around the axes α_1,α_2,z; ι – is intensity of the free rotation ω_3 around the axis z; Γ_{ii} – are elongation deformations in the directions α_1,α_2; $\Gamma_{ii},\Gamma_{ij},\Gamma_{i3},\Gamma_{3i}$ – are shears in the corresponding planes; K_{ii} – are flexures of the plate median plane caused by the stresses; K_{ij} – are torsions of the plate median plane caused by the stresses; κ_{ii},κ_{33} – are flexures of the plate median plane caused by the couple stresses; κ_{ij} – are torsions of the plate median plane caused by the couple stresses; l_{i3} – are hyper shears of the plate median plane caused by the couple stresses; $T_{ii},S_{ij},N_{i3},N_{3i}$ – are averaged forces from the stresses, M_{ii},H_{ij} – are averaged moments from the stresses; $L_{ii},L_{ij},L_{i3},L_{33}$ – are averaged moments from the couple stresses and Λ_{i3} – are hyper moments from the couple stresses; $E,\nu,\alpha,\beta,\gamma,\varepsilon,\mu$ – are physical elasticity parameters of material; p_i^+,p_i^-,m_i^+,m_i^- – are external forces and moments on the planes $z=\pm h$; A_i – are Lame coefficients [8].

Let us note that from this model we can obtain the corresponding geometrically linear model by discarding nonlinear terms [8]. We also can get a geometrically nonlinear classical Timoshenko-type model, if to put $\alpha=0$.

3. Geometrically nonlinear applied model of micropolar elastic thin round plates

In the case of round plates in Equations (1) - (4) we will accept $A_1=1,\ \ A_2=r$ and use the polar coordinates r,θ. As a result, we get:

Balance Equations

$$\frac{\partial T_{11}}{\partial r}+\frac{1}{r}(T_{11}-T_{22})+\frac{1}{r}\frac{\partial S_{21}}{\partial\theta}=-(p_1^+-p_1^-),\ \ \frac{1}{r}\frac{\partial T_{22}}{\partial\theta}+\frac{\partial S_{ji}}{\partial r}+\frac{1}{r}(S_{12}+S_{21})=-(p_2^+-p_2^-),$$

$$\frac{\partial L_{13}}{\partial r}+\frac{1}{r}L_{13}+\frac{1}{r}\frac{\partial L_{23}}{\partial\theta}+(S_{12}-S_{21})=-(m_3^+-m_3^-),$$

$$\frac{\partial N_{13}}{\partial r}+\frac{1}{r}N_{13}+\frac{1}{r}\frac{\partial N_{23}}{\partial\theta}+\frac{\partial^2 w}{\partial r^2}T_{11}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}T_{22}+\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta}\left(S_{12}+S_{21}\right)+$$

$$+\frac{\partial w}{\partial r}\left[\frac{\partial T_{11}}{\partial r}+\frac{1}{r}T_{11}+\frac{1}{2r}\frac{\partial\left(S_{12}+S_{21}\right)}{\partial\theta}\right]+\frac{\partial w}{\partial\theta}\left[\frac{1}{r^2}\frac{\partial T_{22}}{\partial\theta}+\frac{1}{r}T_{11}+\frac{1}{2r}\frac{\partial\left(S_{12}+S_{21}\right)}{\partial r}\right]=-(p_3^+-p_3^-),$$

$$\frac{\partial L_{11}}{\partial r}+\frac{1}{r}(L_{11}-L_{22})+\frac{1}{r}\frac{\partial L_{21}}{\partial\theta}+N_{23}-N_{32}=-(m_1^+-m_1^-),$$

$$\frac{\partial L_{12}}{\partial r}+\frac{1}{r}\frac{\partial L_{22}}{\partial\theta}+\frac{1}{r}(L_{12}+L_{21})+N_{31}-N_{13}=-(m_2^+-m_2^-),$$

$$N_{31}-\left(\frac{\partial M_{11}}{\partial r}+\frac{1}{r}(M_{11}-M_{22})+\frac{1}{r}\frac{\partial M_{21}}{\partial\theta}\right)=h(p_1^++p_1^-),$$

$$N_{32}-\left(\frac{1}{r}\frac{\partial M_{22}}{\partial\theta}+\frac{\partial M_{12}}{\partial r}+\frac{1}{r}(M_{12}+M_{21})\right)=h(p_2^++p_2^-),$$

$$L_{33}-\frac{\partial\Lambda_{13}}{\partial r}-\frac{1}{r}\Lambda_{13}-\frac{1}{r}\frac{\partial\Lambda_{23}}{\partial\theta}-(M_{12}-M_{21})=h(m_3^++m_3^-). \tag{5}$$

