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STABILITY OF STRATIFIED RIVLIN-ERICKSEN (MODEL) FLUID IN MAGNETIZED QUANTUM PLASMA SATURATING A POROUS MEDIUM

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Abstract. The present investigation is to focus on the quantum effects on the Rayleigh –Taylor instability in an infinitely electrically conducting inhomogeneous stratified incompressible, viscoelastic fluid/plasma through a porous medium in the presence of a vertical magnetic field. After developing a mathematical formulation, the linear magneto hydrodynamic equations are solved by normal mode analysis to obtain the velocity perturbation. The linear growth rate is derived for the case when the plasma with exponential density, viscosity, viscoelasticity, quantum parameter distribution is confined between two rigid planes at $z = 0,\ z = d$. The behaviour of growth rates with respect to the kinematic viscoelasticity and the simultaneous presence of quantum effect and magnetic field are obtained in the presence of porous medium, the medium permeability and kinematic viscosity. It is observed that the vertical magnetic field beside the quantum effect yield more stability on the considered system.

1. Introduction

Rayleigh-Taylor instability arises from the character of equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid). The simplest, nevertheless important, for example demonstrating the Rayleigh-Taylor instability is when we consider two fluids of different densities superposed one over the other (or accelerated towards each other); the instability of the plane interface between the two fluids, if it occurs, is known as Rayleigh-Taylor instability. Rayleigh [17] was the first to investigate the character of equilibrium of an in viscid, non- heat conducting as well as incompressible heavy fluid of variable density which is continuously stratified in the vertical direction. The case of (i) two uniform fluids of different densities superposed one over the other and (ii) an exponentially varying density distribution was also treated by him. The main result in all cases is that the configuration is stable or unstable with respect to infinitesimal small perturbations according as the higher density fluid underlies or overlies the lower density fluid. Taylor [13] carried out the theoretical investigation further and studied the instability of liquid surfaces when accelerated in a direction perpendicular to their planes. The experimental demonstration of the development of the Rayleigh –Taylor instability (in case of heavier fluid overlaying a lighter one, is accelerated towards it) is described by Lewis [3]. This instability has been further studied by many authors Kruskal and Schwarzschild [19], Hide [24], Chandrasekhar [27], Joseph [2], and Drazin and Reid [20] to include various parameters. Rayleigh-Taylor instability is mainly used to analyze the frequency of gravity waves in deep oceans, liquid vapour/globe, to extract oil from the earth to eliminate water drops, lazer and inertial confinement fusion etc.

Quantum plasma can be composed of electrons, ions, positrons, holes and or grains, which plays an important role in ultra small electronic devices which has been given by Dutta and McLennan [28], dense astrophysical plasmas system has been given by Madappa et al. [22], intense laser-matter experiments has been investigated by Remington et al. [1], and non-linear quantum optics has been given by Brambilla et al. [18]. The pressure term in such plasmas is divided to two terms $p = p^C + p^Q$ (classical $(p^C)$and quantum (p^Q) pressure) and has been investigated by Gardner [4, 16] for the quantum hydrodynamic model. In the momentum equation, the classical pressure rises in the form $(-\nabla p)$, while the quantum pressure rises in the form $Q = \frac{h^{\sim 2}}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$, where $h^{\sim}$ is the Plank constant, m_e is the mass of electron and m_i is the mass of ion. The linear quantum growth rate of a finite layer plasma in which the density is continuously stratified exponentially along the vertical is studied by Goldston and Rutherford [23]. Nuclear fusion, which is plasma based, is one of the most promising candidates for the energy needs of the future when fossil fuels finally run out. It is well known that quantum effects become important in the behavior of charged plasma particles when the de Broglie wavelength of charge carriers become equal to or greater than the dimension of the quantum plasma system, which has been investigated by Manfredi and Haas [14]. Two models are used to study quantum plasmas systems. The first one is the Wigner-Poisson and the other is the Schrodinger-Poisson approaches [15, 16] they have been widely used to describe the statistical and hydrodynamic behavior of the plasma particles at quantum scales in quantum plasma. The quantum hydrodynamic model was introduced in semiconductor physics to describe the transport of charge, momentum and energy in plasma [17].

