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DISLOCATION PILE-UPS, STRENGTH PROPERTIES AND FRACTURING

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Abstract. A review is given in honor of Ilya A. Ovid'ko beginning from a mutual interest in disclinations. The main part of the review, however, is based on association of dislocation pile-ups and the Hall-Petch relation for an inverse square root of grain size dependence of cleavage fracturing. Important thermal influence is considered. Special attention is given to small pile-up characteristics associated with nanopolycrystalline material behaviors. Other topics include: (1) the ductile-brittle transition; (2) the ductile true fracture strain; (3) the fracture mechanics stress intensity; (4) hardness; (5) creep; and, (6) fatigue behaviors. Experimental measurements are presented for a wide range of materials.

1. INTRODUCTION

Early connection of the author with the researches of Ilya Ovid'ko involved a mutual interest in the application of crystal disclinations to better understand material strength properties. It had been proposed in yesteryear that a (wedge) disclination could serve as a model for a deformation twin [1]. An interesting consequence was that the mechanical force on a twin in such model description depends on the twin thickness. Later investigation with colleagues dealt with evaluation of the generalized force on disclinations of different types [2] and also with development of an analogous Frank-Read model for deformation twin nucleation [3]. Ovid'ko, in turn, has dealt comprehensively with a wider role for disclinations in variously modeled circumstances, for example within: amorphous glass materials [4]; quasi-periodic structures [5]; thin film/substrate interfaces [6]; and, especially, for an important role in controlling the mechanisms of deformation and cracking within nanopolycrystalline grain boundaries [7,8], including wedge-disclination-associated grain boundary cracking [9] as compared with disclination dipoles hampering microcrack growth [10] and with formation of deformation twins [11]. An important update of disclination model considerations has been given by Ovid'ko's colleagues, Romanov and Kolesnikova [12]. Otherwise, Ovid'ko's concern with nanopolycrystal strength levels has led naturally to important interest also in dislocation pile-up model calculations relating to the grain size dependence of those same nanopolycrystal strength properties and particularly relating to the brittleness of ceramic materials [13]. A unique feature of Ovid'ko's concern with nanopolycrystal strength properties has been with dislocation pile-ups that are contained within the nano-grain boundary interfaces and are proposed to play a role in intergranular fracturing.

First personal contact between us came from Ovid'ko's co-editing, with Pande, Krishnamoorti, Lavernia and Skandan, of the Materials Research Society volume "Mechanical Properties of Nanostructured Materials and Nanocomposites" [14]. Later interaction involved providing an article for the RU journal "Reviews on Advanced Materials Science" [15] and most recently has included

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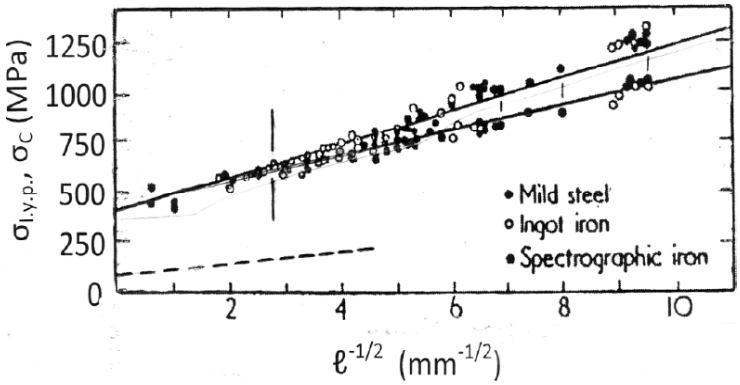

Fig. 1. Combined Petch measurements of lower yield point stress, $\sigma_{l.y.p.}$, and cleavage stress, σ_C, tested at liquid nitrogen temperature, see our work [21].

Ovid'ko's participation in the UK Royal Society theme issue on "Fracturing across the multi-scales of diverse materials", co-edited with S.D. Antolovich, J.R. Griffiths and J.F. Knott [16]. In the present article that is dedicated to Ilya Ovid'ko, a review is given of several dislocation pile-up and Hall-Petch type grain size dependent aspects of metal and ceramic material failure behaviors, especially relating to a mutual interest in nanopolycrystal material strength properties.

The review begins with a brief description of Norman Petch's historical dislocation pile-up reports on both cleavage fracturing and consequent important thermal influence on the ductile-brittle transition exhibited by steel and related materials.

Then, in contrast with continuum expectation, the discreteness of small dislocation pile-ups in nano-materials is shown to make cleavage unlikely in otherwise cleavage-prone conventional grain size materials in agreement with latest experiments. Further brief assessments of rather less well-known pile-up/grain size associations with material properties are then described, including such measurements as: (1) the true tensile ductile fracture stress and true fracture strain; (2) the fracture mechanics stress intensity; (3) hardness dependence on grain size; (4) thermal description of creep life-times; and (5) certain fatigue characteristics.

2. CLEAVAGE FRACTURING

The model of a dislocation pile-up leading to cleavage fracturing was initially described by Zener who noted that coalescence of the leading dislocations could form a crack nucleus [17]. As Hall had done before him, Petch employed the dislocation pile-up parameters reported by Eshelby, Frank and Nabarro to predict an inverse square root of grain size dependence for both the yield and cleavage fracture stresses of iron and steel materials [18,19]. The grain size dependence is now eponymously credited as the Hall-Petch (H-P) relation. For example, Ovid'ko and Sheinerman have employed the same pile-up model in a description of crack nucleation at an internal grain boundary near to a free surface of a nano-metal having a bimodal grain size structure [20].