Physical-Geometrically Relations

$$T_{11}=\frac{2Eh}{1-\nu^2}\left[\frac{\partial u_1}{\partial r}+\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2+\nu\left(\frac{1}{r}\frac{\partial u_2}{\partial\theta}+\frac{1}{r}u_1+\frac{1}{2}\frac{1}{r^2}\left(\frac{\partial w}{\partial\theta}\right)^2\right)\right],$$

$$T_{22}=\frac{2Eh}{1-\nu^2}\left[\frac{1}{r}\frac{\partial u_2}{\partial\theta}+\frac{1}{r}u_1+\frac{1}{2}\frac{1}{r^2}\left(\frac{\partial w}{\partial\theta}\right)^2+\nu\left(\frac{\partial u_1}{\partial r}+\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2\right)\right],$$

$$S_{12}=2h\left[(\mu+\alpha)\frac{\partial u_2}{\partial r}+(\mu-\alpha)\left(\frac{1}{r}\frac{\partial u_1}{\partial\theta}-\frac{1}{r}u_2\right)-2\alpha\Omega_3+\mu\frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial\theta}\right],$$

$$S_{21}=2h\left[(\mu+\alpha)\left(\frac{1}{r}\frac{\partial u_1}{\partial\theta}-\frac{1}{r}u_2\right)+(\mu-\alpha)\frac{\partial u_2}{\partial r}+2\alpha\Omega_3+\mu\frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial\theta}\right],$$

$$N_{13}=2h\left[(\mu+\alpha)\frac{\partial w}{\partial r}+(\mu-\alpha)\psi_1+2\alpha\Omega_2\right],\ N_{23}=2h\left[(\mu+\alpha)\frac{1}{r}\frac{\partial w}{\partial\theta}+(\mu-\alpha)\psi_2-2\alpha\Omega_1\right],$$

$$N_{31}=2h\left[(\mu+\alpha)\psi_1+(\mu-\alpha)\frac{\partial w}{\partial r}-2\alpha\Omega_2\right],\ N_{32}=2h\left[(\mu+\alpha)\psi_2+(\mu-\alpha)\frac{1}{r}\frac{\partial w}{\partial\theta}+2\alpha\Omega_1\right],$$

$$M_{11}=\frac{2Eh^3}{3\left(1-\nu^2\right)}\left[\frac{\partial\psi_1}{\partial r}+\nu\left(\frac{1}{r}\frac{\partial\psi_2}{\partial\theta}+\frac{1}{r}\psi_1\right)\right],M_{22}=\frac{2Eh^3}{3\left(1-\nu^2\right)}\left[\frac{1}{r}\frac{\partial\psi_2}{\partial\theta}+\frac{1}{r}\psi_1+\nu\frac{\partial\psi_1}{\partial r}\right],$$

$$M_{12}=\frac{2h^3}{3}\left[(\mu+\alpha)\frac{\partial\psi_2}{\partial r}+(\mu-\alpha)\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}-\frac{1}{r}\psi_2\right)-2\alpha\iota\right],$$

$$M_{21}=\frac{2h^3}{3}\left[(\mu+\alpha)\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}-\frac{1}{r}\psi_2\right)+(\mu-\alpha)\frac{\partial\psi_2}{\partial r}+2\alpha\iota\right],$$

$$L_{11}=2h\left[(\beta+2\gamma)\frac{\partial\Omega_1}{\partial r}+\beta\left(\frac{1}{r}\frac{\partial\Omega_2}{\partial\theta}+\frac{1}{r}\Omega_1\right)+\beta\iota\right],\ L_{13}=\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{\partial\Omega_3}{\partial r},$$

$$L_{22}=2h\left[(\beta+2\gamma)\left(\frac{1}{r}\frac{\partial\Omega_2}{\partial\theta}+\frac{1}{r}\Omega_1\right)+\beta\frac{\partial\Omega_1}{\partial r}-\beta\iota\right],\ L_{23}=\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{1}{r}\frac{\partial\Omega_3}{\partial\theta},$$

