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STUDY OF RADIAL VIBRATIONS IN THICK WALLED HOLLOW DISSIPATIVE POROELASTIC SPHERICAL SHELL ON ELASTIC FOUNDATION

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Abstract. This paper deals with radial vibrations of dissipative poroelastic spherical shell embedded on the elastic foundation. The case of dissipation results in a transcendental, complex valued frequency equation, and the numerical results are not possible. Hence, the limiting case is considered. When the argument is small, the asymptotic expansions of Bessel functions can be employed and consequently frequency equation can be separated into two real valued equations which in turn give phase velocity and attenuation. In this case, a thick walled hollow spherical shell becomes a thin spherical shell. Phase velocity is computed as a function of the wavenumber, and attenuation is computed against the ratio of outer and inner radii. The results with the elastic foundation are compared with that of without elastic foundation. In the absence of dissipation, the phase velocity is computed and the comparison is made between the present work and earlier works.

Keywords: poroelastic spherical shell, frequency equation, phase velocity, attenuation, elastic foundation

1. Introduction

From our real-time experiences, we find that natural structures such as water saturated sedimentary rocks, saturated soil, manmade engineering structures, osseous tissues, and bones in animal and the human body are might be nearly spherical or a part of spherical shape. Some components in Mechanical Engineering, Aerospace Engineering, and the other areas may have unusual shapes like the spherical shell and elliptical cone and might be poroelastic in nature. Even in the man's own body, bones at the skull, knees, and shoulder joints are of nearly spherical shell or elliptical cone shape. The reason is rotational flexibility is required at the joint connection. The dynamic response of vibrations in the said elements can be of great importance, particularly, in the field of non-destructive evaluation (NDE). In Structural Engineering, spherical shell shape solids trim the internal volume and minimize the surface area, that is it saves material cost. In the above-mentioned areas, poroelastic spherical solids and shells are abundant and the study of vibrations in them is important on Research and Development (R & D) front for the reasons mentioned above. On the other hand, in many Engineering applications, spherical shells resting on elastic foundations are of considerable importance, because they represent a class of commonly used structural elements that serve as bearing components in rigid pavements, bridge decks, storage tanks, highways, airfield pavements, constructing gas cylinders, pressure vessels, boilers, and railway tracks. In addition, radial vibration is the backbone of the entire machinery condition monitoring ethos. Radial vibrations of the arterial wall can help to describe the relationship between the heart rate, the blood pressure, and the properties of an arterial wall when different types of

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stimulation are applied to the cardiovascular system [1]. Spherical shells resting on elastic foundations are used in various kinds of industrial applications such as analysis of reinforced concrete pavement of roads, airports, runways, and foundation of buildings. As far as elastic foundations and spherical structures are concerned, much research work is available. Here are a few papers. The static and dynamic behavior of shallow spherical shell on Winkler foundation is discussed by Paliwal [2]. Dempsey et al. studied axisymmetric indentation of an elastic layer supported by a Winkler foundation [3]. Rajib et al. [4] investigated a beam subjected to moving load and moving mass supported by Pasternak foundation. Free vibrations of circular cylindrical shell on Winkler and Pasternak foundations are studied by Paliwal et al. [5]. Heyliger and Pan [6] discussed free vibrations of layered magneto electro elastic spheres. Effect of rotation and magnetic field on free vibrations in a spherical non-homogeneous solid embedded in an elastic medium is investigated by Bayones et al. [7]. In the said paper, waves are significantly influenced by the magnetic field and rotation of the elastic sphere. Yildirim [8] investigated the exact radial free vibrations of power law graded spheres. In the said paper, the free vibrations of a functionally graded material hollow sphere are investigated. In the paper [9], authors derived frequency equations for the free radial vibrations of a sphere and a spherical shell. In the framework of Biot's theory [10], torsional vibrations of thick walled hollow poroelastic sphere are studied by Ahmed Shah and Tajuddin [11]. In the said paper, authors studied frequency equations for limiting cases namely poroelastic thin spherical shell, poroelastic thick spherical shell, and poroelastic solid sphere. Shah and Tajuddin [12] discussed axially symmetric vibrations in fluid filled poroelastic spherical shells. Vibration analysis of a poroelastic composite hollow sphere and free vibrations of a fluid loaded poroelastic hollow sphere surrounded by a fluid is discussed by Shanker et al. [13,14]. In the above three papers, radial and rotatory vibrations of fluid filled and empty poroelastic shells are studied as special cases. Rajitha and Malla Reddy [15,16] discussed vibrations in poroelastic elliptic cone against the angle made by the major axis of the cone in the spheroconal coordinate system. A comparative study is made between the modes of composite spherical shell and its ring modes [17]. Spherical wave propagation in a poroelastic medium with infinite permeability is investigated by Mehmut Ozyazicioglu [18]. In the paper [19], governing equations are derived in the presence of static stress for a poroelastic solid sphere, which was not available in the earlier literature. Free vibrations in an isotropic poroelastic solid sphere with rigidly fixed conditions are studied by Ramesh et al. [20]. In the paper [21], the governing equations of transversely isotropic poroelastic dissipative solids are formulated for the case of radial vibrations. Yongjio Song et al. [22] studied shear properties of heterogeneous fluid filled porous media with spherical inclusions. In the said paper, the analytical solution for the dynamic shear modulus of a heterogeneous poroelastic material containing macroscopic scale spherical inclusions is presented. In the paper [23], authors concluded that the torsional waves are non dispersive in a thin coated hollow poroelastic sphere. Effect of elastic foundation on thick walled hollow poroelastic spherical shell in absence of dissipation is discussed by Rajitha et al. [24]. However, in the above-aforementioned papers, the case of dissipation is not considered. Hence in the present work, the same is taken into an account for radial vibrations which leads to a complex valued frequency equation. Limiting case when the ratio between thickness and inner radius is very small is investigated numerically so that the asymptotic expansions of Bessel functions are employed and frequency equation can be separated into two real parts which in turn give phase velocity and attenuation coefficient. If the dissipation is neglected, then the problem reduces to that of the particular case of published results [24].