2.1. Historical Petch result

Fig. 1, taken from [21], is a summary figure of Petch's lower yield point, $\sigma_{l.y.p.}$, and cleavage, σ_C, stresses reported for the listed iron and steel materials tested at liquid nitrogen temperature. In the figure, the left-side coincident points corresponded to cleavage occurring at the lower yield point. The separated right-side dependencies involved correction of the cleavage stress for an increasing fracture strain with decreasing grain diameter, l. The correction, that was based on experiment, reduced the microstructural stress intensity, k_C, from ~100 to ~80 MPa mm$^{1/2}$ in the oft-quoted H-P expression

$$\sigma_C = \sigma_{0C} + k_C l^{-1/2}. \quad (1)$$

In Eq. (1), σ_{0C} was taken to be the friction stress for the dislocations piled-up in the pre-yielded grain volumes.

2.2. Thermal activation

The lower dashed line in Fig. 1 applies for the ambient temperature $\sigma_{l.y.p.}$ for mild steel having experimental constants $\sigma_{0l.y.p.}$ and $k_{l.y.p.}$, in the latter case with value of $k_{l.y.p.}$ = ~24 MPa mm$^{1/2}$ [22]. Both $k_{l.y.p.}$ and k_C are known to be athermal. The $k_{l.y.p.}$ in Fig. 1 is indicated to be somewhat higher for the liquid nitrogen temperature test result and probably involves a contribution of needed higher stress con-

centration for deformation twinning, otherwise the important influence of a decreasing test temperature, T, raises $\sigma_{0l.y.p.}$. Petch made use of the thermal influence being in the friction stress to obtain an explicit evaluation of the ductile-brittle transition temperature, T_C, for such steel materials in the relationship [23]

$$T_C = (1/\beta)\left[\ln B_0 - \ln\{(CG\gamma / k_f) - k_f\} - \ln l^{-1/2}\right]. \quad (2)$$

In Eq. (2), β is the exponential temperature coefficient of the thermal component of $\sigma_{l.y.p.}$ whose (extrapolated) value at $T = 0$ is B_0, C is a numerical constant, G is shear modulus, γ is fracture surface energy, and k_f is the H-P microstructural stress intensity (slope value) for the true ductile fracture stress dependence on grain size, as had been previously reported by Petch [24], with $k_f < k_C$.

Whereas emphasis had been given previously to the predicted reduction in T_C achieved with logarithmic increase in $l^{-1/2}$ in Eq. (2), attention is directed here to the importance of the exponential thermal stress factor, β. Zerilli and Armstrong included the same β factor in the following so-called Z-A constitutive relation for the flow stress, σ_ε, of bcc and certain related hcp metals, with or without a lower yield point [25]

$$\sigma_\varepsilon = \sigma_G + B_0 \exp\left[-\beta T\right] + A\varepsilon^n + k_\varepsilon l^{-1/2}. \quad (3)$$

In Eq. (3), σ_G is an athermal stress component dependent on G, and the power law constants A and n describe the material strain hardening. The first three terms on the right-side of Eq. (3) are normally combined in a single term, $\sigma_{0\varepsilon}$, of the H-P relation. The post-lower yield point, k_ε, follows the inequality $k_\varepsilon < k_{l.y.p.}$ [26]. The parameter, β, depends on the applied strain rate in the Z-A description as follows

$$\beta = \beta_0 - \beta_1 \ln[d\varepsilon / dt]. \quad (4)$$

In Eq. (4), β_0 and β_1 are experimental constants rooted in the thermal activation – strain rate analysis (TASRA) and $[d\varepsilon/dt]$ is the unidirectional plastic strain rate [25]. Petch pointed to a low value of β being appropriate for description of the T_C dependence measured in Charpy impact tests [23].

Fig. 2 shows application of such TASRA description to the temperature dependence of σ_y for relatively pure iron material tested at two strain rates as taken from results reported for a titanium-gettered low-carbon steel material by Leslie, Sober, Babcock, and Green [27]. A reduced value of σ_G = 70 MPa, k_y = 5.5 MPa mm$^{1/2}$, $l^{-1/2}$ =5.6 mm$^{-1/2}$ led to the 100 MPa stress increment subtracted from the ordinate σ_y; see Eq. (3). Further analysis of such TASRA-based thermal stress consideration will enter later into particular consideration of higher temperature material creep-type fracturing behavior. As will be seen, other thermal aspects of plastic yielding and subsequent plastic flow often enter into control of even apparent brittle fracturing, especially including the importance of a local plastic zone ahead of a crack tip [28].

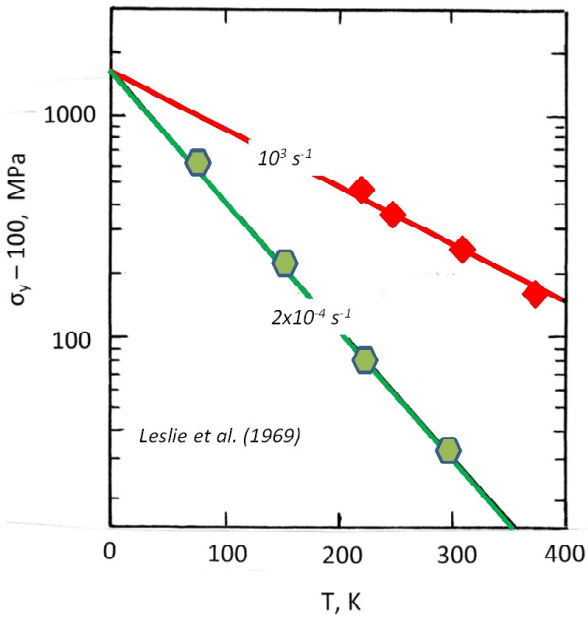

Fig. 2. The thermal component of σ_y for titanium-gettered steel material, adapted from [27].

