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ON INFLUENCE OF SHEAR TRACTION ON HYDRAULIC FRACTURE PROPAGATION

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Abstract. The paper concerns with the problem important for proper simulation of hydraulic fractures. Its objective is to answer the recently raised question: Can we neglect the impact of the hydraulically induced shear stress when using the elasticity equation, which connects the fracture opening with the net-pressure? The estimations, given in the paper, show that the answer is "Yes". The impact can be confidently neglected. Its influence is well beyond physical significance, computational abilities of computers and practical applications of HF.
Keywords: hydraulic fracture, shear traction, elasticity equation.

1. Introduction

It is well known that viscous shear is the main force resisting to fluid flow in a narrow channel of a given small width. Meanwhile, in hydraulic fracture (HF) problems, the width itself depends on the deformation of the channel walls and it is zero at the contour of a propagating fracture. In these problems, the width is defined by the deformation of embedding rock; it is found from the elasticity equation connecting the channel width (opening in this case) with the tractions on the channel (fracture) surfaces.

Starting from the pioneering paper by Spence and Sharp [21], the shear traction, entering the elasticity equation, has been neglected in all papers on HF (e.g. [1-8, 10, 13, 15, 19-21, 23]). However, recently [24] it has been suggested that the shear traction is to be included into the elasticity equation, as well. An example of self-similar solution, given by the authors, shows that the input of the shear term is 10%, at most. Still, the question arises, if it is reasonable always to neglect this input when simulating HF propagation?

The objective of the present paper is to answer the question. We estimate the input of the shear term into the elastic response of embedding rock in HF problems. It is shown that for values of input parameters, typical in HF practice, the influence of the shear term discussed is far-beyond practical significance and computational abilities of modern computers.

2. Problem formulation

The problem formulation is conventional (e.g. [1-8, 10, 13, 15, 19-21, 23]) except for the only difference: the shear traction is accounted for in the elasticity equation. Below we use the formulation employing the fundamental speed equation [9, 12] and the particle velocity [13] rather than the flux (see also [14-17, 23]). Consider the plane strain problem for a straight fracture driven by a viscous fluid, studied, for instance, in the papers [1, 13, 21, 23, 24]. The x-axis is located along the fracture in the propagation direction. The y-axis is directed to the left of x. The quite general power law describes the fluid velocity across the opening:

$$\sigma_{xy} = -M(\dot{\gamma}\,\mathrm{sign}y)^n \mathrm{sign}y, \tag{1}$$

where σ_{xy} is the shear stress; n and M are, respectively, the behavior and consistency indices; $\dot{\gamma} = 2\dot{\varepsilon}_{xy}$; $\dot{\varepsilon}_{xy} = 1/2\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$ is the shear strain rate; v_x, v_y are the components of the fluid velocity. For a Newtonian fluid, $n = 1$ and $M = \mu$ is the dynamic viscosity.

For a narrow channel, the flow is assumed to be of the Poiseuille type, predominanatly in-plane and steady. Then the out-of-plane component v_y is neglected in (1) as compared with v_x, while the movement equation of the Navier-Stokes type yields

$$\sigma_{xy}(y) = -i_* y, \tag{2}$$

where $i_* = -\partial p/dx$ is the pressure gradient taken with the minus sign ($i_* > 0$). Equation (2) implies linear distribution of the shear stress along the channel width w:

$$\sigma_{xy} = -\frac{y}{w/2}\tau, \tag{3}$$

where τ is the shear traction on the lower shore ($y = -w/2$) of the channel. Using (3) in (2) gives (e. g. [22]):

$$\tau = \frac{1}{2} w i_* \tag{4}$$

On the other hand, using (2) in (1) and integration provide the profile of the velocity $v_x(y)$. The latter, being averaged over the cross-section, yields the conventional average particle velocity $v = \int_{-w/2}^{w/2} v_x(y) dy\, /w$ (e.g. [15]):

$$v = w\left(\frac{w}{\mu'} i_*\right)^{1/n}, \tag{5}$$

where $\mu' = 2\left(2\frac{2n+1}{n}\right)^n M$. For a Newtonian fluid, $\mu' = 12\mu$. In view of (4), equation (5) may be re-written in terms of the shear traction as

$$\tau = \frac{\mu'}{2}\left(\frac{v}{w}\right)^n \tag{6}$$

The fracture propagation speed v_*, by the speed equation is $v_* = \lim_{r\to 0} v$, where r is the distance from the fluid front. For a continouos particle velocity v, this infers that in the near-front zone, equation (6) becomes:

$$\tau = \frac{\mu'}{2}\frac{v_*^n}{w^n} \tag{7}$$

Neglect, as usual, the lag (e.g. [1, 13, 21, 23, 24]). Then the propagation speed v_*, being finite and non-zero, equation (7) implies that near the fracture contour, the shear traction behaves as $\tau = O\left(\frac{1}{w^n}\right)$. It is singular, because the opening w goes to zero at the fracture contour.