The rest of the paper is organized as follows. In section 2, Geometry and solution of the problem are given. In section 3, firstly the boundary conditions are prescribed, the next frequency equation is discussed. Numerical results are presented graphically in section 4. Finally, the conclusion is given in section 5.

2. Geometry and solution of the problem

Consider a poroelastic spherical shell with outer and inner radii r_2 and r_1 in the spherical system (r,θ,ϕ). The shell is embedded on an elastic foundation of finite thickness with shear modulus G, which is treated as the Pasternak model. The Winkler foundation consists of an infinite set of uncoupled springs with a spring constant K as given in Fig. 1 [25].

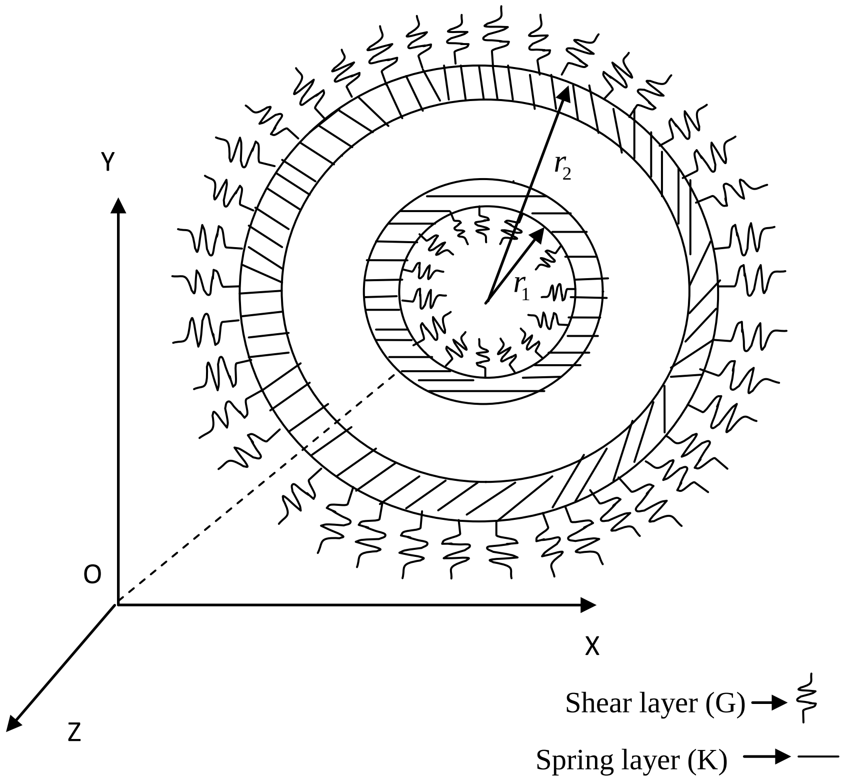

Fig. 1. The geometry of the problem

The equations of motion for a homogeneous, isotropic poroelastic spherical shell in presence of dissipation b for the case of radial vibrations are [10]:

$$N(\nabla^2 - \frac{2}{r^2})u + (A+N)\frac{\partial e}{\partial r} + Q\frac{\partial \varepsilon}{\partial r} = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U) + b\frac{\partial}{\partial t}(u-U),$$
$$Q\frac{\partial e}{\partial r} + R\frac{\partial \varepsilon}{\partial r} = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U) - b\frac{\partial}{\partial t}(u-U), \quad (1)$$

where ∇^2 is the Laplace operator, u and U are radial displacements of solid and fluid. These displacements are functions of r and time t only, e and ε are the dilatations of solid and fluid, A, N, Q, R are all poroelastic constants, b is the dissipative coefficient, and ρ_{ij} are mass coefficients. The solid stresses σ_{ij} and fluid pressure s are

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij} \quad (i, j = 1, 2, 3),$$
$$s = Qe + R\varepsilon. \quad (2)$$