3. SMALL DISLOCATION PILE-UPS

The beneficial influence of grain size reduction on the strength properties of steel and related materials leads naturally to the issue of how far material strength property improvements may be enhanced by grain size reduction, hence, one reason for current interest in nano-scale material technology [29]. From a theoretical viewpoint, Fig. 3 shows comparison of discrete dislocation pile-up types involving small numbers and continuum crack descriptions plotted on the basis of $l = c$, crack length [30]. In an earlier report, the discontinuous nature of the predicted pile-up behaviors was shown to follow a reciprocal n dependence for the reduction occurring in the effective shear stress, $\Delta\tau_e$, needed to overcome a blocking obstacle stress, τ^*, with addition of each dislocation added to the pile-up [31]

$$\Delta\tau_e / \tau_e = -1/n. \quad (5)$$

And, very importantly, the single dislocation pile-up, double-ended one, and internal circular one are matched in Fig. 3 with the out-of-figure continuum

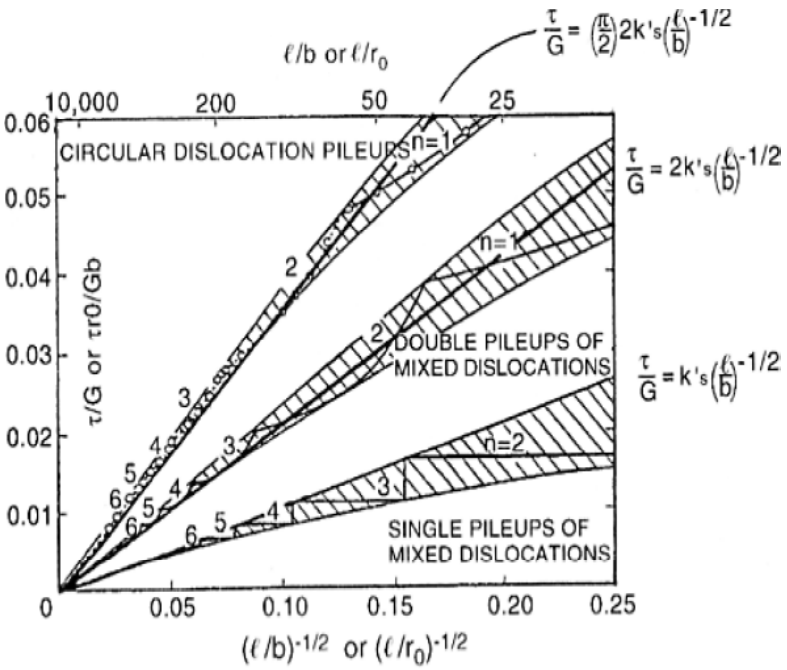

Fig. 3. Theoretical H-P relations for discrete dislocation pile-ups and analogous Griffith-type continuum crack relations on the basis of the grain size, ℓ, taken equal to the crack length, see our work [30].

crack expressions for an edge-type surface crack, internal two-dimensional crack, and one with circular geometry, thus relating to the identical continuum crack and pile-up descriptions on assumption of a continuous distribution of piled-up dislocations with infinitesimal Burgers vectors [30].

3.1. Cleavage difficulty

The indication of divergence between continuum cracks and discrete pile-up results for small pile-ups raised the issue of whether there might be added difficulty for a small pile-up to act as a cleavage crack in generating a similar stress concentration [32]. Fig. 4 shows such stress concentration at distance, *r*, divided by crack size, *a*, ahead of pile-ups of 5 and 49 dislocations in comparison with a Griffith mode II crack result in each case. The individual A_i dislocation positions were taken from the tabulation reported by Chou, Garofalo, and Whitmore [33]. The comparison of crack and pile-up results, that may be seen for the pile-ups at large effective distance to approximate to the stress associated with a multiple-Burgers vector dislocation, gives clear indication of the smaller pile-up not rising to the effectiveness of a crack. The model description appears to be in agreement with experimental results reported by Hohenwarter and Pippan who have reported fractographic observations of intergranular cracking for the low temperature failure of ultrafine grain size α-iron material [34]. These authors have

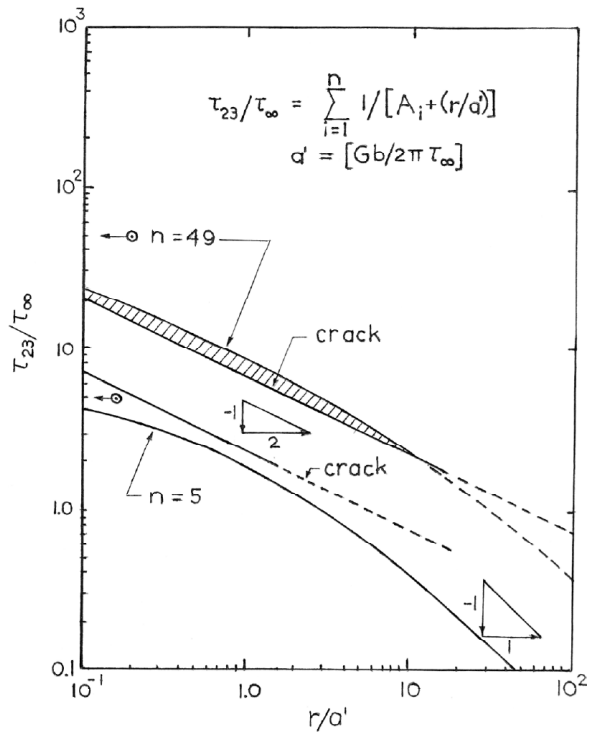

Fig. 4. Comparison of Griffith crack and pile-up stress concentrations for pile-ups involving 5 or 49 dislocations, see our work [36].

also connected such observations with fracture toughness measurements for ultrafine and nanocrystalline grain size materials [35]. Ovid'ko has pointed to the occurrence of 'flat grain boundary cracking' in nano-materials [13].