To the moment, the elasticity equation, defining the opening, hasnot been employed. When accounting for the shear traction, from the classical solution by Muskhelishvili's [18], it follows (e.g. [11]):

$$p(x) = \frac{E'}{4\pi}\int_{-x_*}^{x_*} \frac{\partial w/\partial\xi - 2k_\tau \tau(\xi)/E'}{x-\xi} d\xi\ , \tag{8}$$

where $E' = E/(1-\nu^2)$ is the plane-strain elasticity modulus, E is the Young's modulus, ν is the Poisson's ratio, x_* is the fracture half-length, $k_\tau = \frac{1-2\nu}{1-\nu}$ is the factor depending merely on the Poisson's ratio. For physically significant values of the latter ($0 \le \nu \le 0.5$), the factor k_τ never exceeds 1; it equals to zero when $\nu \le 0.5$ and it reaches its maximal value 1 when $\nu = 0$. For commonly used value $\nu = 0.3$, it is 0.531.

As mentioned, in all the papers on HF, except for the paper [24], the term $2k_\tau\tau(\xi)/E'$, which includes the shear traction, is neglected. Our objective is to compare the input of this term into the net-pressure $p(x)$ with that of the term $\partial w/\partial\xi$ conventionally accounted for.

3. Comparison of conventional and shear terms

The reltive input of the shear traction into the net-pressure, as compared with that of the conventional term $\partial w/\partial\xi$, is given by the ratio

$$R_\tau = -\frac{2k_\tau \tau(x)}{E' \partial w/\partial x} \quad (9)$$

The compared terms are greatest in the near tip zone, where the both of them are singular. In this zone $\partial w/\partial x = -\partial w/\partial r$, where r is the distance from the tip, and equation (7) is applicable. Then the ratio (9) becomes:

$$R_\tau = k_\tau \frac{\mu'}{E'} \frac{v_*^n}{w^n dw/dr} \quad (10)$$

Estimations of R_τ for toughness, viscosity and leak-off dominated regimes may employ well-known asymptotics for these regimes (e.g. [4, 7, 8, 10, 15, 21]). Commonly the asymptotics of the opening are of the form

$$w = A_w r^\alpha \quad (11)$$

where explicit formulae for the factor A_w and the exponent α are given in the cited papers. Then $dw/dr = \alpha w/r$, and (10) becomes

$$R_\tau = k_\tau \frac{\mu'}{E'} \frac{v_*^n}{\alpha A_w^{n+1} r^{\alpha(n+1)-1}} \quad (12)$$

Consider the regimes studied in [24], which are (i) viscosity dominated, and (ii) toughness dominated.

(i) Viscosity dominated regime. In this case, $\alpha = 2/(n+2)$ and

$$A_w = A_\mu (t_n' v_*)^{1-\alpha}, \quad (13)$$

where $A_\mu = [(1-\alpha)B(\alpha)]^{1/(n+2)}$, $B(\alpha) = \frac{\alpha}{4}\cot[\pi(1-\alpha)]$, $t_n' = (\mu'/E')^{1/n}$. Then $\alpha(n+1) - 1 = n/(n+2)$, and substitution (13) into (10) yields:

$$R_\tau = k_\tau \frac{1}{\alpha A_\mu^{n+1}} \left(\frac{t_n' v_*}{r}\right)^{n/(n+2)} \quad (14)$$

Consider, for certainty, a Newtonian fluid ($n = 1$).
Then $\alpha = 2/3$, $A_\mu = 2^{1/3} 3^{5/6} = 3.1473$, $t_n' = \mu'/E' = 12\mu/E'$. Using these values in (13) and substitution into (14) gives:

$$R_\tau = 0.1514\, k_\tau \left(\frac{12\mu}{E'} \frac{v_*}{r}\right)^{1/3} \quad (15)$$