$$L_{12}=2h\left[(\gamma+\varepsilon)\frac{\partial\Omega_2}{\partial r}+(\gamma-\varepsilon)\left(\frac{1}{r}\frac{\partial\Omega_1}{\partial\theta}-\frac{1}{r}\Omega_2\right)\right],\ \Lambda_{13}=\frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{\partial\iota}{\partial r},$$

$$L_{21}=2h\left[(\gamma+\varepsilon)\left(\frac{1}{r}\frac{\partial\Omega_1}{\partial\theta}-\frac{1}{r}\Omega_2\right)+(\gamma-\varepsilon)\frac{\partial\Omega_2}{\partial r}\right],\ \Lambda_{23}=\frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{1}{r}\frac{\partial\iota}{\partial\theta},$$

$$L_{33}=2h\left[(\beta+2\gamma)\iota+\beta\left(\frac{\partial\Omega_1}{\partial r}+\frac{1}{r}\frac{\partial\Omega_2}{\partial\theta}+\frac{1}{r}\Omega_1\right)\right]. \tag{6}$$

Now we consider the axisymmetric problem, in this case, we will have:

Balance Equations

$$\frac{dT_{11}}{dr}+\frac{1}{r}\left(T_{11}-T_{22}\right)=-\left(p_1^+-p_1^-\right),\ \frac{dS_{12}}{dr}+\frac{1}{r}\left(S_{12}+S_{21}\right)=-\left(p_2^+-p_2^-\right),$$

$$\frac{dL_{13}}{dr}+\frac{1}{r}L_{13}+\left(S_{12}-S_{21}\right)=-\left(m_3^+-m_3^-\right),$$

$$\frac{dN_{13}}{dr}+\frac{1}{r}N_{13}+\frac{1}{r}\frac{d}{dr}\left(rT_{11}\frac{dw}{dr}\right)=-\left(p_3^+-p_3^-\right),$$

$$\frac{dL_{11}}{dr}+\frac{1}{r}\left(L_{11}-L_{22}\right)+N_{23}-N_{32}=-\left(m_1^+-m_1^-\right),$$

$$\frac{dL_{12}}{dr}+\frac{1}{r}\left(L_{12}+L_{21}\right)+N_{31}-N_{13}=-\left(m_2^+-m_2^-\right),$$

$$N_{31}-\left(\frac{dM_{11}}{dr}+\frac{1}{r}\left(M_{11}-M_{22}\right)\right)=h\left(p_1^++p_1^-\right),$$

$$N_{32}-\left(\frac{dM_{12}}{dr}+\frac{1}{r}\left(M_{12}+M_{21}\right)\right)=h(p_2^++p_2^-),$$

$$L_{33}-\frac{d\Lambda_{13}}{dr}-\left(M_{12}-M_{21}\right)=h(m_3^++m_3^-). \tag{7}$$

Physical-Geometrically Relations

$$T_{11}=\frac{2Eh}{1-\nu^2}\left[\frac{du_1}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2+\nu\frac{1}{r}u_1\right],S_{12}=2h\left[(\mu+\alpha)\frac{du_2}{dr}-(\mu-\alpha)\frac{1}{r}u_2-2\alpha\Omega_3\right],$$

$$T_{22}=\frac{2Eh}{1-\nu^2}\left[\frac{1}{r}u_1+\nu\left(\frac{du_1}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\right)\right],S_{21}=2h\left[-(\mu+\alpha)\frac{1}{r}u_2+(\mu-\alpha)\frac{du_2}{dr}+2\alpha\Omega_3\right],$$

$$N_{13}=2h\left[(\mu+\alpha)\frac{dw}{dr}+(\mu-\alpha)\psi_1+2\alpha\Omega_2\right],N_{23}=2h\left[(\mu-\alpha)\psi_2-2\alpha\Omega_1\right],$$

$$N_{31}=2h\left[(\mu+\alpha)\psi_1+(\mu-\alpha)\frac{dw}{dr}-2\alpha\Omega_2\right],\ N_{32}=2h\left[(\mu+\alpha)\psi_2+2\alpha\Omega_1\right],$$

