Materials Physics and Mechanics 16 (2013) 135-143 Received: February 20, 2013

PROPAGATION OF WAVE THROUGH CYLINDRICAL BORE IN A SWELLING POROUS ELASTIC MEDIA

Rajneesh Kumar[1*], Divya[2], Kuldeep Kumar[2]**

[1]Department of Mathematics, Kurukshetra University ,Kurukshetra, Haryana, 136119, India

[2]Department of Mathematics, N.I.T., Kurukshetra, Haryana, 136119, India

*e-mail: rajneesh_kuk@rediffmail.com

**e-mail: divyataneja82@yahoo.com

Abstract. The present paper deals with the axial symmetric cylindrical waves propagating through a cylindrical bore in swelling porous elastic medium. The secular equations, connecting the phase velocity with wave number, radius of bore and other parameters for empty and liquid filled bore are derived. A particular case of interest has also been deduced. Numerical computations have been performed and have also been shown graphically to understand behavior of phase velocity and attenuation coefficient in swelling porous (SP) and elastic medium (EL).

1. Introduction

The dynamic response of porous media is of great interest in various areas such as geophysics, soil-mechanics, civil engineering, petroleum engineering and environmental engineering. As most of the modern engineering structures are generally made up of multiphase porous continuum, the classical theory, which represents a fluid saturated porous medium as a single phase material, is inadequate to represent the mechanical behavior of such materials especially when the pores are filled with liquid. Due to these different motions, the different material properties and the complicated geometry of pore structures; the mechanical behavior of a fluid saturated porous medium is very complex and difficult. So from time to time, researchers have tried to overcome this difficulty and considerable work has been done in this regard. Biot [1] was first who studied the propagation of elastic waves in cylindrical borehole containing fluid. Biot [2, 3] developed a linear theory for a fluid saturated porous elastic solid. Eringen [4] pointed out the importance of the theory of mixtures to the applied field of swelling. He developed a continuum theory of mixtures for porous elastic solids filled with fluid and gas. Tomar and Kumar [5], Deswal et al. [6], and Kumar et al. [7] studied problems of wave propagation through a cylindrical bore in a micropolar elastic medium with a stretch micropolar elastic medium. Kumar and Deswal [8] studied the wave propagation through cylindrical bore contained in a microstretch elastic medium. Kumar and Deswal [9] discussed surface wave propagation through a cylindrical bore in a micro stretch generalized thermoelastic medium without energy dissipation. Bofill and Quintanilla [10] studied anti plane shear deformations of swelling porous elastic soils in case of fluid saturation and gas saturation. Gales [11] investigated some theoretical problems concerning waves and vibrations within the context of the isothermal linear theory of swelling porous elastic soils with fluid or gas saturation. Tersa and Bennethum [12] derived transport equations for porous swelling materials that undergo finite deformations. Kleintelter, Park, and Cushman [13] discussed the various aspects of mixture theory applied to unsaturated/saturated swelling soils and studied the two and three phase problems. Gales [14] studied the asymptotic spatial behavior of solutions in a mixture consisting of two thermo elastic solids. Gales [15] studied the spatial behavior of the harmonic vibrations in thermal viscoelastic mixtures.

The cylindrical bore may be realized by a borehole or a mine gallery. Borehole studied are of great interest in exploration seismology, e.g. in the exploration of oils, gases, hydrocarbons etc. In the oils Industry, acoustic borehole logging is commonly practiced. A bore hole is drilled in a potential hydrocarbon reservoir and then probed with an acoustic tool. Almost all oil companies rely on seismic interpretation for selecting the sites for exploratory oil wells. Seismic wave methods also have higher accuracy, higher resolutions and are more economical as compared to drilling which is costly and time consuming. Kumar and Panchal [16] studied the propagation of waves through cylindrical bore in a cubic micropolar generalized thermoelastic medium. Kessler and Kosloffs [17] studied the elastic wave propagation in cylindrical coordinates in geophysics.

In the present paper we have studied the propagation of waves through a cylindrical bore in a swelling porous elastic medium. Secular equations relating the phase velocity and wave number are derived for empty and filled cylindrical bore in SP and EL media. The results so obtained are compared and have also been shown graphically for both SP and EL media. The problem has immense application in mines, oil slurries, and structure problems.