In Eq. (2), δ_{ij} is the well-known Kronecker delta function. Assume that the radial displacements of solid and fluid satisfy:

$$u(r,t) = f(r)e^{i\omega t}, \quad U(r,t) = F(r)e^{i\omega t}, \tag{3}$$

here ω is the frequency of the wave, i is the complex unity, and t is time. Eq. (1) and Eq. (3) yield the displacement components

$$\begin{aligned} u &= (c_1 J_1(\xi_1 r) + c_2 Y_1(\xi_1 r) + c_3 J_1(\xi_2 r) + c_4 Y_1(\xi_2 r))\, e^{i\omega t}, \\ U &= -(c_1\delta_1^2 J_1(\xi_1 r) + c_2\delta_1^2 Y_1(\xi_1 r) + c_3\delta_2^2 J_1(\xi_2 r) + c_4\delta_2^2 Y_1(\xi_2 r))\, e^{i\omega t}, \end{aligned} \tag{4}$$

where c_1, c_2, c_3 and c_4 are arbitrary constants, ω is the frequency of the wave, J_n and Y_n are spherical Bessel functions of first and second kind of order n, respectively, and δ_q, ξ_q $(q = 1,2)$ are given by

$$\delta_q^2 = \frac{-(PR - Q^2)V_q^{-2}}{RM_{12} - QM_{22}}(\xi_q^2) - \frac{RM_{11} - QM_{12}}{RM_{12} - QM_{22}}, \quad q = 1,\ 2,$$

$$P = A + 2N, M_{11} = \rho_{11} - \frac{ib}{\omega},\ M_{12} = \rho_{12} + \frac{ib}{\omega},\ M_{22} = \rho_{22} - \frac{ib}{\omega},\ \xi_q = \frac{\omega}{V_q}(q = 1,\ 2).$$

The notations V_q $(q = 1,2)$ are dilatational wave velocities of the first and second kind, respectively. Using the displacement components in Eq. (2), the pertinent stress σ_{rr} and fluid pressure s are obtained as,

$$\begin{aligned} \sigma_{rr} &= \left(c_1 A_{11}(r) + c_2 A_{12}(r) + c_3 A_{13}(r) + c_4 A_{14}(r)\right)e^{i\omega t}, \\ s &= \left(c_1 A_{21}(r) + c_2 A_{22}(r) + c_3 A_{23}(r) + c_4 A_{24}(r)\right)e^{i\omega t}, \end{aligned} \tag{5}$$

where

$$A_{11}(r) = ((P+Q) + (Q+R)\delta_1^2)((-\xi_1^2)J_1(\xi_1 r) + \frac{\xi_1}{r}J_2(\xi_1 r)) - ((A+Q) + (Q+R)\delta_1^2)(2J_1(\xi_1 r) + \frac{2\xi_1}{r}J_2(\xi_1 r)),$$

$A_{21}(r) = -(Q + R\delta_1^2)\ ((\frac{\xi_1^2}{r^2})J_1(\xi_1 r) + (\frac{2\xi_1}{r})\ J_2(\xi_1 r)),$ $A_{12}(r), A_{22}(r)$ are similar expressions as $A_{11}(r), A_{21}(r)$ with J_1, J_2 replaced by Y_1, Y_2, respectively, $A_{13}(r), A_{23}(r)$ are similar expressions as $A_{11}(r), A_{21}(r)$ with ξ_1, δ_1 replaced by ξ_2, δ_2, respectively, $A_{14}(r), A_{24}(r)$ are similar expressions as $A_{11}(r), A_{21}(r)$ with $J_1, J_2, \xi_1, \delta_1$ replaced by $Y_1, Y_2, \xi_2, \delta_2$ respectively.

3. Boundary conditions and frequency equation

Stress-free boundary conditions are given by

$$\sigma_{rr} + s + Ku + G\Delta u = 0 \text{ at } r = r_1,$$

$$\sigma_{rr} + s + Ku + G\Delta u = 0 \text{ at } r = r_2.$$

Pores at the inner and outer surface assumed to be sealed i.e.

$$\begin{aligned} s &= 0 \text{ at } r = r_1, \\ s &= 0 \text{ at } r = r_2. \end{aligned} \tag{6}$$

In Eq. (6), K is the foundation modulus and G is the shear modulus of the foundation [25], $\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$. Poroelastic spherical shell resting on elastic foundations with different shapes, sizes, and boundary conditions has been the subject of numerous investigations and these structures play a vital role in aerospace, marine, mechanical, civil and nuclear engineering problems. Winkler and Pasternak have developed the foundations with different types of elements such as beams, discs, and shells. The problems of Winkler model (one parameter model) have been solved for static, buckling, and vibration analysis. The Pasternak model (two parameter model) assumes the existence of shear interaction between the spring elements. If the second parameter is neglected, the modeling of the foundation using the Pasternak's formulation converts to that of the Winkler's formulation as a special case. In Eq. (6), if the shear modulus G goes to zero, the Pasternak foundation will be reduced to Winkler foundation and the boundary conditions lead to the following system of homogeneous equations:

$$[A_{lm}][c_l]=0,\ \ l,m=1,2,3,4. \tag{7}$$

Eq. (7) results in a system of four homogeneous equations in four arbitrary constants c_1,c_2,c_3,c_4. For a nontrivial solution, the determinant of coefficients must be zero. Because of dissipation**,** the complex valued frequency equation is obtained which is transcendental in nature and numerical solutions are not possible. Hence, the limiting case $\frac{h}{r_1}<<1$ (i.e. $\xi_1 r_1,\xi_1 r_2,\ \xi_2 r_1,\ \xi_2 r_2>>1$) is considered. In this case, thick walled hollow spherical shell becomes a thin spherical shell so that the following asymptotic approximations for Bessel's function [26] can be used.

$$J_n(x)\approx\sqrt{\frac{2}{\pi x}}\left[\cos\left(x-\left(\frac{n}{2}+\frac{1}{4}\right)\pi\right)-\frac{15}{8x}\sin\left(x-\left(\frac{n}{2}+\frac{1}{4}\right)\pi\right)\right],$$

$$Y_n(x)\approx\sqrt{\frac{2}{\pi x}}\left[\sin\left(x-\left(\frac{n}{2}+\frac{1}{4}\right)\pi\right)+\frac{15}{8x}\cos\left(x-\left(\frac{n}{2}+\frac{1}{4}\right)\pi\right)\right].$$

In this case, the frequency equation is resolved as:

$$\left|c_{lm}\right|+i\left|d_{lm}\right|=0\quad (l,m=1,2,3,4). \tag{8}$$

In Eq. (8), the elements c_{lm} and d_{lm} are real valued and are given by

$$c_{11}=(P+Q+(Q+R)A_{10})(R_1N_{10}-R_2N_{20}-R_3N_1+R_4N_2)-((Q+R)A_{11})(3R_1N_{20}+2R_2N_{10}-R_4N_1$$
$$-R_3N_2-2N_2)-(A+Q+(Q+R)A_{10})(2N_1+2R_1N_{20}-R_2N_{10})+r_1K(N_1-R_1N_{10}+R_2N_{20}),$$

c_{12} is a similar expression as c_{11} with N_1,N_2,N_{10},N_{20} replaced by N_3,N_4,N_{30},N_{40} respectively,

c_{13} is a similar expression as c_{11} with $A_{10},A_{11},R_1,R_2,R_3,R_4$ replaced by $E_{10},E_{11},R_{10},R_{20},R_{30},R_{40}$ respectively,

c_{14} is a similar expression as c_{11} with $N_1,N_2,N_{10},N_{20},\ A_{10},A_{11},R_1,R_2,R_3,R_4$ replaced by $N_3,N_4,N_{30},N_{40},\ E_{10},E_{11},R_{10},R_{20},R_{30},R_{40}$ respectively,

$$c_{21}=RA_{11}(R_3N_2+R_4N_1)+2R_1N_{10}-2R_2N_{20}-(Q+RA_{10})(R_3N_1-R_4N_2),$$

c_{22} is a similar expression as c_{21} with N_1,N_2,N_{10},N_{20} replaced by N_3,N_4,N_{30},N_{40} respectively,

c_{23} is a similar expression as c_{21} with $A_{10},A_{11},R_1,R_2,R_3,R_4$ replaced by $E_{10},E_{11},R_{10},R_{20},R_{30},R_{40}$ respectively,

c_{24} is a similar expression as c_{21} with $N_1,N_2,N_{10},N_{20},\ A_{10},A_{11},R_1,R_2,R_3,R_4$ replaced by $N_3,N_4,N_{30},N_{40},\ E_{10},E_{11},R_{10},R_{20},R_{30},R_{40}$ respectively,

$c_{31},c_{32},c_{33},c_{34}$ are similar expressions as those of $c_{11},c_{12},c_{13},c_{14}$ with r_1 replaced by r_2,

$c_{41}, c_{42}, c_{43}, c_{44}$ are similar expressions as those of $c_{21}, c_{22}, c_{23}, c_{24}$ with r_1 replaced by r_2,

$$d_{11} = (P + Q + (Q+R)A_{10})(R_1 N_{20} + R_2 N_{10} - R_4 N_1 - R_3 N_2) + ((Q+R)A_{11})(-R_1 N_{10} + R_2 N_{20} - R_3 N_1 + R_4 N_2 - 2N_1) - (A + Q + (Q+R)A_{10})(2N_2 + 2R_1 N_{20} + R_2 N_{10}) + r_1 K(N_2 - R_1 N_{20} - R_2 N_{10}),$$

d_{12} is a similar expression as d_{11} with N_1, N_2, N_{10}, N_{20} replaced by N_3, N_4, N_{30}, N_{40} respectively,

d_{13} is a similar expression as d_{11} with $A_{10}, A_{11}, R_1, R_2, R_3, R_4$ replaced by $E_{10}, E_{11}, R_{10}, R_{20}, R_{30}, R_{40}$ respectively,