$$M_{12}=\frac{2h^3}{3}\left[(\mu+\alpha)\frac{d\psi_2}{dr}-(\mu-\alpha)\frac{1}{r}\psi_2-2\alpha\iota\right],\ M_{11}=\frac{2Eh^3}{3(1-\nu^2)}\left[\frac{d\psi_1}{dr}+\nu\frac{1}{r}\psi_1\right],$$

$$M_{21}=\frac{2h^3}{3}\left[-(\mu+\alpha)\frac{1}{r}\psi_2+(\mu-\alpha)\frac{d\psi_2}{dr}+2\alpha\iota\right],\ M_{22}=\frac{2Eh^3}{3(1-\nu^2)}\left[\frac{1}{r}\psi_1+\nu\frac{d\psi_1}{dr}\right],$$

$$L_{11}=2h\left[(\beta+2\gamma)\frac{d\Omega_1}{dr}+\beta\frac{1}{r}\Omega_1+\beta\iota\right],\ L_{22}=2h\left[(\beta+2\gamma)\frac{1}{r}\Omega_1+\beta\frac{d\Omega_1}{dr}-\beta\iota\right],$$

$$L_{12}=2h\left[(\gamma+\varepsilon)\frac{d\Omega_2}{dr}-(\gamma-\varepsilon)\frac{1}{r}\Omega_2\right],\ L_{21}=2h\left[-(\gamma+\varepsilon)\frac{1}{r}\Omega_2+(\gamma-\varepsilon)\frac{d\Omega_2}{dr}\right],$$

$$L_{33}=2h\left[(\beta+2\gamma)\iota+\beta\left(\frac{d\Omega_1}{dr}+\frac{1}{r}\Omega_1\right)\right],\ \Lambda_{13}=\frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{d\iota}{dr},L_{13}=\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{d\Omega_3}{dr}. \tag{8}$$

To the Equations (5), (6), or (7), (8) need to join the corresponding boundary conditions (4). From systems (5), (6), and (7), (8) we can obtain the basic equations regarding the displacements and rotations.

4. The linearized equations of stability of micropolar elastic round plates. The critical force of compressed round micropolar plate

Let us consider micropolar elastic round solid plate under the action of radial compression P, which is uniformly distributed along the contour. The flat stress-strain state (SSS) has been implemented, we will call it the main subcritical state, and it is the solution to the following linear system of equations of the generalized plane stress state of micropolar elastic thin round plate (as in the case of the classical theory [2]):

Balance Equations

$$\frac{dT_{11}^0}{dr}+\frac{1}{r}\left(T_{11}^0-T_{22}^0\right)=0,\ \frac{dS_{12}^0}{dr}+\frac{1}{r}\left(S_{12}^0+S_{21}^0\right)=0,\ \frac{dL_{13}^0}{dr}+\frac{1}{r}L_{13}^0+\left(S_{12}^0-S_{21}^0\right)=0. \tag{9}$$

Physical-Geometrically Relations

$$T_{11}^0=\frac{2Eh}{1-\nu^2}\left[\frac{du_1^0}{dr}+\nu\frac{1}{r}u_1^0\right],\ T_{22}^0=\frac{2Eh}{1-\nu^2}\left[\frac{1}{r}u_1^0+\nu\frac{du_1^0}{dr}\right],$$

$$S_{12}^0=2h\left[(\mu+\alpha)\frac{du_2^0}{dr}-(\mu-\alpha)\frac{1}{r}u_2^0-2\alpha\Omega_3^0\right],$$

$$S_{21}^0=2h\left[-(\mu+\alpha)\frac{1}{r}u_2^0+(\mu-\alpha)\frac{du_2^0}{dr}+2\alpha\Omega_3^0\right],L_{13}^0=\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{d\Omega_3^0}{dr}. \tag{10}$$

The following boundary conditions must be satisfied on the contour of round solid plate (Fig. 1):

$$T_{11}^0=-P,\ u_2^0=0,\ L_{13}^0=0,\ \text{at } r=R,\ \text{where } p=const. \tag{11}$$