2. Basic equations:

Following Eringen [4], the field equations in linear theory of swelling porous elastic soils are

$$\mu u^s_{i,jj} + (\lambda+\mu)u^s_{j,ji} - \sigma^f u^f_{j,ji} + \xi^{ff}(\dot{u}^f_i - \dot{u}^s_i) + f^s_i = \rho^s_0 \ddot{u}^s_i, \tag{1}$$

$$\mu_v \dot{u}^f_{i,jj} + (\lambda_v+\mu_v)\dot{u}^f_{j,ji} - \sigma^f u^s_{j,ji} - \sigma^{ff} u^f_{j,ji} - \xi^{ff}(\dot{u}^f_i - \dot{u}^s_i) + f^f_i = \rho^f_0 \ddot{u}^f_i, \tag{2}$$

$$t^s_{ij} = (-\sigma^f u^f_{r,r} + \lambda u^s_{r,r})\delta_{ij} + \mu(u^s_{i,j} + u^s_{j,i}), \tag{3}$$

$$t^f_{ij} = (-\sigma^f u^s_{r,r} - \sigma^{ff} u^f_{r,r} + \lambda_v \dot{u}^f_{r,r})\delta_{ij} + \mu_v(\dot{u}^f_{i,j} + \dot{u}^f_{j,i}), \qquad i,j=1,2,3 \tag{4}$$

where, the superscripts *s* and *f* denote respectively , the elastic solid and the fluid; u^s_i and u^f_i are the displacement components of solid and fluid respectively. The functions (f^s_i, f^f_i) are the body forces, ρ^s_0, ρ^f_0 are the densities of each constituent and $\lambda, \mu, \lambda_v, \mu_v, \sigma^f, \sigma^{ff}, \xi^{ff}$ are constitutive constants. Subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate, and a superposed dot denotes time differentiation, t^s_{ij}, t^f_{ij} are the partial stress tensors.

3. Problem formulation and its solution

We consider a cylindrical bore of radius a^* having circular cross section in a swelling porous elastic medium. We use cylindrical polar coordinates (r, θ,z) with z-axis pointing upwards along axis of cylinder as shown in Fig. 1. The propagation of axial symmetric waves is considered near the bore hole and these waves are the analogue of Rayleigh wave propagation at a traction free boundary of a swelling porous elastic medium. This section deals with the situation when bore does not contain any fluid. We are discussing a two dimensional problem with z-axis coincides with the axis of plate i.e. $\partial/\partial\theta = 0$ Therefore, we take

$$\vec{u}^i = (u^i_r, u^i_\theta, 0),\ \vec{\phi}^i = (0, \phi^i_\theta, 0),\ \vec{\psi}^i = (0, \psi^i_\theta, 0) \qquad \text{(where } i=s, f\text{)}. \tag{5}$$

We define the non-dimensional quantities:

$$r' = \frac{r}{a^*},\ z' = \frac{z}{a^*},\ u_r'^i = \frac{u_r^i}{a^*},\ u_z'^i = \frac{u_z^i}{a^*},\ t_{ij}'^s = \frac{t_{ij}^s}{\mu},\ t_{ij}'^f = \frac{t_{ij}^f}{\mu},\ c_1^2 = \frac{\lambda+2\mu}{\rho_0^s},\quad t' = \frac{c_1}{a^*}t,\ \omega^* = \frac{\xi^{ff}}{\rho_0^s}. \tag{6}$$