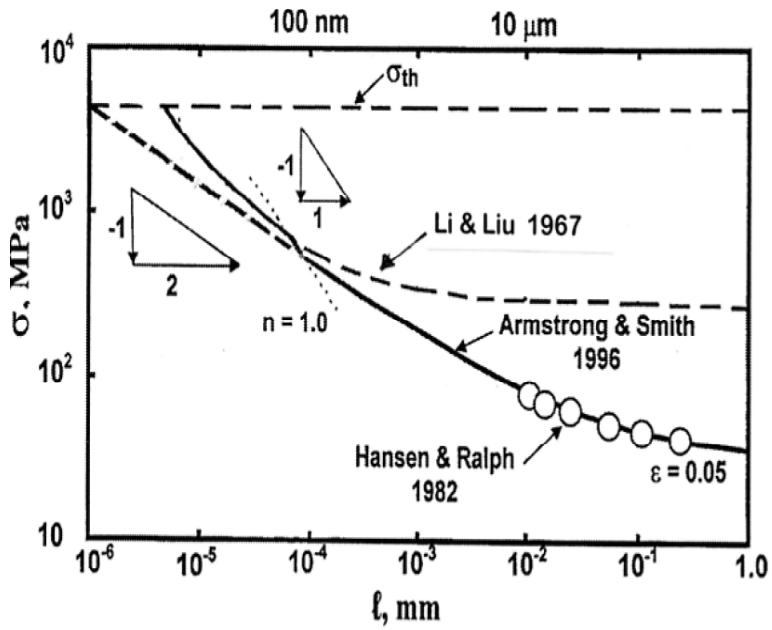

Fig. 5. H-P dependence for conventional grain size copper material extrapolated to nano-scale grain sizes where transition occurs to the stress required for a single dislocation loop to expand against the grain boundary resistance, see our work [36].

3.2. Transition to a single loop

A single dislocation loop expanding against the grain boundary resistance naturally presents itself as the lower limit of pile-up behavior. Such condition has also been investigated [36]. Fig. 5 shows a result for copper that had been predicted by Armstrong and Smith [37]. The experimental H-P results shown for conventional grain size material are often-quoted measurements reported by Hansen and Ralph [38]. The single loop curve which meets the extrapolated H-P curve at a value of ℓ just less than 100 nm had been taken from circular pile-up calculations reported by Li and Liu [39] but with addition here of the friction stress and H-P determined obstacle stress at the grain boundary. Later results reported by L. Lu, Chang, Huang, and K. Lu have included micro-, nano-grain size and nano-twinned boundary measurements falling near to the H-P extrapolation [36, 40]. Recent nano-scale H-P measurements reported for nano-Al material by Choi, Lee, Park, and Bae have fallen above extrapolated Al results [41]. The limiting transition condition has been suggested to explain the measurements [42]. Additional evidence for the lower single loop explanation was provided by an H-P interpretation of corresponding thermal-type strain rate sensitivity measurements.

4. OTHER STRENGTH APPLICATIONS

After Petch's extension of the cleavage fracturing dependence to a same description for ductile fracturing of mild steel material, a report had been made to include the "in-between" flow stress, σ_ε, measurements in a combined figure [22] and thus establishing an H-P dependence for the full stress-strain behavior, see above $k_\varepsilon < k_{\text{l.y.p.}}$ [26]. In addition, further H-P connections were established with accumulated metal hardness, creep and fatigue properties. An update is provided here of the same type H-P connections and, as well, on an H-P dependence for the true fracture strain (with qualification), and on the fracture mechanics stress intensity, also connecting in this case with steel, (ceramic) Al_2O_3 and (cermet) WC-Co material properties.

4.1. The true (ductile) fracture strain

The observation of H-P dependence for the stress-strain behavior between the yield stress and true (ductile) fracture stress was shown to provide an H-P type expression for the true fracture strain, for example, of magnesium material as shown in Fig. 6 in accordance with [43]:

$$\varepsilon_f = \varepsilon_0 + \left(\Delta k / h\right) l^{-1/2}. \tag{6}$$

In Eq. (6), $\varepsilon_0 = \varepsilon_{0f} - \varepsilon_{0y}$, $\Delta k = k_f - k_y$, and h is average strain hardening coefficient. One might note the reasonably large range in grain size covered in the figure, including the finer grain size (dashed line and curve) measurements for Wilson and Chapman [44]. They suggested that superplasticity was operative in their finer grain size material. Fedorov, Gutkin,