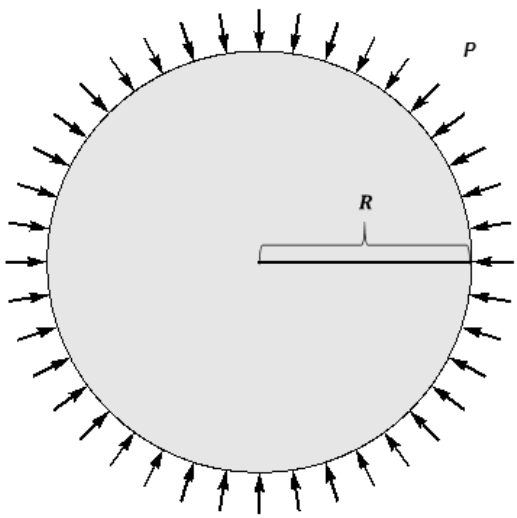

Fig. 1. Middle surface of the round plate

The components of displacements, deformations, force, and moment stresses corresponding to this SSS are marked with superscripts zero. In case of loss of stability, subcritical SSS will receive some disturbances. The values characterizing the SSS caused by these disturbances, we will imagine that they are marked with the asterisk at the top. The perturbed SSS in the plate is characterized by the values of the corresponding sums with indices of zero and asterisk.

Perturbations (i.e. the values with asterisks at the top) are small and during the transformations, we will neglect their powers higher than the first.

It is easy to notice that the solution of the subcritical boundary value problem (9) - (11) has the form (which satisfy the indicated equations and boundary conditions):

$$\overset{0}{T}_{11} = const = -P\,,\ \overset{0}{T}_{22} = const = -P\,, \overset{0}{u}_2 \equiv 0\,, \overset{0}{S}_{12} = \overset{0}{S}_{21} \equiv 0\,, \overset{0}{L}_{13} = \overset{0}{L}_{31} \equiv 0,\ \overset{0}{\Omega}_3 \equiv 0\ \text{ю} \tag{12}$$

The displacement u_1^0 is determined on the basis of the expressions for T_{11}^0 and T_{22}^0 from (10).

We investigate the stability of a round solid plate of radius R, with the hinge supported contour and under the action of radial compressive forces P uniformly distributed along the contour of the plate. We assume that the curved surface is axisymmetric [2].

We substitute the noted total relations into the system of equations of the geometrically nonlinear theory of micropolar plates (7), (8), then we obtain the equations of the perturbed stress state in the form:

Balance Equations

$$\frac{dT_{11}}{dr} + \frac{1}{r}\left(T_{11} - T_{22}\right) = 0\,,\ \frac{dS_{12}}{dr} + \frac{1}{r}\left(S_{12} + S_{21}\right) = 0\,,$$

$$N_{32} - \left(\frac{dM_{12}}{dr} + \frac{1}{r}\left(M_{12} + M_{21}\right)\right) = 0\,,\ \frac{dL_{13}}{dr} + \frac{1}{r}L_{13} + \left(S_{12} - S_{21}\right) = 0\,,$$

$$\frac{dL_{11}}{dr} + \frac{1}{r}\left(L_{11} - L_{22}\right) + N_{23} - N_{32} = 0\,,\ L_{33} - \frac{d\Lambda_{13}}{dr} - \left(M_{12} - M_{21}\right) = 0\,, \tag{13}$$

$$\frac{dN_{13}}{dr} + \frac{1}{r}N_{13} + \frac{1}{r}\frac{d}{dr}\left(rP\frac{dw}{dr}\right) = 0\,, N_{31} - \left(\frac{dM_{11}}{dr} + \frac{1}{r}\left(M_{11} - M_{22}\right)\right) = 0,$$

$$\frac{dL_{12}}{dr} + \frac{1}{r}\left(L_{12} + L_{21}\right) + N_{31} - N_{13} = 0\,. \tag{14}$$

Physical-Geometrically Relations

$$T_{11} = \frac{2Eh}{1-\nu^2}\left[\frac{du_1}{dr} + \nu\frac{1}{r}u_1\right],\ S_{12} = 2h\left[(\mu+\alpha)\frac{du_2}{dr} - (\mu-\alpha)\frac{1}{r}u_2 - 2\alpha\Omega_3\right],$$

$$T_{22} = \frac{2Eh}{1-\nu^2}\left[\frac{1}{r}u_1 + \nu\frac{du_1}{dr}\right],\ S_{21} = 2h\left[-(\mu+\alpha)\frac{1}{r}u_2 + (\mu-\alpha)\frac{du_2}{dr} + 2\alpha\Omega_3\right],$$