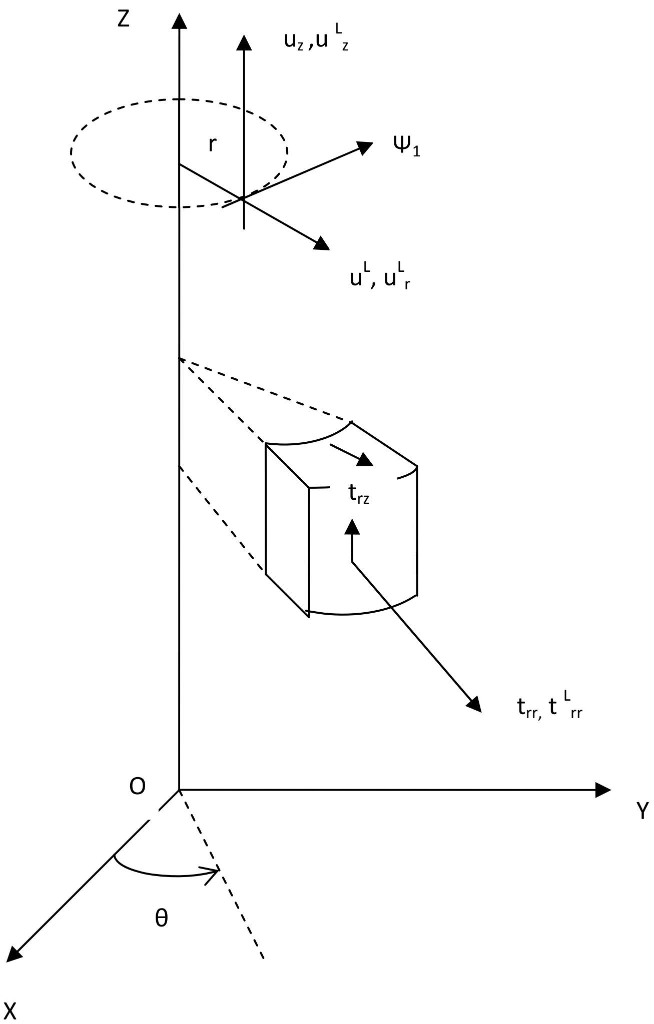

Fig. 1. Geometry of the problem.

The displacement components u_r^i, u_z^i are connected by the potential functions

$$u_r^i = \frac{\partial \phi^i}{\partial r} + \frac{\partial \psi^i}{\partial z},\qquad u_z^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial r} - \frac{\psi^i}{r}\quad , \tag{7}$$

Equations (1)-(4) with the aid of Eqs. (5)-(7), and after suppressing primes reduces to

$$\left(\nabla^2 - a_2\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\phi^s + \left(-a_1\nabla^2 + a_2\frac{\partial}{\partial t}\right)\phi^f = 0\,, \tag{8}$$

$$\left(\delta^2\left(\nabla^2 - \frac{1}{r^2}\right) - a_2\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\psi^s + a_2\frac{\partial}{\partial t}\psi^f = 0\,, \tag{9}$$

$$\left(-h_1\nabla^2+h_3\frac{\partial}{\partial t}\right)\phi^s+\left(\nabla^2\frac{\partial}{\partial t}-h_2\nabla^2-h_3\frac{\partial}{\partial t}-h_4\frac{\partial^2}{\partial t^2}\right)\phi^f=0, \tag{10}$$

$$h_3\frac{\partial}{\partial t}\psi^s+\left(\delta_1^2\left(\nabla^2-\frac{1}{r^2}\right)\frac{\partial}{\partial t}-h_3\frac{\partial}{\partial t}-h_4\frac{\partial^2}{\partial t^2}\right)\psi^f=0, \tag{11}$$

$$t_{rr}^s=\frac{\sigma^f}{\mu}\nabla^2\phi^f+\frac{\lambda}{\mu}\nabla^2\phi^s+\left(2\frac{\partial^2\phi^s}{\partial r^2}+\frac{\partial^2\psi^s}{\partial r\partial z}\right), \tag{12}$$

$$t_{rr}^f=-\frac{\sigma^f}{\mu}\nabla^2\phi^s-\frac{\sigma^{ff}}{\mu}\nabla^2\phi^f+\frac{\lambda_v c_1}{\mu a^*}\frac{\partial}{\partial t}\nabla^2\phi^f+\frac{2\mu_v c_1}{\mu a^*}\frac{\partial}{\partial t}\left(\frac{\partial^2\phi^f}{\partial r^2}+\frac{\partial^2\psi^f}{\partial r\partial z}\right), \tag{13}$$

$$t_{rz}^s=2\frac{\partial^2\phi^s}{\partial r\partial z}-\frac{\partial^2\psi^s}{\partial r^2}+\frac{\partial^2\psi^s}{\partial z^2}+\frac{\psi^s}{r^2}-\frac{1}{r}\frac{\partial\psi^s}{\partial z}, \tag{14}$$

$$t_{rz}^f=\frac{\mu_v c_1}{\mu a^*}\frac{\partial}{\partial t}\left(2\frac{\partial^2\phi^f}{\partial r\partial z}+\frac{\partial^2\psi^f}{\partial z^2}-\frac{\partial^2\psi^f}{\partial r^2}-\frac{1}{r}\frac{\partial\psi^f}{\partial r}+\frac{\psi^f}{r^2}\right), \tag{15}$$