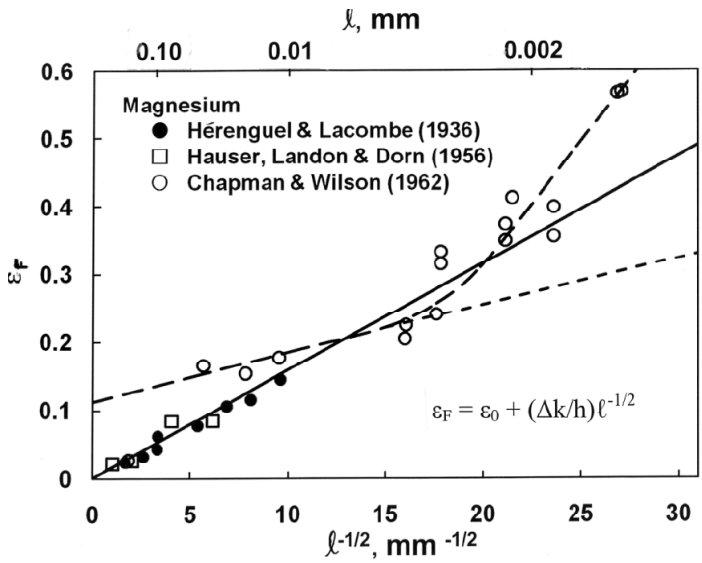

Fig. 6. An H-P dependence for the true fracture strain, ε_f, of a number of magnesium materials, see our work [43].

and Ovid'ko have connected their model of dislocation pile-ups within grain boundaries both with grain boundary sliding and superplasticity behavior [45].

Fig. 7 provides some qualification for the application of Eq. (6) as shown in connected measurements reported for a broad range of carbon steel materials [46]. In the top left-side part of Fig. 7, a linear H-P type dependence of ε_f is shown for measurements tabulated for Armco iron material by Srinivas, Malakondaiah, Armstrong, and Rama Rao, to be discussed later in connection with corresponding fracture mechanics stress intensity measurements [47].

On the top right-side are ε_f measurements reported by Liu and Gurland who took the mean free path among particles and grain boundaries of spheroidized steel materials of different carbon contents as the effective grain size in an analogous H-P description of the material strength properties [48]. In this case, the smallest value of $\ell^{1/2}$ applies for a lowest 0.065 carbon concentration. The right-side curve through these data was derived by Petch and Armstrong on the basis of ε_f being determined by bringing the particles to a critical separation [49]. The lower measurements shown in Fig. 7 are for similar copper ε_f measurements determined for intergranular cavitation results reported in creep tests by Fleck, Cocks and, Taplin also relating to later discussion of failure behavior in creep testing [50].

4.2. Fracture mechanics

An early description of grain size dependence in the fracture mechanics stress intensity was given by Armstrong [51]. The connection was made in pointing to close approximation to the Griffith equation being obtained from an analysis by Bilby, Cottrell, and Swinden for a crack of length, *c*, with a strip-type plastic zone size, *s*, at the crack tip [52]. Both crack and plastic zone had been modeled in terms of continuous distributions of dislocations, relating to the demonstration given here in Fig. 3. Fig. 8 shows a later compilation of measurements exhibiting an H-P type dependence for the thus-derived H-P relation for the fracture mechanics stress intensity [53]. An evaluation of the plane strain stress intensity, K_{Ic}, was given more recently as [28]

$$K_{IC} = (8/3\pi)\left[\sigma_{0C} + k_C l^{-1/2}\right]s^{1/2}. \qquad (7)$$

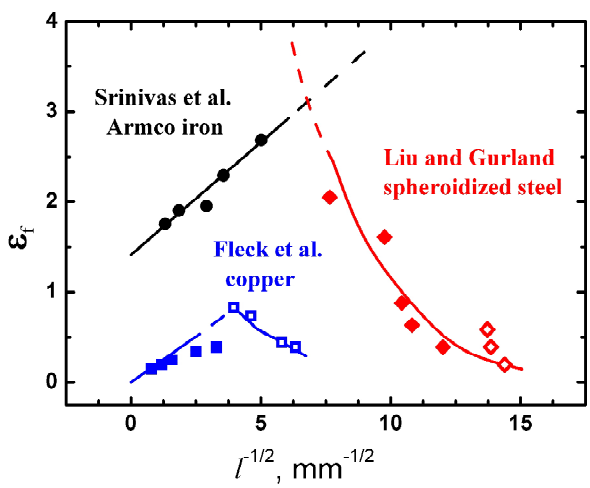

Fig. 7. The true fracture strain dependence on grain size and, on an effective grain size basis, mean free path of particle separations, adapted from [46].

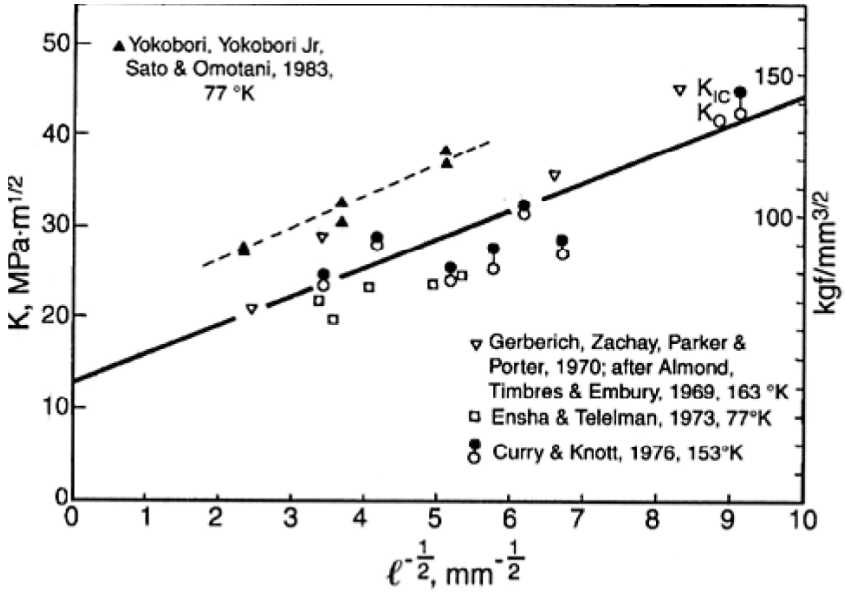

Fig. 8. An H-P dependence for compiled measurements of fracture mechanics stress intensity, K and K_{Ic}, made on various steel materials, see our work [53].