A magnetohydrodynamic model for semiconductor devices was investigated by Haas [4], which is an important model in astrophysics, space physics and dusty plasmas. The effect of quantum term on Rayleigh-Taylor instability in the presence of vertical and horizontal magnetic field, separately, has been studied by Hoshoudy [5, 6]. The Rayleigh-Taylor instability in a non-uniform dense quantum magneto-plasma has been studied by Ali et al. [26]. Hoshoudy [7] has studied quantum effects on Rayleigh-Taylor instability of incompressible plasma in a vertical magnetic field. Rayleigh-Taylor instability in quantum magnetized viscous plasma has been studied by Hoshoudy [9]. External magnetic field effects on the Rayleigh-Taylor instability in an inhomogeneous rotating quantum plasma has been studied by Hoshoudy [10]. In all the above studies, the plasma/fluids have been considered to be Newtonian. With the growing importance of the non-Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. There are many elastico-viscous constitutive relations or Oldroyd constitutive relations. We are interested there in Rivlin-Ericksen Model. Rivlin-Ericksen Model [25] has proposed a theoretical model for such elastic-viscous fluid. Molten plastics, petroleum oil additives and whipped cream are examples of incompressible viscoelastic fluids. Such types of polymers are used in agriculture, communication appliances and in bio-medical applications. Previous work on the effects of incompressible quantum plasma on Rayleigh-Taylor instability of Oldroyd model through a porous medium has been investigated by Hoshoudy [8], where the author has shown that both maximum k^*_{max} and critical k^*_c point for the instability are unchanged by the addition of the strain retardation and the stress relaxation. All growth rates are reduced in the presence of porosity of the medium, the medium permeability, the strain retardation time and the stress relaxation time. This paper aims at numerical analysis of the effect of the quantum mechanism on Rayleigh-Taylor instability for a finite thickness layer of incompressible viscoelastic plasma in a porous medium. Hoshoudy [11] studied Quantum effects on Rayleigh-Taylor instability of a plasma-vacuum. Hoshoudy [12] also investigated Rayleigh-Taylor instability of Magnetized plasma through Darcy porous medium. Sharma et al. [21] discussed the Rayleigh-Taylor instability of two superposed compressible fluids in un- magnetized plasma.

In this paper, quantum effects on the Rayleigh –Taylor instability in an infinitely electrically conducting inhomogeneous stratified incompressible, viscoelastic fluid/plasma through a porous medium in the presence of a vertical magnetic field has been investigated.

2. Formulation of the physical problem and perturbation equations

The initial stationary state whose stability we wish to examine is that of an incompressible, heterogeneous infinitely conducting viscoelastic Rivlin–Ericksen (Model) fluid of thickness h bounded by the planes $z = 0$ and $z = d$; of variable density, kinematic viscosity, kinematic viscoelasticity, magnetic field and quantum pressure arranged in horizontal strata electrons and immobile ions in a homogeneous, saturated, isotropic porous medium with the Oberbeck–Boussinesq approximation for density variation is considered, so that the free surfaces almost horizontal. The fluid is acted on by gravity force $\boldsymbol{g} = (0,0,-g)$and the plasma is immersed in magnetic field as shown in Fig. 1.

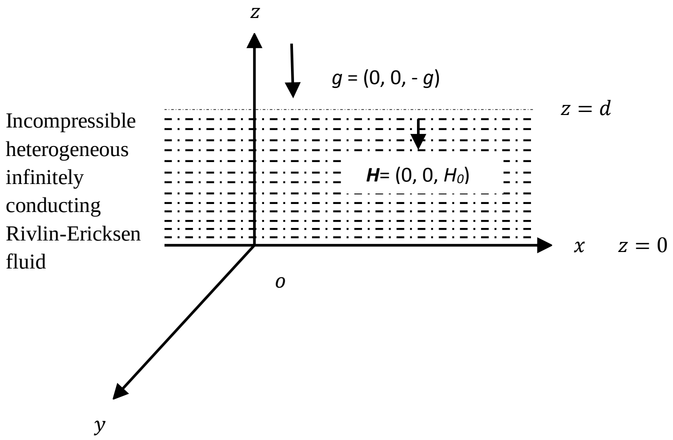

Fig. 1. Schematic diagram of finite quantum plasma layer.

The relevant equations of motion, continuity (conservation of mass), incompressibility, Gauss divergence equation and Magnetic induction equations are Hoshoudy [5, 6]

$$\frac{\rho}{\varepsilon}\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathrm{q}.\nabla)\right]\boldsymbol{q} = -\nabla p + \rho\boldsymbol{g} + \frac{\mu_e}{4\pi}(\nabla \times H) \times H - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \boldsymbol{Q} \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

$$\varepsilon\frac{\partial\rho}{\partial t} + (\boldsymbol{u} \cdot \nabla)\rho = 0, \tag{3}$$

$$\nabla \cdot H = 0, \tag{4}$$

$$\varepsilon\frac{\partial H}{\partial t} = \nabla \times (u \times H), \tag{5}$$

where $\boldsymbol{u}$, ρ, p, μ, μ', k_1, μ_e, $\varepsilon, H, \boldsymbol{Q}$ represents velocity, density, pressure, viscosity, viscoelasticity, medium permeability, magnetic permeability, medium porosity, magnetic field and Bohr vector potential, respectively. Equation (3) ensures that the density of a particle remains unchanged as we follow with its motion.
Then equilibrium profiles are expressed in the form

$$\boldsymbol{u_0} = (0,0,0), \rho_0 = \rho_0(z), p = p_0(z),\ \boldsymbol{H} = H_0(z) \text{ and } \boldsymbol{Q} = \boldsymbol{Q}_0(z).$$