where,

$$a_1=\frac{\sigma^f}{\lambda+2\mu},\quad \delta^2=\frac{\mu}{\lambda+2\mu},\quad a_2=\frac{\xi^{ff}c_1a^*}{\lambda+2\mu},\quad h_1=\frac{\sigma^f a^*}{(\lambda_v+2\mu_v)c_1},\quad h_2=\frac{\sigma^{ff}a^*}{(\lambda_v+2\mu_v)c_1},$$

$$h_3=\frac{\xi^{ff}a^{*2}}{(\lambda_v+2\mu_v)},\ h_4=\frac{\rho_0^f c_1 a^*}{(\lambda_v+2\mu_v)},\ \nabla^2=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right),\ \delta_1^2=\frac{\mu_v}{(\lambda_v+2\mu_v)}.$$

4. Boundary conditions

At the surface r=1 the appropriate boundary conditions are

$(i)\quad t_{rr}^s+t_{rr}^f=0,$

$(ii)\quad t_{zr}^s+t_{zr}^f=0,$

$(iii)\quad \dot{u}_r^s-\dot{u}_r^f=0,$

$$(iv)\quad \dot{u}_z^s-\dot{u}_z^f=0. \tag{16}$$

5. Formal solution of the problem

Let solution of equations (8)-(11) as

$$(\phi^s,\phi^f,\psi^s,\psi^f)=(b_1K_0(mr),b_2K_0(mr),b_3K_1(mr),b_4K_1(mr))e^{i(kz-\omega t)}, \tag{17}$$

where k is wave number and ω is angular frequency.

Using equation (17) in Eqs. (8)-(11) and solving the resulting differential equations, the expressions for $\phi^s,\phi^f,\psi^s,\psi^f$ are obtained as

$$\phi^s=\{A_1K_0(m_1r)+A_2K_0(m_1r)\}e^{i(kz-\omega t)}, \tag{18}$$

$$\phi^f=\{\alpha_1A_1K_0(m_1r)+\alpha_2A_2K_0(m_1r)\}e^{i(kz-\omega t)}, \tag{19}$$

$$\psi^s = \{A_3K_1(m_3r) + A_4K_1(m_4r)\}e^{i(kz-\omega t)}\,, \tag{20}$$

$$\psi^f = \{\alpha_3A_3K_1(m_3r) + \alpha_4A_4K_1(m_4r)\}e^{i(kz-\omega t)}\,, \tag{21}$$

where $m_k^2 = -k^2(c^2q_k^2 - 1)$, $(q_1^2, q_2^2) = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$, $A = -i\omega - h_2 - h_1a_1$,

$B = \tau_2 - \tau_1(i\omega + h_2) - h_1\tau_0 - a_1\tau_3$, $C = \tau_1\tau_2 - \tau_0\tau_3$, $\tau_0 = \dfrac{ia_2}{\omega}$, $\tau_1 = 1 + \tau_0$, $\tau_2 = \dfrac{ih_3}{\omega}$, $\tau_3 = h_4 + \tau_2$,

$$(q_3^2, q_4^2) = \frac{-(\tau_2\delta^2 - i\omega\delta_1^2\tau_0) \pm \sqrt{(\tau_2\delta^2 - i\omega\delta_1^2\tau_0)^2 - 4(-i\omega\delta^2\delta_1^2)(\tau_1\tau_2 - \tau_0\tau_3)}}{-2i\omega\delta^2\delta_1^2}\,,$$

$$\alpha_{l'} = \frac{(m_{l'}^2 - k^2) + \omega^2\tau_1}{a_1(m_{l'}^2 - k^2) + \omega^2\tau_0},\quad \alpha_{r'} = \frac{\delta^2(m_{r'}^2 - k^2) + \omega^2\tau_1}{\omega^2\tau_0},\quad l' = 1,2;\ r' = 3,4$$