$$M_{12} = \frac{2h^3}{3}\left[(\mu+\alpha)\frac{d\psi_2}{dr} - (\mu-\alpha)\frac{1}{r}\psi_2 - 2\alpha\iota\right],\ N_{23} = 2h\left[(\mu-\alpha)\psi_2 - 2\alpha\Omega_1\right],$$

$$M_{21} = \frac{2h^3}{3}\left[-(\mu+\alpha)\frac{1}{r}\psi_2 + (\mu-\alpha)\frac{d\psi_2}{dr} + 2\alpha\iota\right], N_{32} = 2h\left[(\mu+\alpha)\psi_2 + 2\alpha\Omega_1\right],$$

$$L_{11} = 2h\left[(\beta+2\gamma)\frac{d\Omega_1}{dr} + \beta\frac{1}{r}\Omega_1 + \beta\iota\right],\ L_{22} = 2h\left[(\beta+2\gamma)\frac{1}{r}\Omega_1 + \beta\frac{d\Omega_1}{dr} - \beta\iota\right],$$

$$L_{33} = 2h\left[(\beta+2\gamma)\iota + \beta\left(\frac{d\Omega_1}{dr} + \frac{1}{r}\Omega_1\right)\right], \Lambda_{13} = \frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{d\iota}{dr}, L_{13} = \frac{4\gamma\varepsilon}{\gamma+\varepsilon}\frac{d\Omega_3}{dr}, \tag{15}$$

$$M_{11}=\frac{2Eh^3}{3(1-\nu^2)}\left[\frac{d\psi_1}{dr}+\nu\frac{1}{r}\psi_1\right],\ N_{13}=2h\left[(\mu+\alpha)\frac{dw}{dr}+(\mu-\alpha)\psi_1+2\alpha\Omega_2\right],$$

$$M_{22}=\frac{2Eh^3}{3(1-\nu^2)}\left[\frac{1}{r}\psi_1+\nu\frac{d\psi_1}{dr}\right],\ N_{31}=2h\left[(\mu+\alpha)\psi_1+(\mu-\alpha)\frac{dw}{dr}-2\alpha\Omega_2\right],$$

$$L_{12}=2h\left[(\gamma+\varepsilon)\frac{d\Omega_2}{dr}-(\gamma-\varepsilon)\frac{1}{r}\Omega_2\right],\ L_{21}=2h\left[-(\gamma+\varepsilon)\frac{1}{r}\Omega_2+(\gamma-\varepsilon)\frac{d\Omega_2}{dr}\right] \tag{16}$$

Hinge supported boundary conditions:

$$\Omega_1=0,\quad \psi_2=0,\quad \Lambda_{13}=0,\ \text{at}\ r=R, \tag{17}$$

$$w=0,\quad M_{11}=0,\quad L_{12}=0,\ \text{at}\ r=R. \tag{18}$$

Problem (13), (15), (17) has a trivial solution, and problem (14), (16), (18) can be reduced to the following form with respect to displacement w and rotations ψ_1 and Ω_2:

$$\left(\frac{d}{dr}+\frac{1}{r}\right)\left(\left((\mu+\alpha)-\frac{P}{2h}\right)\frac{dw}{dr}+(\mu-\alpha)\psi_1+2\alpha\Omega_2\right)=0, \tag{19}$$

$$(\mu-\alpha)\frac{dw}{dr}-2\alpha\Omega_2+(\mu+\alpha)\psi_1-\frac{Eh^2}{3(1-\nu^2)}\Delta\psi_1=0, \tag{20}$$

$$(\gamma+\varepsilon)\Delta\Omega_2-4\alpha\Omega_2+2\alpha\left(\psi_1-\frac{dw}{dr}\right)=0, \tag{21}$$

$$w=0,\quad \frac{\partial\psi_1}{\partial r}+\frac{\nu}{r}\psi_1=0,\quad (\gamma+\varepsilon)\frac{\partial\Omega_2}{\partial r}-(\gamma-\varepsilon)\frac{1}{r}\Omega_2=0,\ \text{at}\ r=R. \tag{22}$$

Here we note

$$\Delta=\frac{d}{dr}\left(\frac{d}{dr}+\frac{1}{r}\right). \tag{23}$$

By integrating equation (19) we can obtain:

$$\left((\mu+\alpha)-\frac{P}{2h}\right)\frac{dw}{dr}+(\mu-\alpha)\psi_1+2\alpha\Omega_2=\frac{C_1}{r}. \tag{24}$$

In the case of the solid plate we have $C_1=0$.