To investigate the stability of hydromagnetic motion, it is necessary to see how the motion responds to a small fluctuation in the value of any flow of the variables. Let the infinitesimal perturbations in fluid velocity, density, pressure, magnetic field and quantum pressure be given by

$$u = (u, v, w), \rho = \rho_0 + \delta\rho, p = p_0 + \delta p, H = H_0 + H_1(h_x, h_y, h_z)$$
and $Q = Q_0 + Q_1(Q_x, h_y, h_z)$. (6)

Using these perturbations and linear theory (neglecting the products and higher order perturbations because their contributions are infinitesimally very small), equations (1) - (5) in the linearized perturbation form become

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} = -\nabla\delta p + g\delta\rho + \frac{\mu_e}{4\pi}[(\nabla \times H_0) \times H_1 + (\nabla \times H_1) \times H_0] - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)u + Q_1, \quad (7)$$

$$\nabla . u = 0, \quad (8)$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho + w\frac{d\rho_0}{dz} = 0, \quad (9)$$

$$\nabla . H_1 = 0, \quad \varepsilon\frac{\partial H_1}{\partial t} = \nabla \times (u \times H_0), \quad (10)$$

where

$$\boldsymbol{Q}_1 = \frac{h^2}{2m_e m_i}\begin{bmatrix} \frac{1}{2}\nabla(\nabla^2\delta\rho) - \frac{1}{2\rho_0}\nabla\delta\rho\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla\rho_0\nabla^2\delta\rho + \\ \frac{\delta\rho}{2\rho_0^2}\nabla\rho_0\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla(\nabla\rho_0\nabla\delta\rho) + \frac{\delta\rho}{4\rho_0^2}\nabla(\nabla\rho_0)^2 + \\ \frac{1}{2\rho_0^2}(\nabla\rho_0)^2\nabla\delta\rho + \frac{1}{\rho_0^2}(\nabla\rho_0\nabla\delta\rho)\nabla\rho_0 - \frac{\delta\rho}{\rho_0^3}(\nabla\rho_0)^3 \end{bmatrix}.$$

3. Solution of the problem

To investigate the stability of the system, we analyze an arbitrary perturbation into a complex set of normal modes individually. For the present problem, analysis is made in terms of two dimensional periodic waves of assigned wavenumber. Thus to all quantities are ascribed describing the perturbation dependence on x, y and t of the forms

$$f_1(x, y, z, t) = f(z)exp i(k_x x + k_y y - nt), \quad (11)$$

where k_x and k_y are wavenumbers along x and y directions, $k = (k_x^2 + k_y^2)^{\frac{1}{2}}$ is the resultant wavenumber and n is the growth rate which is, in general a complex constant.
Since the boundaries are assumed to be rigid. Therefore the boundary conditions appropriate to the problem are

$w = 0$, $Dw = 0$ at $z = 0$ and $z = h$, on a rigid surface. (12)

Using the expression (11) and eliminating variables $u, v, h_x, h_y, h_z, \delta\rho, \delta p, Q_x, Q_y$ from the resulting equations, we obtain the characteristic equation:

$$(-a_1 H_0^2)D^4 w + [-3a_1 H_0 DH_0 - a_1 D(H_0)^2]D^3 w + \Big[(-in) + \frac{\mu + \mu'(-in)\varepsilon}{\rho_0 k_1} - 4a_1 D(H_0)^2 - 4a_1 H_0 D^2 H_0 + a_1 k^2 H_0^2 + a_2(D\rho_0)^2\Big] D^2 w + \Big[\frac{(-in)D\rho_0}{\rho_0} + \frac{(\mu + \mu'(-in))\varepsilon k^2}{\rho_0 k_1} - a_1 H_0 D^3 H_0 - 3a_1 DH_0 D^2 H_0 + a_1 k^2 D(H_0)^2 - \frac{a_2(D\rho_0)^2}{\rho_0} - 2D\rho_0 D^2\rho_0\Big] Dw + \Big[-(-in)\rho_0 k^2 + \frac{gk^2 D\rho_0}{\rho_0(-in)} - \frac{(\mu + \mu'(-in))\varepsilon k^2}{\rho_0 k_1} - a_2 k^2 (D\rho_0)^2\Big] w = 0. \quad (13)$$

where $a_1 = \frac{\mu_e}{4\pi\varepsilon(-in)}$ and $a_2 = \frac{k^2h^2}{4inm_e m_i \rho_0^2 \varepsilon}$.