6. Derivation of secular equation

Making use of equations (18)-(21) in equations (12)-(15) and using boundary conditions (16), we obtain four homogeneous equations in four unknowns. The elimination of these unknowns gives the frequency equation:

$$(P_{33}P_{44} - P_{43}P_{34})(P_{11}P_{22} - P_{12}P_{21}) + (P_{32}P_{44} - P_{34}P_{42})(P_{13}P_{21} - P_{11}P_{23}) + (P_{32}P_{43} - P_{33}P_{42})(P_{11}P_{24} - P_{14}P_{21})$$

$$+(P_{31}P_{44} - P_{41}P_{34})(P_{12}P_{23} - P_{13}P_{22}) + (P_{31}P_{43} - P_{33}P_{41})(P_{13}P_{24} - P_{24}P_{12}) = 0\,, \tag{22}$$

where,

$$P_{1l'} = \left(\left(\left(\frac{-\sigma^f}{\mu} - \frac{\sigma^{ff}}{\mu} - \frac{i\lambda_v c_1\omega}{\mu a^*}\right)\alpha_{l'} + \frac{\lambda}{\mu} - \frac{\sigma^f}{\mu}\right)\left(m_{l'}^2 - k^2\right) + 2m_{l'}^2\left(1 - \frac{i\mu_v c_1\omega\alpha_{l'}}{\mu a^*}\right)\right)K_0(m_{l'}) + 2m_{l'}K_1(m_{l'})\left(1 - \frac{i\mu_v c_1\omega\alpha_{l'}}{\mu a^*}\right),$$

$P_{2l'} = -2ikm_{l'}\alpha_{l'}K_1(m_{l'})\left(1 - \dfrac{i\omega\mu_v c_1}{\mu a^*}\right)$, $P_{1r'} = 2ik\left(-1 + \dfrac{\mu_v c_1\omega\alpha_{r'}}{a^*\mu}\right)(m_{r'}K_0(m_{r'}) + K_1(m_{r'}))$,

$P_{2r'} = -(k^2 + m_{r'}^2)\alpha_{r'}K_1(m_{r'})\left(1 - \dfrac{i\omega\mu_v c_1}{\mu a^*}\right)$, $P_{3l'} = i\omega m_{l'}(1 - \alpha_{l'})K_1(m_{l'})$, $P_{3r'} = K_1(m_{r'})(1 - \alpha_{r'})ik$,

$P_{4l'} = k\omega(1 - \alpha_{l'})K_0(m_{l'})$ $P_{4r'} = i\omega m_{r'}(1 - \alpha_{r'})K_0(m_{r'})$, where $l' = 1,2$ and $r' = 3,4$.

7. Propagation of waves in a cylindrical bore filled with liquid

Here, we consider the same problem as in the previous section with the additional constraint that the borehole is filled with homogeneous inviscid liquid.

The field equation and constitutive relations for homogeneous inviscid liquid are

$$\lambda^L\nabla(\nabla.\vec{u}^L) = \rho^L\frac{\partial^2\vec{u}^L}{\partial t^2}\mathfrak{A}\,, \tag{23}$$

$$t_{ij}^L = \lambda^L(\nabla.\vec{u}^L)\delta_{ij}\,, \tag{24}$$

where $\vec{u}^L$ is the displacement vector, λ^L and ρ^L are respectively the bulk modulus and density of liquid. Other symbols have their usual meaning as defined earlier. For two dimensional problems we take

$$\vec{u}^L = (u_r^L, 0, u_z^L) \text{ and } \partial/\partial\theta = 0. \tag{25}$$

The dimensionless variables defined in this case, in addition to those defined by (6), are

$$u_r^{L'} = \frac{u_r^L}{a^*},\ u_z^{L'} = \frac{u_z^L}{a^*},\ t_{rr}^{L'} = \frac{t_{rr}^L}{\mu}. \tag{26}$$

We relate the dimensionless displacement components and potential function ϕ^L as

$$u_r^L = \frac{\partial \phi^L}{\partial r},\ \ u_z^L = \frac{\partial \phi^L}{\partial z}. \tag{27}$$

Making use of equation (27) in equations (23), (24), with the help of equations (26), (27) after suppressing the primes yields

$$\frac{\partial^2 \phi^L}{\partial r^2} + \frac{1}{r}\frac{\partial \phi^L}{\partial r} + \frac{\partial^2 \phi^L}{\partial z^2} = \delta_L^2 \frac{\partial^2 \phi^L}{\partial t^2}, \tag{28}$$

$$t_{rr}^L = \frac{\lambda^L}{C_{11}} \nabla^2 \phi^L, \tag{29}$$

where,

$$\delta^L = \frac{c_1}{c_L}, \qquad c^L = \sqrt{\frac{\lambda^L}{\rho^L}}. \tag{30}$$