Then we can obtain a differential equation along the ψ_1:

$$B_1\Delta\Delta\psi_1+B_2\Delta\psi_1+B_3=0, \tag{25}$$

where

$$B_1=-\frac{(\gamma+\varepsilon)}{2\alpha}\left[(\mu+\alpha)-\frac{P}{2h}\right]\frac{1}{2\mu-\frac{P}{2h}}\frac{Eh^2}{3(1-\nu^2)},\ B_3=\frac{P}{2h}\frac{2\mu}{2\mu-\frac{P}{2h}},$$

$$B_2=\frac{(\gamma+\varepsilon)}{2\alpha}\left[(\mu+\alpha)-\frac{P}{2h}\right]\frac{2\mu}{2\mu-\frac{P}{2h}}-\frac{(\gamma+\varepsilon)}{2\alpha}(\mu-\alpha)+\left(2\mu-\frac{P}{h}\right)\frac{1}{2\mu-\frac{P}{2h}}\frac{Eh^2}{3(1-\nu^2)}. \tag{26}$$

If we have a solution of differential equation (25), then w and Ω_2 are defined as follows:

$$\Omega_2=\frac{1}{2\alpha}\left((\mu+\alpha)-\frac{P}{2h}\right)\frac{dw}{dr}+\frac{(\mu-\alpha)}{2\alpha}\psi_1, \tag{27}$$

$$\frac{dw}{dr}=-\frac{2\mu}{2\mu-\frac{P}{2h}}\psi_1+\frac{1}{2\mu-\frac{P}{2h}}\frac{Eh^2}{3(1-\nu^2)}\frac{d}{dr}\left(\frac{d}{dr}+\frac{1}{r}\right)\psi_1. \tag{28}$$

Differential equation (25) can be represented as:

$$(\Delta-\lambda_1)(\Delta-\lambda_2)\psi_1=0\,, \tag{29}$$

where λ_1,λ_2 are the roots of the next second-degree equation:

$$B_1\lambda^3+B_2\lambda^2+B_3=0\,, \tag{30}$$

or

$$\lambda_{1,2}=\frac{-B_2\pm\sqrt{(B_2)^2-4B_1B_3}}{2B_1}\,. \tag{31}$$

The solution of differential equation (29) for a solid plate has the form:

$$\psi_1(r)=C_1^1J_1\left(-i\sqrt{\lambda_1}r\right)+C_2^1J_1\left(-i\sqrt{\lambda_2}r\right). \tag{32}$$

Substituting (32) into (17) and (28) and integrating the result, we obtain:

$$w(r)=C_1^2J_0\left(-i\sqrt{\lambda_1}r\right)+C_2^2J_0\left(-i\sqrt{\lambda_2}r\right)+C_3^2\,, \tag{33}$$

$$\tilde{\Omega}_2(r)=C_1^3J_1\left(-i\sqrt{\lambda_1}r\right)+C_2^3J_1\left(-i\sqrt{\lambda_2}r\right), \tag{34}$$

where $J_0(x), J_1(x)$ -are the Bessel functions of the valid argument of the zero and first orders.

$$C_n^2=\frac{\left(\dfrac{Eh^2}{3\left(1-\nu^2\right)}\lambda_n+2\mu\right)}{\left(\dfrac{P}{2h}-2\mu\right)}C_n^1\,, \tag{35}$$

$$C_n^3=-\frac{\dfrac{Eh^2}{3\left(1-\nu^2\right)}\dfrac{\mu+\alpha}{2\alpha}\lambda_n+2\mu+\dfrac{P}{2h}\left(\dfrac{\mu+\alpha}{2\alpha}-\dfrac{Eh^2}{3\left(1-\nu^2\right)}\lambda_n\right)}{\dfrac{P}{2h}-2\mu}C_n^1. \tag{36}$$