Fig. 8 gives indication, in line with Eq. (7), that the plastic zone size is relatively constant in many cases but there are interesting exceptions, for example, of special circumstances to be described in which s is proportional to l and thus leading to a reduction in K_{Ic} with decrease in grain size. Srinivas et al. have made comparison with ductile fracture toughness measurements made on Armco iron material [47].

4.2.1. Crack tip blunting

Eq. (7) provides a role for dealing with crack blunting caused by local plastic deformation at the crack tip. In one case, Ovid'ko and Sheinerman have attributed a decrease in fracture toughness observed for nanocrystalline and ultra-fine grained aluminum and α-iron materials to their grain boundaries limiting the extension of dislocations being emitted from a crack tip [54,55], hence a case of $s \approx l$ in Eq. (7). In another case, the same authors have described a model situation in which a blunted crack enhances additional nano-crack nucleation and growth at smaller nano-grain sizes, thus also producing lower ductility and reduced fracture toughness [56].

4.2.2. Indentation fracture mechanics

Indentation fracture mechanics provides another method of determining the fracture mechanics stress intensity of relatively brittle materials such as ceramics. And here again, Fig. 9 provides evidence for an H-P dependence of fracture mechanics stress intensity, K_{Ic}, measurements made via indentation testing of relatively fine grain size Al_2O_3 materials [57].

In this case, the large and small filled-circle measurements were taken from detailed measurements reported in two investigations [58,59]. Armstrong and Cazacu provided further interpretation of these results and extended the analysis to similar measurements made on cermet WC-Co materials. For the latter system the plastic zone size was taken to be controlled by the mean free path, λ, of the cobalt phase thus leading also to a positive K dependence on $\lambda^{1/2}$ [60].

4.3. Hardness

Hardness testing is often employed to measure the plastic strength properties of nano-scale grain size material and recently to do so by means of nano-indentations. A recent review has been given of the elastic, plastic and cracking aspects of hardness testing, including the method of indentation fracture mechanics described in the preceding section [61].

Hall was first to suggest that the metal hardness should follow an $l^{-1/2}$ dependence [62]. Armstrong and Jindal showed that compiled hardness measurements reported in a study of titanium material recrystallization and grain growth followed an H-P dependence that at the time applied for the largest range in grain size that had been measured [63,64]. Fig. 10 shows a compilation of later measurements reported for electrodeposited nickel material extending downward to a grain size of 12 nm

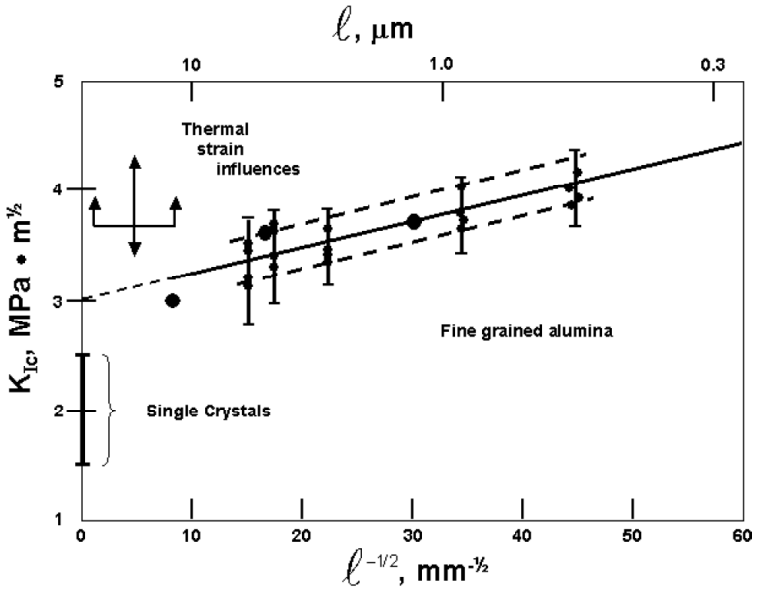

Fig. 9. An H-P dependence for a compilation of K_{Ic} measurements determined via indentation fracture mechanics tests made on Al_2O_3 materials, data from [57-59].

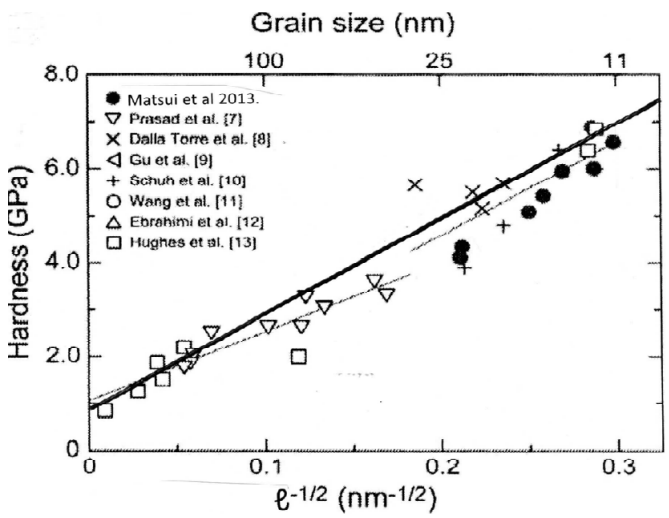

Fig. 10. An H-P dependence for a compilation of hardness measurements for nickel materials covering conventional to nano-scale grain sizes, see our work [65].