d_{14} is a similar expression as d_{11} with N_1, N_2, N_{10}, N_{20}, $A_{10}, A_{11}, R_1, R_2, R_3, R_4$ replaced by N_3, N_4, N_{30}, N_{40}, $E_{10}, E_{11}, R_{10}, R_{20}, R_{30}, R_{40}$ respectively,

$$d_{21} = -RA_{11}(R_3 N_1 - R_4 N_2) + 2R_1 N_{20} + 2R_2 N_{10} - (Q + RA_{10})(R_3 N_2 + R_4 N_1),$$

d_{22} is a similar expression as d_{21} with N_1, N_2, N_{10}, N_{20} replaced by N_3, N_4, N_{30}, N_{40} respectively,

d_{23} is a similar expression as d_{21} with $A_{10}, A_{11}, R_1, R_2, R_3, R_4$ replaced by $E_{10}, E_{11}, R_{10}, R_{20}, R_{30}, R_{40}$ respectively,

d_{24} is a similar expression as d_{21} with N_1, N_2, N_{10}, N_{20}, $A_{10}, A_{11}, R_1, R_2, R_3, R_4$ replaced by N_3, N_4, N_{30}, N_{40}, $E_{10}, E_{11}, R_{10}, R_{20}, R_{30}, R_{40}$ respectively,

$d_{31}, d_{32}, d_{33}, d_{34}$ are similar expressions as those of $d_{11}, d_{12}, d_{13}, d_{14}$ with r_1 replaced by r_2,

$d_{41}, d_{42}, d_{43}, d_{44}$ are similar expressions as those of $d_{21}, d_{22}, d_{23}, d_{24}$ with r_1 replaced by r_2. (9)

The notations involved in the above are given in Appendix (A).

4. Numerical results

The complex valued frequency equation (8) gives attenuation (Q_1^{-1}), and phase velocity (C_p). These values are computed by using the following equations [21]:

$$Q_1^{-1} = \frac{2(\omega \text{ of imaginary part of Eq.}(8))}{\omega \text{ of real part of Eq.}(8)}, \quad C_p = \frac{(\omega \text{ of real part of Eq.}(8))}{\text{Wavenumber}}. \tag{10}$$

For the illustration purpose, three types of poroelastic solids, namely, spherical shell-I made up of sandstone saturated with kerosene [27], spherical shell-II made up of sandstone saturated with water [28], and the spherical bone is used. The parameter values of bone are computed as in the paper [29]. The physical parameter values are given in Table 1.

Table 1. Material Parameters

Material Parameters	P (Gpa)	A (Gpa)	N (Gpa)	Q (Gpa)	R (Gpa)	ρ_{11} (Kg/m^3)	ρ_{12} (Kg/m^3)	ρ_{22} (Kg/m^3)
Spherical shell-I	8.966	3.436	2.765	0.076	0.326	1.926×10^3	-0.002×10^3	0.215×10^3
Spherical shell-II	21.5	3.06	9.22	0.13	0.637	1.903×10^3	0	0.226×10^3
Bone	21.812	10.766	5.522	-8.820	36.779	1763	0	1496

The dissipative coefficient is taken to be 0.17×10^{-6} Gpa/m^2 as in the paper [21]. Winkler elastic modulus value is taken to be 1.5×10^{-9} Gpa/m as in the paper [25]. Employing these values in Eq. (8), the phase velocity is computed as a function of wavenumber and the attenuation is computed as a function of the ratio $(g=\frac{r_2}{r_1})$. The values are computed using the bisection method implemented in MATLAB, and the results are depicted in Figures 2 to 4. The notations SS-I, SS-II, RoEF, FfEF used in the figures represent the spherical shell I, spherical shell II, resting on elastic foundation, and free from the elastic foundation, respectively. Figure 2 depicts a variation of phase velocity against wavenumber. The comparison is made between spherical shell resting on elastic foundation with that of free from the elastic foundation. From Figure 2, it is seen that as wavenumber increases, phase velocity, in general, increases for all the solids resting on elastic foundation. The phase velocity of spherical shell II is higher than that of the spherical shell I and bone. Spherical shell I and II differ in only the fluid part. This discrepancy is due to the influence of the fluid part. When the wavenumber is greater than 6, the phase velocity is almost linear in nature for all the three poroelastic solids. Whereas in the case of free from the elastic foundation, phase velocity, in general, decreases for all the solids as wavenumber increases. The trend is reversed for solid is free from the elastic foundation. Figure 3 depicts the variation of attenuation with the ratio (g) for spherical shell resting on elastic foundation with that of free from the elastic foundation. From Figure 3, it is clear that as g increases, in general, attenuation decreases for all the solids resting on elastic foundation. When the solids are free from the elastic foundation as g increases, in general, attenuation increases for all the solids. Attenuation in the case of bone, in general, higher than that of the remaining two solids. If the dissipation is ignored, the problem reduces to that of the particular case of the published work [24]. Figure 4 depicts the variation of phase velocity with wavenumber in the absence of dissipation. From Figure 4, it is seen that as wavenumber increases, phase velocity, in general, decreases for all the solids. Present results are in agreement with that of earlier results.