Now the case of incompressible continuously stratified viscoelastic plasma layer is considered in a porous medium the density, viscosity, viscoelasticity, magnetic field and quantum pressure are given by

$\rho_0(z) = \rho_0(0)exp\left(\frac{z}{L_D}\right), \mu(z) = \mu_0(0)exp\left(\frac{z}{L_D}\right), \mu'(z) = \mu_0'(0)exp\left(\frac{z}{L_D}\right), k_1(z) = k_{10}(0)exp\left(\frac{z}{L_D}\right), H_0(z) = H_0(0)exp\left(\frac{z}{L_D}\right), n_q(z) = n_{q_0}(0)exp\left(\frac{z}{L_D}\right), \varepsilon(z) = \varepsilon_0(0)exp\left(\frac{z}{L_D}\right),$

where $\rho_0(0), \mu_0(0), \mu_0'(0), n_{q_0}(0), H_0(0), k_{10}(0), \varepsilon_0(0)$ and L_D are constants.

Using the above expression in equation (14), we get

$$\left(\frac{-V_A^2}{(-in)}\right) D^4 w + \left[\frac{-3V_A^2}{(-in)2L_D} - \frac{V_A^2}{4L_D^2(-in)}\right] D^3 w + \left[(-in) + \frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1} - \frac{2V_A^2}{L_D^2(-in)} + \frac{V_A^2k^2}{(-in)} + \frac{n_q^2}{(in)}\right] D^2 w + \left[\frac{(-in)}{L_D} - \frac{V_A^2}{2L_D^3(-in)} + \frac{V_A^2k^2}{(-in)2L_D^2} + \frac{n_q^2}{(in)L_D} + \frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1}\right] Dw + \left[-(-in)k^2 + \frac{gk^2}{(-in)L_D} - \frac{\left(\nu+\nu'(-in)\right)\varepsilon k^2}{k_1} - \frac{n_q^2k^2}{(in)}\right] w = 0, \tag{14}$$

where $n_q^2 = \frac{h^2k^2}{4m_e m_i L_D^2}$, $V_A^2 = \frac{\mu_e H_0^2}{4\pi\rho_0}$, represents quantum effect and square of Alfven velocity.

The equation (14) implies by applying the boundary conditions (11) that

$$D^4w = 0, D^2w = 0 \; at \; z = 0 \text{ and } z = h. \tag{15}$$

The exact solution of the eigen-value problem (13) satisfying the boundary conditions (15), is chosen to be $w = sin(nz)exp(\lambda z)$, where $n = \left(\frac{n_1\pi}{h} z\right)$.

Substituting this solution into equation (14), we obtain

$$sin(nz)(\lambda^4 - 6\lambda^2n^2 + n^4) + cos(nz)[\lambda^3 n - \lambda n^3]4V + \left(sin(nz)(\lambda^3 - 3\lambda n^2) + cos(nz)(3\lambda^2 n - n^3)\right)\left[\frac{3V}{2L_D} + \frac{V}{L_D^2}\right] + [(\lambda^2 - n^2)sin(nz) + 2\lambda n\, cos(nz)]\left[(-in) + \frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1} + \frac{2V}{L_D^2} - Vk^2 + \frac{n_q^2}{(in)}\right] + \left(\lambda\, sin(nz) + n\, cos(nz)\right)\left[\frac{(-in)}{L_D} + \frac{V}{2L_D^3} - \frac{k^2V_A^2}{2L_D^2} + \frac{n_q^2}{(in)L_D} + \frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1}\right] + sin(nz)\left[-(-in)k^2 + \frac{gk^2}{(-in)L_D} - \frac{k^2\varepsilon\left(\nu+\nu'(-in)\right)}{k_1} - \frac{n_q^2k^2}{(in)}\right] = 0. \tag{16}$$

Equating the coefficients of $sin(nz)$ and $cos(nz)$ in equation (16) yield that

$$[(\lambda^4 - 6\lambda^2n^2 + n^4)V] + \left[(\lambda^3 - 3\lambda n^2)\left(\frac{3V}{2L_D} + \frac{V}{L_D^2}\right)\right] + (\lambda^2 - n^2)\left[(-in) + \frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1} + \frac{2V}{L_D^2} - Vk^2 + \frac{n_q^2}{(in)}\right] + \lambda\left[\frac{(-in)}{L_D} + \frac{V}{2L_D^3} - \frac{k^2V_A^2}{2L_D^2} + \frac{n_q^2}{(in)L_D} + \frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1}\right] + \left[-(-in)k^2 + \frac{gk^2}{(-in)L_D} - \frac{\left(\nu+\nu'(-in)\right)\varepsilon k^2}{k_1} - \frac{n_q^2k^2}{(in)}\right] = 0 \tag{17}$$

and

$$[(\lambda^3 n-\lambda n^3)4V]+(\lambda^3-3\lambda n^2)\left(\frac{3V}{2L_D}+\frac{V}{L_D^2}\right)+2\lambda\left(\frac{n_1\pi}{h}\right)\left[(-in)+\frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1}+\frac{2V}{L_D^2}-Vk^2+\frac{n_q^2}{(in)}\right]+n\left[\frac{(-in)}{L_D}+\frac{V}{2L_D^3}-\frac{k^2V_A^2}{2L_D^2}+\frac{n_q^2}{(in)L_D}+\frac{\left(\nu+\nu'(-in)\right)\varepsilon}{k_1}\right]=0. \quad (18)$$