The solution of (30) corresponding to surface waves may be written as

$$\phi^L = A_0 I_0(m_0 r) e^{i(kz-\omega t)}. \tag{31}$$

After some simplification, the pressure and radial displacement of liquid are given by

$$p^L = -t_{rr}^L = \delta^{*2}\omega^2 A_0 I_0(m_0 r) e^{i(kz-\omega t)}, \tag{32}$$

$$u^L = m_0 A_0 I_1(m_0 r) e^{i(kz-\omega t)}, \tag{33}$$

where,

$$\delta^{*2} = \frac{\rho^L c_1^2}{\mu}, \qquad m_0^2 = k^2(1-\delta_L^2 c^2), \tag{34}$$

where $I_0()$ and $I_1()$ are modified Bessel functions of first kind and of order zero and one respectively.

8. Derivation of secular equation

The appropriate boundary conditions for the present situation at $r=1$ are

$$t_{rr}^s + t_{rr}^f = -p^L,$$

$$t_{rz}^s + t_{rz}^f = 0,$$

$$\dot{u}_z^s - \dot{u}_z^f = 0,$$

$$\dot{u}_r^s - \dot{u}_r^f = 0,$$

$$u_r^s + u_r^f = u_r^L . \tag{35}$$

Making use of (31)-(33) and (18)-(21), in the boundary conditions (35), with the help of (12)-(15), we obtain five homogeneous equations in five unknowns. The conditions for the non-trivial solution yields the frequency equation

$$\begin{aligned}
&\delta^{*2}\omega^2 I_0(m_0)[-m_1K_1(m_1)\{P_{22}(P_{33}P_{44}-P_{34}P_{43})-P_{23}(P_{32}P_{44}-P_{34}P_{42})+P_{24}(P_{32}P_{43}-P_{33}P_{42})\}\\
&+m_2K_1(m_2)\{P_{21}(P_{33}P_{44}-P_{34}P_{43})-P_{23}(P_{31}P_{44}-P_{34}P_{41})+P_{24}(P_{31}P_{43}-P_{33}P_{41})\}\\
&+ikK_1(m_3)\{P_{21}(P_{32}P_{44}-P_{34}P_{42})-P_{22}(P_{31}P_{44}-P_{34}P_{41})+P_{24}(P_{31}P_{42}-P_{32}P_{41})\}\\
&-ikK_1(m_4)\{P_{21}(P_{32}P_{43}-P_{33}P_{42})-P_{22}(P_{31}P_{43}-P_{33}P_{41})+P_{23}(P_{31}P_{42}-P_{32}P_{41})\}]\\
&+m_0I_1(m_0)[P_{11}\{P_{22}(P_{33}P_{44}-P_{34}P_{43})-P_{23}(P_{32}P_{44}-P_{34}P_{42})+P_{24}(P_{32}P_{43}-P_{33}P_{42})\}\\
&-P_{12}\{P_{21}(P_{33}P_{44}-P_{34}P_{43})-P_{23}(P_{31}P_{44}-P_{34}P_{41})+P_{24}(P_{31}P_{43}-P_{33}P_{41})\}\\
&+P_{13}\{P_{21}(P_{32}P_{44}-P_{34}P_{42})-P_{22}(P_{31}P_{44}-P_{34}P_{41})+P_{24}(P_{31}P_{42}-P_{32}P_{41})\}\\
&-P_{14}\{P_{21}(P_{32}P_{43}-P_{33}P_{42})-P_{22}(P_{31}P_{43}-P_{33}P_{41})+P_{23}(P_{31}P_{42}-P_{32}P_{41})\}]=0 .
\end{aligned} \tag{36}$$

9. Particular case

In absence of swelling porous i.e., taking $\lambda_v=\mu_v=\sigma^f=\xi^{ff}=\sigma^{ff}=0$, we obtain the corresponding expressions for the elastic medium (EL) with the changed values of m_i (i=1, 2, 3, 4).

10. Numerical results and discussion

For numerical computation, we take the following values of parameters $\lambda = 2.238\times10^{10}\, N/m^2$, $\lambda_v = 2.05\times10^{10}\, N\,Sec/m^2$, $\rho_0^s = 2.65\times10^3\, N\,Sec^2/m^4$, $\mu = 2.992\times10^{10}\, N/m^2$, $\mu_v = 2.5\times10^{10}\, N\,Sec/m^2$, $\rho_0^f = 1.92\times10^3\, N\,Sec^2/m^4$, $\sigma^f = 1.21\times10^{10}\, N/m^2$, $\sigma^{ff} = 0.20\times10^{10}\, N/m^2$, $\xi^{ff} = 1.13\times10^3\, N\,Sec/m^4$, $a^* = 15m$, $\lambda^L = 2.1904\times10^{10}\, N/m^2$, $\rho^L = 1.01\times10^3\, Kg/m^3$. Equations (22) and (36) determine the phase velocity c of the axial symmetric surface waves as a function of wave number k, radius of bore and various physical parameters in complex form.