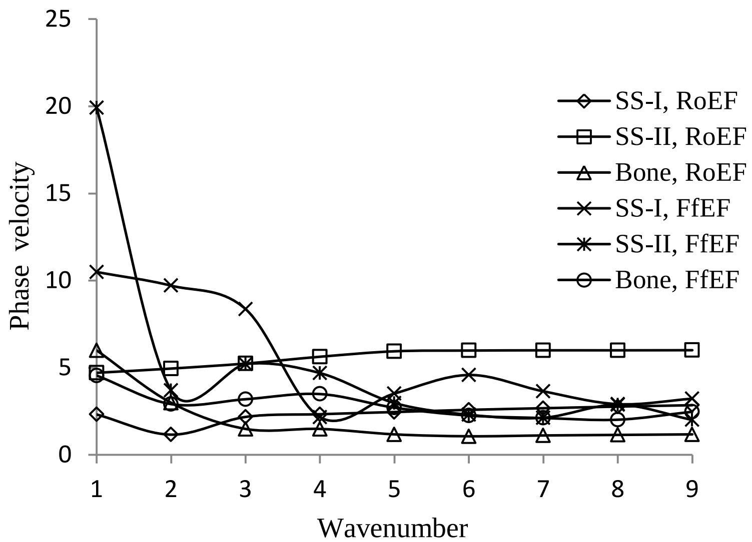

Fig. 2. Variation of phase velocity with wavenumber

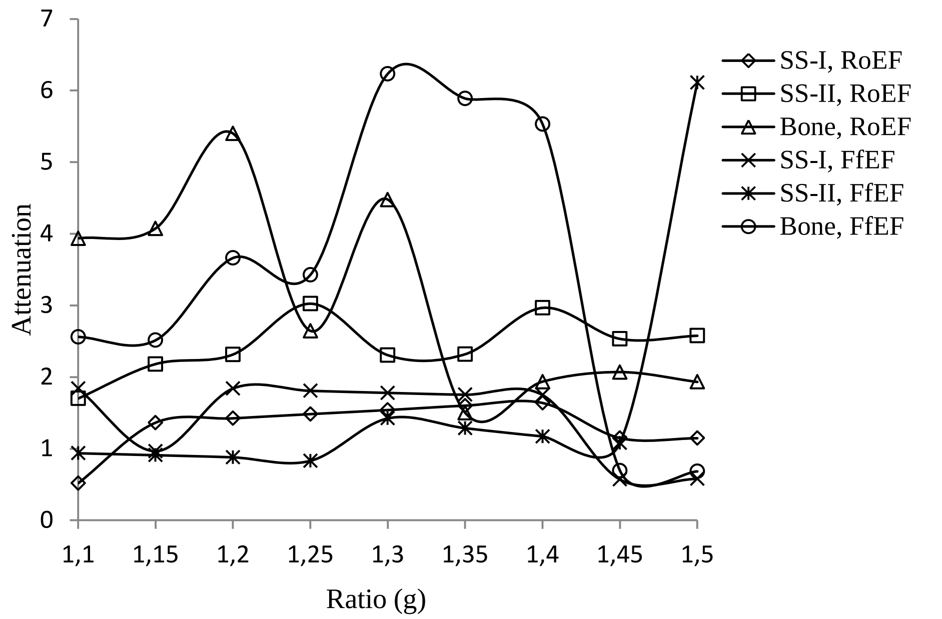

Fig. 3. Variation of attenuation with the ratio (g)