Now, we introducing the non-dimensional quantities

$$n^*=\frac{n}{n_{pe}},n_q^{*2}=\frac{n_q^2}{n_{pe}^2k^{*2}L_D^2},n_\varepsilon^*=\frac{\varepsilon}{n_{pe}},n_\nu^*=\frac{\nu}{n_{pe}},n_{\nu'}^*=\nu',n_{k_1}^*=\frac{k_1}{n_{pe}},h^{*2}=\frac{h^2}{L_D^2},k^{*2}=k^2L_D^2,g^*=\frac{g}{n_{pe}^2L_D^2},n_A^{*2}=\frac{v_A^2}{n_{pe}^2L_D},\lambda^{*2}=\lambda^2L_D^2, \text{where } n_{pe}=\left(\frac{\rho_0e^2}{m_0^2\varepsilon_0}\right)^{\frac{1}{2}}$$

is the plasma frequency, then equations (17) and (18), we get

$$n_A^{*2}\left[\left(\frac{\lambda^{*4}}{L_D^4}-\frac{6\lambda^{*2}n_1^2\pi^2}{h^{*2}L_D^4}+\frac{n_1^2\pi^4}{h^{*4}L_D^4}\right)+\left(\frac{\lambda^{*3}}{L_D^3}-\frac{3\lambda^*n_1^2\pi^2}{h^{*2}L_D^3}\right)\left(\frac{3}{2in^*}+\frac{1}{4in^*L_D}\right)+\left(\frac{\lambda^{*2}}{L_D^2}-\frac{n_1^2\pi^2}{h^{*4}L_D^4}\right)\left(\frac{2}{in^*L_D}-\frac{k^{*2}}{in^*L_D}\right)+\frac{\lambda^*}{L_D}\left(\frac{1}{2in^*L_D}-\frac{k^{*2}}{in^*L_D}\right)\right]+n_q^{*2}\left[\frac{k^{*2}L_D^2}{in^*}\left(\frac{\lambda^{*2}}{L_D^2}-\frac{n_1^2\pi^2}{h^{*2}L_D^2}\right)+\frac{k^{*2}L_D}{in^*}\left(\frac{\lambda^*}{L_D}\right)-\frac{k^{*4}}{in^*}\right]+\left[\left\{(-in^*)+\frac{\left(n_\nu^*+n_{\nu'}^*(-in^*)\right)n_\varepsilon^*}{n_{k_1}^*}\right\}\left(\frac{\lambda^{*2}}{L_D^2}-\frac{n_1^2\pi^2}{h^{*2}L_D^2}\right)+\frac{\lambda^*}{L_D}\left(\frac{(-in^*)}{L_D}+\frac{\left(n_\nu^*+n_{\nu'}^*(-in^*)\right)n_\varepsilon^*}{n_{k_1}^*}\right)+\frac{in^*k^{*2}}{L_D^2}+\frac{g^*k^{*2}}{(-in^*)L_D^2}-\frac{\left(n_\nu^*+n_{\nu'}^*(-in^*)\right)n_\varepsilon^*k^{*2}}{n_{k_1}^*L_D^2}\right] \quad (19)$$

and

$$n_A^{*2}\left[\frac{4\lambda n_1\pi}{L_D^3h^*(in^*)}\left(\lambda^{*2}-\frac{n_1^2\pi^2}{h^{*2}}\right)+\frac{n_1\pi}{h^*L_D^3}\left(\frac{3}{2(in^*)}+\frac{1}{4(in^*)L_D}\right)+\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)\left(\frac{2}{(-in^*)L_D}-\frac{k^{*2}}{(in^*)L_D}\right)+\frac{n_1\pi}{h^*L_D}\left(\frac{1}{2(in^*)L_D}+\frac{k^{*2}}{(in^*)L_D^3}\right)\right]+n_q^{*2}\left[\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)\left(\frac{-k^{*2}L_D^2}{in^*}\right)+\frac{n_1\pi}{h^*L_D}\left(\frac{k^{*2}L_D}{in^*}\right)\right]+(-in^*)\left[\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)+\frac{n_1\pi}{h^*L_D^2}\right]+\left[\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)\left(\frac{n_\nu^*+n_{\nu'}^*(-in^*)n_\varepsilon^*}{n_{k_1}^*}\right)+\frac{n_1\pi}{h^*L_D}\left(\frac{n_\nu^*+n_{\nu'}^*(-in^*)n_\varepsilon^*}{n_{k_1}^*}\right)\right]=0, \quad (20)$$