[65]. The original figure was reported by Matsui, Uesugi, Takigawa, and Higashi [66]; and, the solid line added to the figure was taken from [67].

4.4. Creep

Creep testing, generally done at the opposite higher temperature end of the spectrum away from cleavage temperatures, provides an additional example of potential grain size strengthening or weakening depending on test conditions and, in particular, on the temperature, *T* [68]. Pioneering results on influence of the material grain size were first reported by Jeffries [69], including disputed report of an increased true fracture strain with decrease in grain size as shown in the later results of Fleck, Cocks, and Taplin [50] at larger grain size in the lower part of Fig. 7. Ovid'ko and Sheinerman have attributed an enhanced ductility of nano-materials to a combination of grain boundary sliding and (temperature-aided) diffusion processes, also relating to the occurrence of superplasticity [70], as mentioned above with respect to the magnesium results shown in Fig. 6. In an earlier report, Gutkin, Ovid'ko, and Skiba proposed that nano-material softening could be attributed in part to grain boundary pile-ups pushing lattice dislocations into the grain volumes [71].

4.4.1. Z-A, Z-H, and L-M relations

There is connection of the TASRA-based Eq. (3) and the higher temperature equations employed to

describe thermal aspects of creep deformation. An often employed beginning is provided by the eponymous Zener-Hollomon (Z-H) parameter, expressed in terms of a limiting plastic strain rate, $(d\varepsilon/dt)_{0S}$, as [72]

$$(d\varepsilon/dt)_{0S} = (d\varepsilon/dt)_S \exp[G\{\tau_{Th}\}/k_B T]. \quad (8)$$

In Eq. (8), $(d\varepsilon/dt)_S$ is the steady-state creep rate; $G\{\tau_{Th}\}$ is the Gibbs free energy taken to be a function of the thermal component of stress, $\tau_{Th} = (\sigma_{Th}/m_T)$, for which m_T is the Taylor factor; and k_B is Boltzmann's constant. The Z-A description of Eq. (3) leads to evaluation of $G\{\tau_{Th}\}$ as [46]

$$G\{\tau_{Th}\} = G_0 - W_0 \ln(\tau_{Th}/\tau_{Th0}). \quad (9)$$

In Eq. (9), G_0 is the reference Gibbs free energy; $W_0 \approx 3.1 \times 10^{-20}$ J is the product of τ_{Th} and the thermal activation volume, $v^* = (k_B T)[\partial \mathbf{ln}(d\varepsilon/dt)/\partial\tau_{Th}]_T$; and, τ_{Th0} is a lower limiting value of thermal shear stress. Thus, the Z-H parameter, equal to $(d\varepsilon/dt)_{0S}$, is obtained as an explicit function of stress as

$$Z-H = (d\varepsilon/dt)_S \exp[G_0/k_B T] \times (\tau_{Th}/\tau_{Th0})^{-W0/kBT}. \quad (10)$$

As will be seen there is connection to be made with a pioneering analysis by Larson and Miller of empirical prediction of creep lifetimes determined as a function of the applied stress [73].

4.4.2. L-M prediction

Larson and Miller had developed a pioneering graphical description of the stress-dependent time-temperature relations for life-times of creep-determined material failure properties [73]. Tamura, Abe, Shiba, Sakasegawa, and Tanigawa have produced a comprehensive report on the topic [74]. Fig. 11 shows one display of their own measurements. Application of the Z-A description to the logarithmic dependence of the creep stress on failure time led to the relationship [46]

$$\ln(\tau_{Th}) = -(k_B/W_0)\left(T\left[\ln\Delta t - \left\{\ln\Delta t_0 + (G_0/kT) + (W_0/k_B T)\ln\tau_{Th0}\right\}\right]\right). \quad (11)$$

In Eq. (11), the term in curly brackets, with reference time, Δt_0, is taken as approximately constant, thus establishing connection with the abscissa scale of Fig. 11. An exponential slope of -3.4×10^{-4} K applies for the 'All data' line in the figure and compares with the Z-A based slope prediction from Eq. (11) of $-(k_B/W_0) = -4.2 \times 10^{-4}$ K. Similar agreement

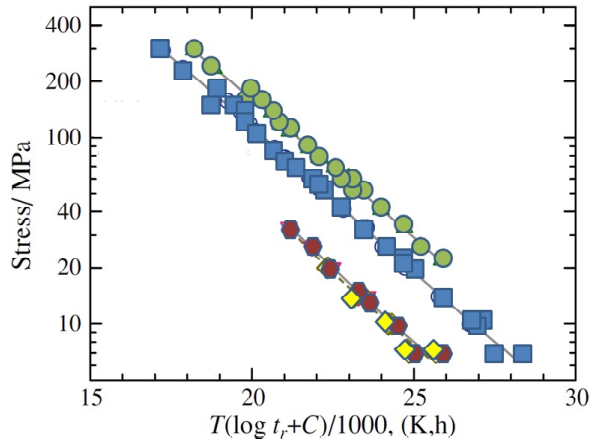

Fig. 11. Larson-Miller measurements of creep-lifetimes for an Fe-Cr-Ni-AlTi material (Fe-21Cr-32NiAlTi tube): All data – C=17.88, *Q'*= 441.8 kJ/mole; ● 873-1073K, *C*=18.96, *Q'*=417.9 kJ/mole; ⬢ 1173-1323K, *C*=15.93, *Q'*=447.1 kJ/mole; ◆ X<1, *C*=15.83, *Q'*= 455.9 kJ/mole, adapted from [74].

had been shown to apply for the low carbon, C-Mo, and stainless steel materials previously investigated by Larson and Miller [73].