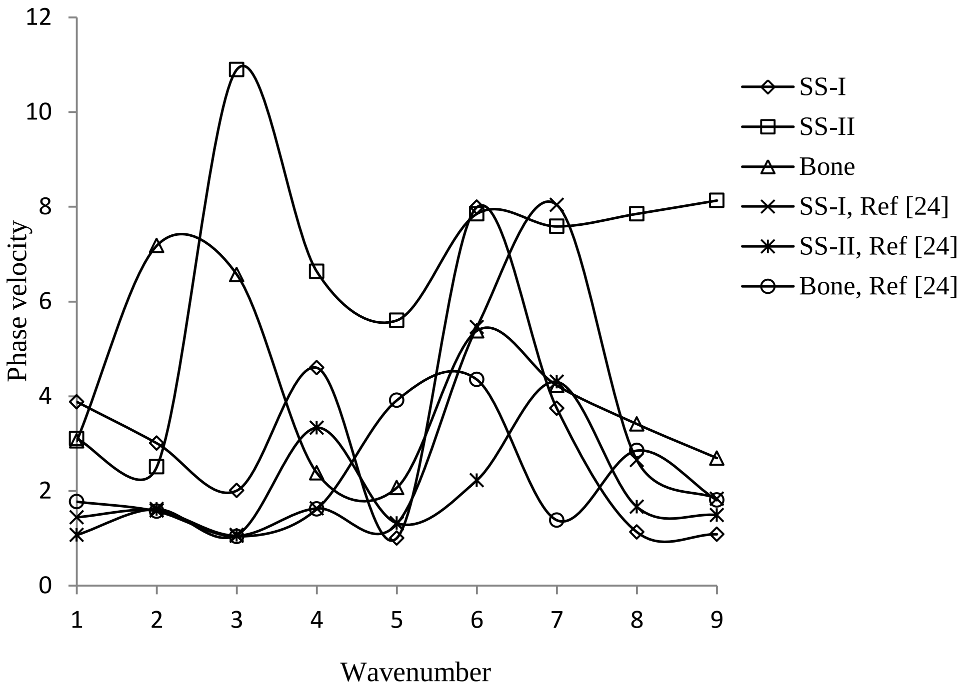

Fig. 4. Variation of phase velocity with wavenumber in the absence of dissipation

5. Conclusion

Employing the Biot's poroelasticity theory, radial vibrations of dissipative thick walled hollow poroelastic spherical shell resting on elastic foundation is investigated. The frequency equations are discussed when the solids are resting on an elastic foundation with that of free from the elastic foundation. Phase velocity is computed as a function of wavenumber and attenuation is computed against the ratio of outer and inner radii. For the numerical process three types of solids are considered namely, sandstone saturated with kerosene (say spherical shell I), sandstone saturated with water (say spherical shell II), and bone is employed and the results are presented graphically. From the numerical results, it is clear that wave characteristics are strongly affected when a spherical shell is attached with an elastic foundation. If the dissipation is ignored, the problem reduces to that of the particular case of the earlier works. The efficiency and validity of this approach are confirmed by comparing the present results with that of earlier results. This type of analysis can be made for any poroelastic spherical shell if the values of parameters of that poroelastic material are available. Since all the bones are not isotropic in nature, this model is not exactly one to study the bones, and one has to go for a general case namely transversely isotropic. Moreover, this model can be extended to relatively more general plane strain and plane stress problems.

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Appendix A

$$A_{10} = \frac{A_0 C + BD}{C^2 + D^2}, \quad A_{11} = \frac{BC - A_0 D}{C^2 + D^2}, E_{10} = \frac{EG + FH}{G^2 + H^2}, E_{11} = \frac{FG - EH}{G^2 + H^2},$$

$$A_0 = (PR - Q^2) - V_{10}(R\rho_{11} - Q\rho_{12}) - \frac{V_{11} b(R-Q)}{\omega}, \quad B = \frac{V_{10} b(R-Q)}{\omega} - V_{11}(R\rho_{11} + Q\rho_{12}),$$

$$C = V_{10}(R\rho_{12} - Q\rho_{22}) - \frac{V_{11} b(R+Q)}{\omega}, \quad D = V_{11}(R\rho_{12} - Q\rho_{22}) + \frac{V_{10} b(R+Q)}{\omega},$$

$$V_{10} = \frac{2Q\rho_{12} - P\rho_{22} - R\rho_{11} + X_{11}}{2(PR - Q^2)}, V_{11} = \frac{1}{2(PR - Q^2)}\left(\frac{Hb}{\omega} + X_{22}\right),$$

$$X_{11}=\left(x^2+y^2\right)^{1/2}\cos\left[\frac{1}{2}\tan^{-1}\left(\frac{y}{x}\right)\right],\qquad X_{22}=\left(x^2+y^2\right)^{1/2}\sin\left[\frac{1}{2}\tan^{-1}\left(\frac{y}{x}\right)\right],$$

$$x=(P\rho_{22}-2Q\rho_{12}+R\rho_{11})^2-\left(\frac{bH}{\omega}\right)^2+2T_1(\rho_{11}\rho_{22}-\rho_{12}^2),$$

$$y=(2Q\rho_{12}-P\rho_{22}-R\rho_{11})\left(\frac{2bH}{\omega}\right)-2T_1\frac{b\rho}{\omega},$$

E,F,G,H are similar expressions as those of A_0,B,C,D with V_{10},V_{11} replaced by V_{20},V_{21}, respectively.

$$V_{20}=\frac{2Q\rho_{12}-P\rho_{22}-R\rho_{11}-X_{11}}{2(PR-Q^2)},\ V_{21}=\frac{1}{2(PR-Q^2)}\left(\frac{Hb}{\omega}-X_{22}\right),$$

$$N_1=T_1D_{11}+T_2D_{21},\quad N_2=T_1D_{21}-T_2D_{11},\quad N_{10}=T_1D_1+T_2D_2,\quad N_{20}=T_1D_2-T_2D_1,$$

$$N_3=T_1D_{31}+T_2D_{41},\quad N_4=T_1D_{41}-T_2D_{31},\quad N_{30}=T_1D_3+T_2D_4,\quad N_{40}=T_1D_4-T_2D_3,$$

$$M_1=T_3D_{11}+T_4D_{21},\quad M_2=T_3D_{21}-T_4D_{11},\quad M_{10}=T_3D_1+T_4D_2,\quad M_{20}=T_3D_2-T_4D_1,$$