where $n^*=n_r^*+i\gamma$ and for $n_r^*=0$ (stable oscillations). Thus substituting $n^*=i\gamma$ in equation (19) and (20), the resulting equations describing the square of normalized growth rate are:

$$n_A^{*2}\left[\left(\frac{\lambda^{*4}}{L_D^4}-\frac{6\lambda^{*2}n_1^2\pi^2}{h^{*2}L_D^4}+\frac{n_1^4\pi^4}{h^{*4}L_D^4}\right)+\left(\frac{\lambda^{*3}}{L_D^3}-\frac{3\lambda^*n_1^2\pi^2}{h^{*2}L_D^3}\right)\left(-\frac{3}{2\gamma}-\frac{1}{4\gamma L_D}\right)+\left(\frac{\lambda^{*2}}{L_D^2}-\frac{n_1^2\pi^2}{h^{*2}L_D^4}\right)\left(-\frac{2}{\gamma L_D}+\frac{k^{*2}}{\gamma L_D}\right)+\frac{\lambda^*}{L_D}\left(-\frac{1}{2\gamma L_D}+\frac{k^{*2}}{\gamma L_D}\right)\right]+n_q^{*2}\left[-\frac{k^{*2}L_D^2}{\gamma}\left(\frac{\lambda^{*2}}{L_D^2}-\frac{n_1^2\pi^2}{h^{*2}L_D^2}\right)-\frac{k^{*2}\lambda^*}{\gamma}+\frac{k^{*4}}{\gamma}\right]+\left[\left(\gamma+\frac{\left(n_\nu^*+n_{\nu'}^*\gamma\right)n_\varepsilon^*}{n_{k_1}^*}\right)\left(\frac{\lambda^{*2}}{L_D^2}-\frac{n_1^2\pi^2}{h^{*2}L_D^2}\right)+\frac{\lambda^*}{L_D}\left(\frac{\gamma}{L_D}+\frac{\left(n_\nu^*+n_{\nu'}^*\gamma\right)n_\varepsilon^*}{n_{k_1}^*}\right)-\frac{\gamma k^{*2}}{L_D^2}+\frac{g^*k^{*2}}{\gamma L_D^2}-\frac{\left(n_\nu^*+n_{\nu'}^*\gamma\right)n_\varepsilon^*k^{*2}}{n_{k_1}^*L_D^2}\right], \quad (21)$$

and

$$n_A^{*2}\left[-\frac{4\lambda n_1\pi}{L_D^3h^*\gamma}\left(\lambda^{*2}-\frac{n_1^2\pi^2}{h^{*2}}\right)+\frac{n_1\pi}{h^*L_D^3}\left(-\frac{3}{2\gamma}-\frac{1}{4\gamma L_D}\right)+\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)\left(\frac{2}{\gamma L_D}+\frac{k^{*2}}{\gamma L_D}\right)+\frac{n_1\pi}{h^*L_D}\left(-\frac{1}{2\gamma L_D}-\frac{k^{*2}}{\gamma L_D^3}\right)\right]+n_q^{*2}\left[\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)\left(\frac{k^{*2}L_D^2}{\gamma}\right)-\frac{n_1\pi}{h^*L_D}\left(\frac{k^{*2}L_D}{\gamma}\right)\right]+\gamma\left[\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)+\frac{n_1\pi}{h^*L_D^2}\right]+\left[\frac{2\lambda^*}{L_D^2}\left(\frac{n_1\pi}{h^*}\right)\left(\frac{\left(n_\nu^*+n_{\nu'}^*\gamma\right)n_\varepsilon^*}{n_{k_1}^*}\right)+\frac{n_1\pi}{h^*L_D}\left(\frac{\left(n_\nu^*+n_{\nu'}^*\gamma\right)n_\varepsilon^*}{n_{k_1}^*}\right)\right]=0. \quad (22)$$

In the absence of the non-dimensional parameters, accounting for vertical magnetic field ($n_A^{*2}=0$), equation (22) yields that $\lambda^*=-\frac{1}{2}$, and substituting this value of λ^* in equation (21), the dispersion relation obtained is

$$\gamma^2\left[\left(\frac{1}{4}-b_1^2\right)(1+n_2)-\frac{L_D}{2}\left(\frac{1}{L_D}+n_2\right)-k^{*2}(1+n_2)\right]+\gamma\left[\left(\frac{1}{4}-b_1^2\right)n_2-\frac{L_D}{2}n_2-n_2k^{*2}\right]+n_A^{*2}\left[\frac{1}{L_D^2}\left(\frac{1}{16}-\frac{3b_1^2}{2}+b_1^4\right)-\frac{1}{L_D}\left(-\frac{1}{8}+\frac{3b_1^2}{2}\right)\left(\frac{3}{2}+\frac{1}{4L_D}\right)+\frac{1}{L_D}\left(\frac{1}{4}-\frac{b_1^2}{L_D^2}\right)(k^{*2}-2)-\frac{1}{2}\left(k^{*2}-\frac{1}{2}\right)\right]+n_q^{*2}k^{*2}L_D^2\left[\frac{1}{4}+b_1^2+k^{*2}\right]g^*k^{*2}=0, \quad (23)$$

where $b_1^2=\frac{n_1^2\pi^2}{h^{*2}}$ and $n_2=\frac{n_\nu^*n_\varepsilon^*}{n_{k_1}^*}$.