For the values $\mu = 10^{-7}$MPa·s, $E' = 2.5 \cdot 10^4$MPa, typical for HF (e.g. [1, 3, 5, 10, 15]), equation (15) becomes $R_\tau = 0.5503 \cdot 10^{-4}\, k_\tau \left(\frac{v^*}{r}\right)^{1/3}$. Take the maximal value $k_\tau = 1$ and quite a large value of the fracture propagation speed $v_* = 0.1$ m/s (360 m/hour). Then equation (15) implies that the input of the shear traction $\tau(x)$ reaches 1% of the input of the conventional term $-\partial w/\partial x$, that is R grows to 0.01, only at the distance r from the tip less than $1.67 \cdot 10^{-8}$ m. This shows that the input of the shear traction reaches the level of 1% only at the distance of atomic sizes. Surely, it is beyond physical significance, computational abilities of computers and practical applications of HF.

(ii) Toughness dominated regime. In this case, $\alpha = 2/3$, $A_w = \sqrt{\frac{32}{\pi}} \frac{K_{IC}}{E'}$, where K_{IC} is the critical stress intensity factor. Then equation (12) reads:

$$R_\tau = 2k_\tau \frac{\mu'}{E'} \left(\sqrt{\frac{\pi}{32}} \frac{E'}{K_{IC}}\right)^{n+1} v_*^n r^{(1-n)/2} \quad (16)$$

For a Newtonian fluid ($n = 1$), equation (16) becomes:

$$R_\tau = 2k_\tau \frac{\pi}{32} \frac{\mu'}{E'} \left(\frac{E'}{K_{IC}}\right)^2 v_* \quad (17)$$

As known (e.g. [7, 8, 15]), the toughness dominated regime occurs when $\left(L_\mu/L_k\right)^{1/2} \ll 1$, where $L_k = \frac{32}{\pi}\left(\frac{K_{IC}}{E'}\right)^2$, $L_\mu = t_n' v_*$. Thus, for the toughness dominated regime, $\sqrt{\frac{\pi}{32}\frac{\mu'}{E'} v_*} \frac{E'}{K_{IC}} \ll 1$. Being squared, the inequality becomes square stronger $\frac{\pi}{32}\frac{\mu'}{E'}\left(\frac{E'}{K_{IC}}\right)^2 v_* \ll 1$.

When used in the right hand side of (17), it implies that $R_\tau \ll 1$. This means that in the toughness dominated regime, the input of the shear traction into elasticity equation is negligible, as well.

Comment on self-similar solution with exponentially growing injection rate. The paper [24] contains an example, which served authors to illustrate the input of the shear traction in the elasticity equation. To employ a self-similar solution, the authors assumed exponential growth of the injection rate. They considered a fracture driven by a Newtonian fluid ($n = 1$). From the numerical results, shown in figures 7 and 8 of this paper, it appears that for regimes with large toughness, the influence of the shear term is indistinguishable. This agrees with the conclusion above for such a regime. However, in the case of the viscosity dominated regime, for which $K_{IC} = 0$, the calculated injection pressure is about 10% greater if the shear term is taken into account (Fig. 8a of the paper). This result disagrees with the estimation (15) for the viscosity dominated regime. According to this estimation, such influence of the shear term may occur merely for very high values of the fracture propagation speed v_*.

Thus, it is reasonable to estimate the physical speed v_* for the example of the paper [24]. Below we employ the notation of this paper and the definitions of the normalized and self-similar quantities given in its equations (53) and (84)-(87). These definitions imply that the physical injection rate $q_0(t)$ and the physical speed $v_*(t)$ of the fracture propagation are:

$$q_0(t) = \frac{1}{t_n}\overline{q_0}e^{2\alpha t/t_n},\ v_*(t) = v(t,l) = \frac{1}{\sqrt{t_n}}\sqrt{\frac{\overline{q_0}}{t_n}e^{2\alpha t/t_n}}\frac{\hat{v}_0}{\sqrt{L_0}}, \qquad (18)$$