$$M_3=T_3D_{31}+T_4D_{41},\quad M_4=T_3D_{41}-T_4D_{31},\quad M_{30}=T_3D_3+T_4D_4,\quad M_{40}=T_3D_4-T_4D_3,$$

$$T_1=\frac{T_{01}}{T_{03}},\ T_2=\frac{T_{02}}{T_{03}},\ T_3=\frac{T_{001}}{T_{003}},\ T_4=\frac{T_{002}}{T_{003}},$$

$$T_{01}=R_1\cos(X)-R_2\sin(X),\ T_{02}=-R_1\sin(X)-R_2\cos(X),$$

$$T_{03}=8\sqrt{\pi}\,r_1\left(R_1^2+R_2^2\right)^{1/2}\left[\left(R_1\cos(X)-R_2\sin(X)\right)^2+\left(R_1\sin(X)+R_2\cos(X)\right)^2\right],$$

$$T_{001}=R_{10}\cos(Y)-R_{20}\sin(Y),\ T_{002}=-R_{10}\sin(Y)-R_{20}\cos(Y),$$

$$T_{003}=8\sqrt{\pi}\,r_1\left(R_{10}^2+R_{20}^2\right)^{1/2}\left[\left(R_{10}\cos(Y)-R_{20}\sin(Y)\right)^2+\left(R_{10}\sin(Y)+R_{20}\cos(Y)\right)^2\right],$$

$$R_1=\frac{S_1\omega r_1}{(S_1^2+S_2^2)},\ R_2=\frac{S_2\omega r_1}{(S_1^2+S_2^2)},\qquad X=\frac{1}{2}\tan^{-1}\left(\frac{R_2}{R_1}\right),\ Y=\frac{1}{2}\tan^{-1}\left(\frac{R_{20}}{R_{10}}\right),$$

$$S_1=\left(V_{10}^2+V_{11}^2\right)\cos\left[\frac{1}{2}\tan^{-1}\left(\frac{V_{11}}{V_{10}}\right)\right],\ S_2=\left(V_{10}^2+V_{11}^2\right)\sin\left[\frac{1}{2}\tan^{-1}\left(\frac{V_{11}}{V_{10}}\right)\right],$$

$$R_{10}=\frac{S_3\omega r_1}{(S_3^2+S_4^2)},\ R_{20}=\frac{S_4\omega r_1}{(S_3^2+S_4^2)},\ S_3=\left(V_{20}^2+V_{21}^2\right)\cos\left[\frac{1}{2}\tan^{-1}\left(\frac{V_{21}}{V_{20}}\right)\right],$$

$$S_4=\left(V_{20}^2+V_{21}^2\right)\sin\left[\frac{1}{2}\tan^{-1}\left(\frac{V_{21}}{V_{20}}\right)\right],\ R_3=\frac{V_{10}(\omega r_1)^2}{V_{10}^2+V_{11}^2},\ R_4=\frac{V_{11}(\omega r_1)^2}{V_{10}^2+V_{11}^2},$$

$$R_{30}=\frac{V_{20}(\omega r_1)^2}{V_{20}^2+V_{21}^2},\ R_{40}=\frac{V_{21}(\omega r_1)^2}{V_{20}^2+V_{21}^2},$$

$$D_1 = 8R_1 \cosh R_2 (\cos R_1 + \sin R_1) + (15\cosh R_2 - 8R_2 \sinh R_2)\ (\cos R_1 - \sin R_1),$$

$$D_2 = 8R_1 \sinh R_2 (\cos R_1 - \sin R_1) + (8R_2 \cosh R_2 - 15\sinh R_2)\ (\cos R_1 + \sin R_1),$$

$$D_3 = 8R_1 \cosh R_2 (\sin R_1 - \cos R_1) + (15\cosh R_2 - 8R_2 \sinh R_2)\ (\cos R_1 + \sin R_1),$$

$$D_4 = 8R_1 \sinh R_2 (\cos R_1 + \sin R_1) + (8R_2 \cosh R_2 - 15\sinh R_2)\ (\sin R_1 - \cos R_1),$$

$$D_{11} = 8R_1 \cosh R_1 (\sin R_1 + \cos R_1) + (3\cosh R_2 - 8R_2 \sinh R_2)\ (\cos R_1 - \sin R_1),$$

$$D_{21} = 8R_1 \sinh R_2 (\cos R_1 - \sin R_1) + (8R_2 \cosh R_2 - 3\sinh R_2)\ (\cos R_1 + \sin R_1),$$

$$D_{31} = 8R_1 \cosh R_2 (\sin R_1 - \cos R_1) + (3\cosh R_2 - 8R_2 \sinh R_2)\ (\cos R_1 + \sin R_1),$$

$$D_{41} = 8R_1 \sinh R_2 (\cos R_1 + \sin R_1) + (3\sinh R_2 - 8R_2 \cosh R_2)\ (\cos R_1 - \sin R_1).$$