$$a_1\gamma^2+a_2\gamma+a_3=0, \quad (24)$$

where

$$a_1=\left[\left(\frac{1}{4}-b_1^2-k^{*2}\right)(1+n_2)-\frac{L_D}{2}\left(\frac{1}{L_D}+n_2\right)\right],$$

$$a_2=\left[\left(\frac{1}{4}-b_1^2-\frac{L_D}{2}\right)n_2-n_2k^{*2}\right],$$

$$a_3=n_A^{*2}\left[\frac{1}{L_D^2}\left(\frac{1}{16}-\frac{3b_1^2}{2}+b_1^4\right)-\frac{1}{L_D}\left(-\frac{1}{8}+\frac{3b_1^2}{2}\right)\left(\frac{3}{2}+\frac{1}{4L_D}\right)+\frac{1}{L_D}\left(\frac{1}{4}-\frac{b_1^2}{L_D^2}\right)(k^{*2}-2)-\frac{1}{2}\left(k^{*2}-\frac{1}{2}\right)\right]+n_q^{*2}k^{*2}L_D^2\left[\frac{1}{4}+b_1^2+k^{*2}\right]g^*k^{*2}.$$

4. Numerical results and discussion

Computations are carried out using the dispersion relation described by equation (24) using the software Mathematica version 5.2. This is to find the role of medium porosity, the medium permeability, kinematic viscosity, kinematic viscoelasticity, quantum plasma and, magnetic field on the square of the normalized growth rate of most unstable mode of perturbation for fixed permissible values of the dimensionless parameters $n_\varepsilon^*=0.5$, $n_{\nu'}^*=0.2$, $n_q^*=0.6$, $n_{k_1}^*=0.4$, $v_f^{*2}=0.2$ and $n_\nu^*=0.1$.

Figures 2 and 3 correspond to the variation of the square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values of Alfvén velocity $v_f^{*2}=0.1,0.3,0.5,0.7$ and medium permeability $n_{k_1}^*=0.2, 0.4, 0.6, 0.8$, respectively. It is clear from the figures that the square of Alfvén velocity has stabilizing effect on the system $k^{*2}>1$ and the critical wavenumberis 1.42, whereas medium permeability has a very little destabilizing effect on the system, however the critical wavenumber k_c^{*2} remains the same i.e. 1.6.

Figures 4 and 5 correspond to the variation of the square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for three different values of medium porosity $n_\varepsilon^*=0.1,0.3,0.5,0.7$ and $n_q^*=0.2, 0.6, 0.9$, respectively. It is clear from the figure n_ε^* that medium porosity has a slight stabilizing effect, whereas the critical wavenumber remains the same i.e. 1.6. It is clear from the figure that in the presence of n_q^* square of the normalized growth rate γ^2 increases with the increasing k^{*2} until arrives at the maximum instability, then returns to decrease with the increasing k^{*2} until arrives at the complete stability, where the maximum instability appears at k_{max}^{*2}= 0.6 and the complete stability appears at k_c^{*2}=1.37. This graph shows that quantum effect play a major role in securing a complete stability.

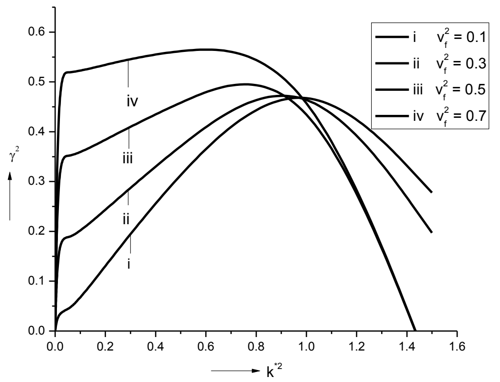

Fig. 2. Square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values of Alfvén velocity $v_f^{*2} = 0.1,0.3,0.5,0.7$.

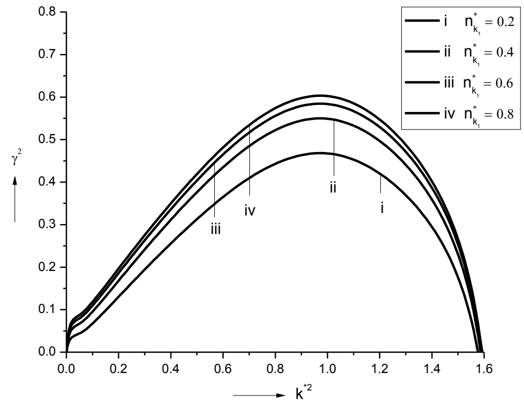

Fig. 3. Square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values of medium permeability $n_{k_1}^* = 0.2, 0.4,0.6, 0.8$.