where $t_n = 2\pi M/E'$, $M = \mu' = 12\mu$, $E' = E/(1-\nu^2)$, $L_0 = \sqrt{\hat{v}_0/\alpha}$, $\hat{v}_0 = \hat{v}(1)$ is the self-similar propagation speed; the factor $\overline{q_0}$ and the exponent α, characterizing the intensity of the flux, are assigned values. The authors of the paper [24] set $\alpha = 1/3$; the values of $\hat{v}_0 = \hat{v}(1)$ of the self-similar propagation speed are defined by Fig. 7b of their paper. From this figure it appears that for the viscosity dominated regime, when $K_{IC} = 0$, the self-similar propagation speed is approximately $\hat{v}(1) = 0.625$. Then $L_0 = 1.369$ and $\hat{v}_0/\sqrt{L_0} = 0.5341$. The parameter $\overline{q_0}$ characterizes the influx at a specified time instant t_0. If at an instant t_0 the influx has a value q_{0HF}, typical for practice of hydraulic fracturing ($q_0(t_0) = q_{0HF}$), then equations (18) become:

$$q_0(t) = q_{0HF}e^{2\alpha(t-t_0)/t_n},\ v_*(t) = 0.5341\sqrt{\frac{q_{0HF}}{t_n}}e^{\alpha(t-t_0)/t_n} \qquad (19)$$

For the typical values $\mu = 10^{-7}$ MPa·s, $E' = 2.5 \cdot 10^4$ MPa (e.g. [1, 3, 5, 10, 15]), the definitions of t_n for $\alpha = 1/3$ yields $t_n = 3.016\ 10^{-10}$ s, $\alpha/t_n = 1.10 \cdot 10^9$ 1/s. Then for the typical influx $q_{0HF} = 0.5 \cdot 10^{-4}$ m^2/s (e.g. [3, 5]), the speed defined by the second of (19) is $v_*(t) = 688 \cdot \exp[(t-t_0)1.10 \cdot 10^9]$ m/s.

Therefore, the solution of the example, considered in the paper [24], implies that if at some time instant $t = t_0$ the influx has a typical order of 10^{-4} m^2/s, then at this instant the propagation speed v_* is of order km/s. Such a speed is much greater than values typical in practice of HF: normally the propagation speed is four orders less. Even more extraordinary is that during a very short time interval $t - t_0 = 10^{-8}$ s after t_0, the propagation speed exceeds the speed of light $c = 3 \cdot 10^8$ m/s. Therefore, the example corresponds to quite exotic, to say the least, problem. This explains why the authors of the paper [24] obtained non-negligible influence of the shear term in the elasticity equation on the calculated injection pressure (some 10%) for the viscosity dominated regime.

4. Reason of different results

It remains to clarify why the authors of the paper [24] came to the different conclusions on the impact of the shear stress? They inferred these conclusions by considering the ratio $\tau/|p|$ in the line of Spence and Sharp [21].

For the magnitude of the pressure $|p|$, the asymptotic equation is (e.g. [15]):

$$|p| = E' A_w B(\alpha) r^{\alpha-1} \quad (20)$$

where α and $B(\alpha)$ are defined as in equation (13). By using (20) and equation (7) for the shear traction, we obtain for the ratio $\tau/|p|$:

$$\frac{\tau}{|p|} = \frac{\alpha}{2B(\alpha)} \frac{\mu'}{E'} \frac{v_*^n}{\alpha A_w^{n+1} r^{\alpha(n+1)-1}} \quad (21)$$

Equation (21) is analogous to equation (12) for the ratio R_τ. The only difference between (12) and (21) is in the factors k_τ and $\frac{\alpha}{2B(\alpha)}$ on the right hand sides. Consider the case of zero toughness ($K_{IC} = 0$), for which the impact of the shear traction is maximal. Then for a Newtonian fluid, considered in [24], $= 2/3$, $B(\alpha) = 1/(6\sqrt{3}$), $\frac{\alpha}{2B(\alpha)} = 2\sqrt{3}$. Hence, similar to (12), the factor in (21) is of order 1. Consequently, using (21) implies the same conclusions as those above.

Unfortunately, the authors of the paper [24] have not derived equation (7), which provided us with quantitative estimations. Not having this equation, they formally tended r to zero when considering the ratio $\tau/|p|$. Clearly, the ratio goes to infinity, what leads to an illusion that the shear stress should be accounted for in the elasticity equation. This explains the reason of erroneous claims made in the cited paper on the impact of the shear traction and on the viscosity dominated regime.

5. Conclusion

The estimations, given for the impact of the shear term in the elasticity equation on the net-pressure, show that it can be confidently neglected when solving practical problems of hydraulic fracturing. Its influnce is well beyond physical significance, computational abilities of computers and practical applications of HF.

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