If we write

$$c^{-1} = v^{-1} + i\omega^{-1}q, \tag{37}$$

then wave number $k=R+iq$, where $R=\omega/v$ *and* q is attenuation coefficient of the surface waves. The graphical representation is given to depict the behavior of phase velocity and attenuation coefficient with respect to R i.e. real part of wave number to compare the results for swelling porous elastic medium (SP) and elastic medium.

Figure 2 depicts the variation of phase velocity with wave number in case of empty and liquid filled bore for SP and EL media. From Fig. 2 we notice, that in case of SP medium, phase velocity for empty cylindrical bore decreases with the little change in the magnitude values as the wave number increases whereas, for liquid filled cylindrical bore it initially increases and then became stationary. In case of elastic medium the phase velocity for both empty and liquid filled cylindrical bore decrease with the increase of wave number. In SP medium the phase velocity of empty cylindrical bore remains less than that of liquid filled cylindrical bore for $k \geq 1.25$, whereas for elastic medium the phase velocity for empty cylindrical bore remains less than the phase velocity of liquid filled cylindrical bore in whole range.

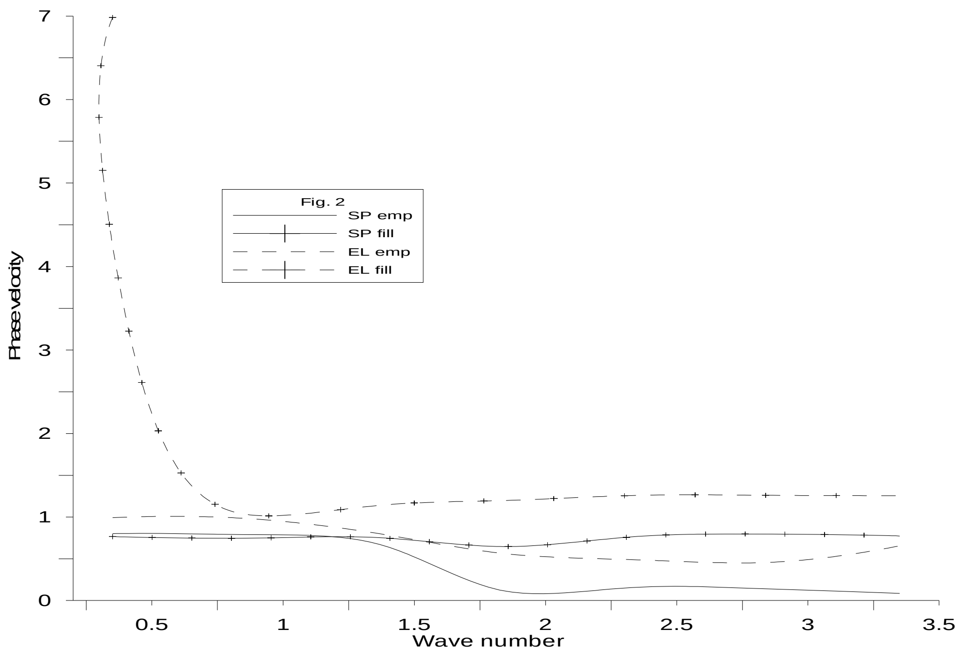

Fig. 2. Variation of phase velocity with wave number.