Figures 6 and 7 correspond to the variation of the square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values of kinematic viscoelasticity $n_{\nu'}^* = 0.1, 0.3, 0.5, 0.9$ and kinematic viscosity $n_\nu^* = 0.1, 0.3, 0.5, 0.9$, respectively. It is clear from the graphs that with the increase in kinematic viscosity and kinematic viscoelasticity the growth rate of the unstable perturbation decreases; thereby stabilizing effect on the system, however the critical wave number $k_c^{*^2}$ remains the same i.e. 1.58.

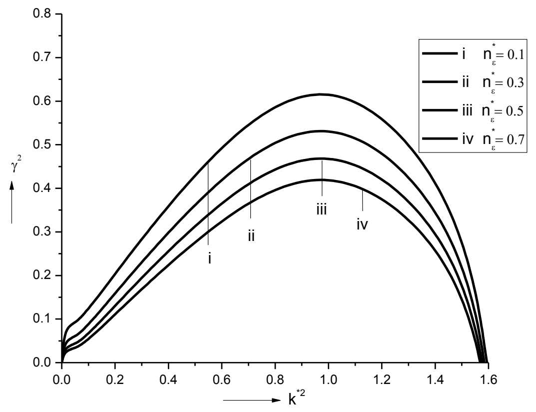

Fig. 4. Square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for different values of medium porosity $n_\varepsilon^* =$ 0.1,0.3,0.5,0.7.

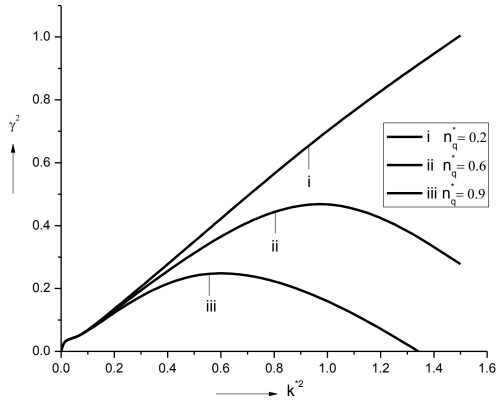

Fig. 5. Square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for different values of medium porosity $n_q^* =$ 0.2, 0.6, 0.9.

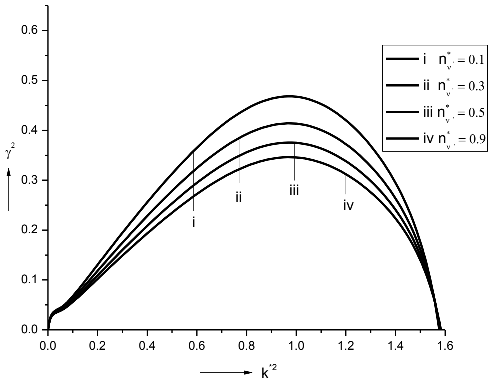

Fig. 6. Square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values of kinematic viscoelasticity $n_{\nu'}^{*} = 0.1, 0.3, 0.5, 0.9$.

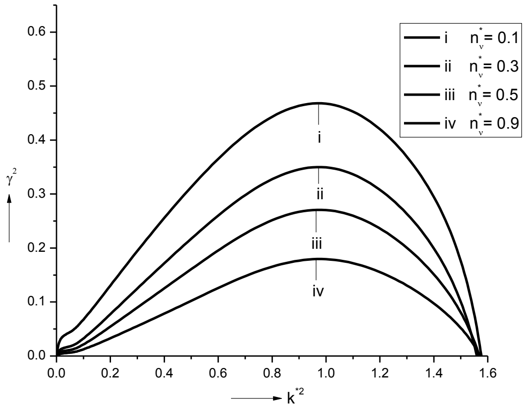

Fig. 7. Square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values of kinematic viscosity $n_{\nu}^{*} = 0.1, 0.3, 0.5, 0.9$.

5. Conclusions

The effect of quantum term on the Rayleigh-Taylor instability of stratified viscoelastic Rivlin –Ericksen (Model) fluid /plasma saturating a porous media has been studied. The principal conclusions of the present analysis are as follows:

1. The kinematic viscoelasticity has stabilizing effect on the system and the critical wavenumber is k_c^{*2}=1.6.
2. The kinematic viscosity has stabilizing effect on the system.
3. The medium porosity has a slight stabilizing effect whereas medium permeability has a very little destabilizing effect on the system, however the critical wavenumber k_c^{*2} remains the same i.e. 1.6.
4. Quantum plasma plays a major role in approaching a complete stability implying thereby the large enough stabilizing effect on the system.

The presence of magnetic field have large enough stabilizing effect on the system as the critical wavenumber decreases from 0.52 to 1.1.

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