To satisfy the boundary conditions (22), we obtain homogeneous algebraic equations along to the C_1^1, C_2^1, C_3^2. Further, demanding a nonzero solution, as a result, we obtain the transcendental equation with respect to P, which we can represent in compact form:

$$Det\left|a_n^m\right|=0\,,\quad n,m=1,2,3\,, \tag{37}$$

where

$$a_i^1=\frac{i2h\left(Eh^2\lambda_i-6\mu\left(1-\nu^2\right)\right)\left(-1+I_0\left(R\sqrt{\lambda_i}\right)\right)}{3\sqrt{\lambda_i}\left(P-4h\mu\right)\left(1-\nu^2\right)},\ a_3^1=1,$$

$$a_i^2=\left(\sqrt{\lambda_i}I_0\left(R\sqrt{\lambda_i}\right)-\frac{(1-\nu)I_1\left(R\sqrt{\lambda_i}\right)}{R}\right),\ a_3^2=a_3^3=0\,,$$

$$\begin{aligned}a_i^3=&\left(R\left(\gamma+\varepsilon\right)\sqrt{\lambda_i}\left(P\left(Kh^2\lambda_i-3\left(\mu+\alpha\right)\left(1-\nu^2\right)\right)-2h\left(Kh^2\lambda_i\left(\mu+\alpha\right)-12\mu\alpha\left(1-\nu^2\right)\right)\right)\times\right.\\&\left.\times I_0\left(R\sqrt{\lambda_i}\right)+2\gamma\left(2Kh^3\lambda_i\left(\mu+\alpha\right)-24h\mu\alpha\left(1-\nu^2\right)-P\left(Kh^2\lambda_i-3\left(\mu+\alpha\right)\left(1-\nu^2\right)\right)\right)I_1\left(R\sqrt{\lambda_i}\right).\end{aligned} \tag{38}$$

Here $I_0(x), I_1(x)$ – Bessel functions of purely imaginary argument of the zero and the first orders.

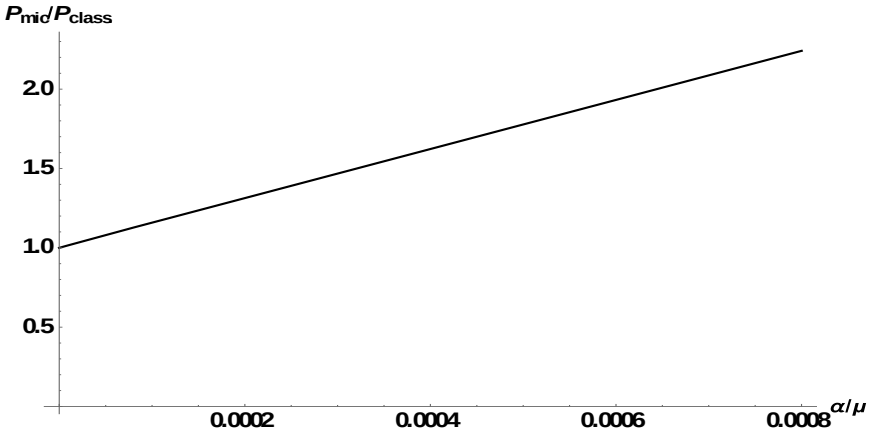

Fig. 2. Dependence of the dimensionless quantity $P_{mic.}/P_{class.}$ on the dimensionless physical parameter α/μ

We consider a specific numerical example for hypothetic material: $R = 0.07\,m$, $h = \frac{a}{40}$, $\mu = 2\,MPa$, $\lambda = 3\,MPa$, $\gamma = \varepsilon = 150$ H. On Fig. 2 the dependence of the dimensionless quantity $P_{mic.}/P_{class.}$ on the dimensionless physical parameter, α/μ is presented, $P_{mic.}$ is the critical value of the external force for micropolar theory, $P_{class.}$ is the same quantity for classical theory. As we note we can get a geometrically nonlinear classical Timoshenko-type model from the micropolar model if to put $\alpha = 0$. For more large values of α, we obtain more large values for the $P_{mic.}$ compared to the value $P_{class.}$. So it is easy to verify that with increasing α/μ, the value $P_{mic.}/P_{class.}$ increases, which means that in the case of micropolar material the round plate is more stable.

5. Conclusion

In the work, the stability problem of micropolar elastic round plates is studied. During the study of the problem, the stability equations are obtained using the linearization procedure in the geometrically nonlinear equations.

The concrete stability problem of round micropolar plate was studied, and it was shown that, with other parameters being equal, the round micropolar plate is more stable than the round plate with the classical material.

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