4.5. Fatigue

Remarkable progress has been made in recent years on understanding and improving the fatigue properties of materials [75,76]. As with the other mechanical properties, the fatigue strength is generally improved by grain size reduction [26].

4.5.1. H-P dependence

An H-P dependence was established previously for low carbon-steel and α-brass materials, in the latter case following the yield stress dependence [77]. Fig. 12 shows the somewhat more complicated H-P description given for low-carbon steel materials. Later demonstration was made of an H-P dependence for commercial aluminum material [78] and for an additional compilation of other α-brass measurements [79]. And most recently clear improvement has been demonstrated for ultrafine grain size copper material [80]. In that case, a lowered fatigue limit, k_{FL}, was determined for the stress dependence that extended to high-cyclic behavior.

4.5.2. Persistent slip bands

Fig. 13 gives indication of the special circumstance of dislocation dipole formation in cyclic deformation [81]. A dislocation pile-up in the reverse direction

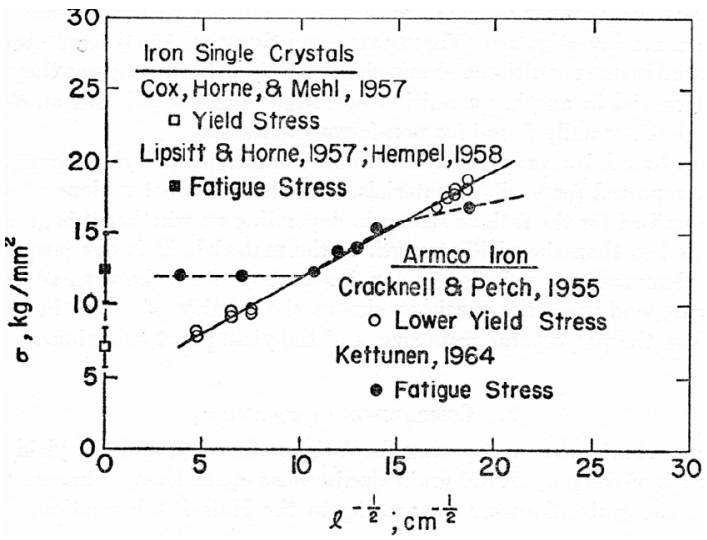

Fig. 12. Compilation of lower yield stress and fatigue limit stresses for α-iron and low carbon steel single crystals and polycrystals; 1 kg/mm^2 = 9.81 MPa, see our work [77].

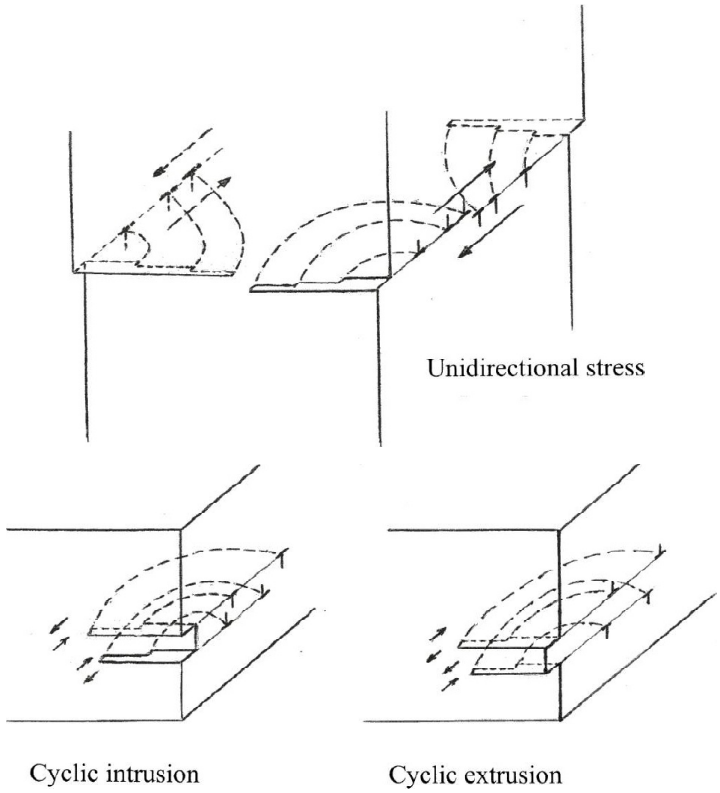

Fig. 13. Cyclic dislocation pile-up action to form a dipole structure, see our work [81].

lowers the H-P microstructural stress intensity. The surface occurrence of slip band intrusions/extrusions is now known to be an important source of damage in association with the presence of 'persistent slip bands' (PSBs) [82,83]. The heavy surface damage is a result ultimately of the greater number of dislocations able to be packed into a slip band because of the steeper force interaction associated with the dipoles compared to the normal reciprocal distance characteristic of a single dislocation.

5. SUMMARY

A review has been given of dislocation pile-ups and their connection via a Hall-Petch description with the grain size dependence of a variety of material strength properties. And it is a great pleasure to have included references in a number of places to the important work of Ilya A. Ovid'ko and his colleagues. Until the present time, the topic of Hall-Petch interpretation of material properties has

spanned ~60 years of research results accumulated in the international literature. Surely, Ilya will agree that there is more to come, especially on his favorite topic of nano-materials.

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