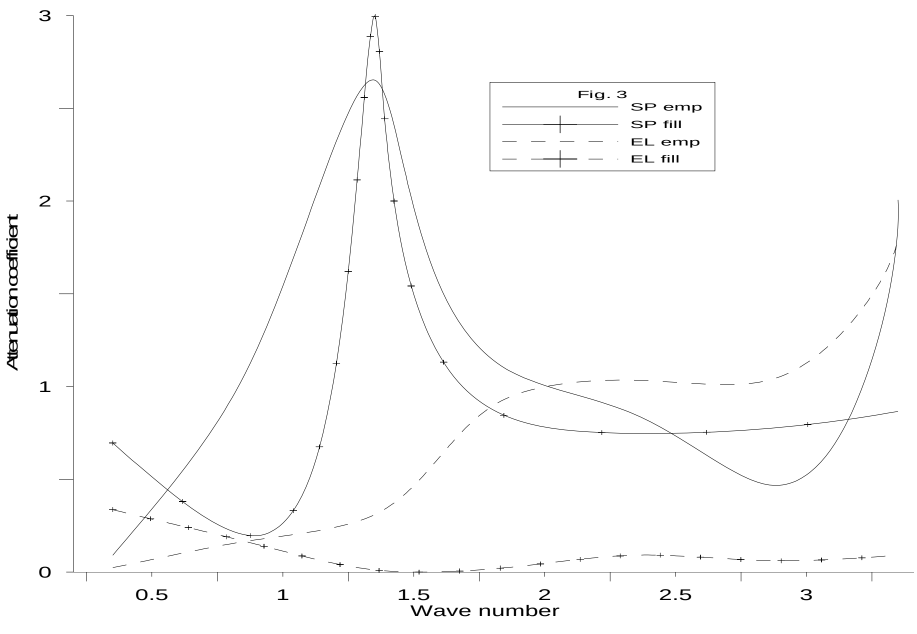

Fig. 3. Variation of attenuation coefficient with wave number.

Figure 3 depicts the variation of attenuation coefficient with wave number in case of empty and liquid filled cylindrical bore for SP and elastic medium. From the figure we notice that attenuation coefficient is of oscillatory behavior for empty and liquid filled cylindrical bore in SP medium. In elastic medium the attenuation coefficient increases with wave number when cylindrical bore is empty, whereas for liquid filled cylindrical bore it is of oscillatory behavior. In EL medium the attenuation coefficient of liquid filled cylindrical bore remains less than the attenuation coefficient in SP medium. The attenuation coefficient for empty cylindrical bore is more than the attenuation coefficient of empty cylindrical bore in SP medium for $k \geq 2$.

11. Conclusions

From the above figures and manipulation we notice that phase velocity in SP medium remains less than the phase velocity of EL medium in case of empty and filled cylindrical bore. In SP medium the phase velocity for liquid filled cylindrical bore is greater than the phase velocity for empty cylindrical bore for $k \geq 1.25$, whereas for elastic medium, the phase velocity of empty cylindrical bore remains less than the phase velocity of liquid filled cylindrical bore in whole range. The oscillation of attenuation coefficient in SP medium is greater than that of elastic medium.

References

[1] M.A. Biot // *J. Applied Phys*. **23** (1952) 937.
[2] M.A. Biot // *J. Acoust. Soc. Am.* **28** (1956) 168.
[3] M.A. Biot // *J. Acoust. Soc. Am.* **28** (1956) 179.
[4] A.C. Eringen // *Int. J. Eng. Sci.* **32** (1994) 1337.
[5] S.K. Tomar, R. Kumar // *Proc. Indian Acad. Sci. (Math. Sci.)* **109** (1999) 425.
[6] S. Deswal, S.K. Tomar, R. Kumar // *Sadhna* **25** (2000) 439.
[7] R. Kumar, S. Deswal, S. Choudhary // *Ganita* **52 (2)** (2001) 161.
[8] R. Kumar, S. Deswal // *J. Sound and Vibration* **250** (2002) 711.
[9] R. Kumar, S. Deswal // *Proc. Nat. Acad. Sci.* **73A (III)** (2003) 315.
[10] F. Bofill, R. Quintanilla // *Int. J. of Engng. Sci*. **41** (2003) 801.
[11] C. Gales // *European Journal of Mechanics A: Solids* **23** (2004) 345.
[12] W. Tersa, S. Bennethum // *Int. J. of Engng. Sci.* **44** (2006) 1408.
[13] N. Kleintelter, M. Park, J. Cushman // *Transp. Porous Med.* **68** (2007) 69.
[14] C. Gales // *Int. J. of Solids and Structures* **45** (2008) 2117.
[15] C. Gales // *Journal of Thermal Stresses* **32** (2009) 512.
[16] R. Kumar, M. Panchal // *Int. J. of Thermophysics* **30** (2009) 2058.
[17] D. Kessler, D. Kosloffs // *Geophysics* **56 (12)** (1991) 2080.