

# COMPOSITE MATERIALS WITH RADIOACTIVE INCLUSIONS AS ARTIFICIAL RADIOABSORBING COVERING

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**Abstract.** New form of composite materials as semiconductor dielectric layer with radioisotope inclusions is presented in this paper. The presence of additional generators of ionization in semiconducting material (due to  $\alpha$ -particles irradiated by radioisotope inclusions) causes the essentially nonequilibrium characteristics of composite materials: increase of absorption factor, changes of real component of permittivity, and nonlinearity phenomenon of conductivity. The generalized kinetic equation is proposed to describe nonequilibrium stationary states of the electronic subsystem of radioisotope composite covering. It is shown that nonequilibrium states of electrons in the covering are solutions of this equation and have the power asymptotic forms.

## 1. INTRODUCTION

Recently the composite materials are widely adopted in many areas of physics and technology (for example as protective covering). Such materials can be considered as materials with complex (fractal) structure. The distinguishing characteristic of fractal media implies that the macroscopic properties of materials (coefficients of thermal conductivity, electro-conductivity, diffusion, etc.) and dielectric properties depend on their fractal characteristics (for example, fractal dimensions of a material or its porosity) [1].

Characteristic features and phenomena in such simulated fractal media, which can be used to operate their electro-dynamical properties are:

- irreversibility and their general property of 'forgetfulness' (or 'loss of information' [1]);
- nonequilibrium medium, creation of states with power distribution functions and power falling of correlations [2]; changing dispersion of medium in these states [3];
- nonlinearity of medium caused by its fractality [4];
- peculiar properties of material due to specific dispersion characteristics of inclusions;
- capability to control stochastic resonance characteristics in simulated fractal media.

The interest to study the use of composite materials in electronics is caused by possibility to control effectively electro-dynamical properties of such materials.

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From our point of view, the fractal radioisotope composite materials, i.e. dielectric materials with fractal inclusions consisting of alpha-radioactive elements, can be used more effectively in future.

The use of alpha-radioactive inclusions gives a possibility to use the nonequilibrium of electrical systems of materials as complementary control factor, i.e. it is possible to control not only the inhomogeneity of material in coordinate space, but the inhomogeneity of electrical subsystem in phase space.

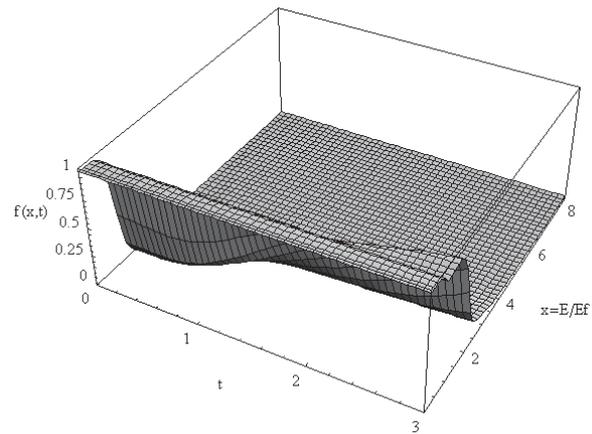
Radioisotope inclusions cause the complementary ionization by  $\alpha$ -particles in a material. For a radioisotope covering with intensity of a radioactive material  $\chi$  (Ku/cm<sup>2</sup>) with energy  $E$  (MeV) of  $\alpha$ -particles in volume of a material with excitation energy  $I$  (eV) there occurs an electron sources with intensity  $Q = 3.7 \cdot 10^{10} \chi E / IR_d$ , where  $R_d$  (cm) is a path length of  $\alpha$ -particles in material [5,6].

The presence of steady sources of ionization and sinks of particles in system through the recombination of surplus charge and emission current from a composite material gives rise to the quasistationary states for electrons [2,3]. Distribution functions of particles on energy in such states possess the power asymptotics and correspond to quasistationary flows in phase space. Kinetics of electrons in plasma of solids and creation of stationary nonequilibrium distributions of electrons due to this ionization provide a theoretical basis to describe the radioisotope coverings; this fact has been first confirmed experimentally in [5,6].

Nonequilibrium states are characterized by nonextensivity and their entropy can be represented in Tsallis form with nonextensivity coefficient  $q$  [7]. We have developed the generalized kinetic equation without sources and sinks with Jackson derivatives which order depends on flow of particles [8]. The stationary solutions of this equation agree with nonequilibrium stationary states of radioisotope composite material.

## 2. METHODOLOGY

It has been first shown in [2,3] that power distribution functions are the exact stationary solution of equation for integrals of collision between Boltzmann and Fermi-particles. It is convenient to present the kinetic equation for isotropic distribution function as continuity equation for distribution function on energy in the form



**Fig. 1.** The dependence of the generalized flow in phase space from energy and parameter  $q$ .

$$\frac{\partial f}{\partial t} = - \frac{\partial}{g(\varepsilon) \partial \varepsilon} \Pi \{f, \varepsilon\} + \Psi(\varepsilon); \quad (1)$$

where  $\Pi \{f, \varepsilon\}$  is a flow in the phase space and  $\Psi(\varepsilon)$  are sources and sinks.

The differential form of the kinetic equation for photons interacting with electrons has been obtained first in paper [9] and the power-series solution of this equation – in [10]. For electrons interacting with phonons, the general kinetic equation in differential form looks like Eq. (1) with flow of particles

$$\Pi \{f, \varepsilon\} = \varepsilon g(\varepsilon) v(\varepsilon) \left[ T_0 \frac{\partial f}{\partial \varepsilon} + f - f^2 \right], \quad (2)$$

where  $v(\varepsilon) = v_0 (\varepsilon / T_0)^q$ , and values  $v_0, q$  are determined by mode of interaction between electrons and phonons. Stationary nonequilibrium solutions correspond to the constant nonzero flow of particles (or energy) in the phase space (see [2,3]). The energy distribution evolution is accompanied by energy flow evolution in the phase space. Usually, the connection between distribution function and flow in the pulse space is determined at the same moment of time (locally versus time). At large source intensities, it is necessary to take into account the specific time of relaxation of particles flow  $t$  and to note the kinetic equation, which represents the diffusion equation for distribution function in the energy space, as generalized equation of hyperbolic

$$\frac{\partial f}{\partial t} = -\frac{1}{4\pi v^2} \frac{\partial}{\partial v} J\{f, v\};$$

$$\tau \frac{\partial J}{\partial t} + J = \prod (v, \{f(v)\}). \quad (3)$$

In the case of the energy sources in the system, Eqs. (2) and (3) has a stationary non-equilibrium solution in the exponential asymptotical form [2,3]. To clarify the influence of radioisotope sources on degree of irregularity of electrons states in solids we give results of numerical solution of system (2),(3) for kinetic equation with localized sources and sinks (see Fig. 1).

The quasi-stationary distributions with the exponential 'tails' are of general character and associated with the generalization on thermodynamics to non-ideal systems.

C. Tsallis [7] has considered the thermodynamics of such systems in which the entropy has not the property of extensiveness. The formalism of Tsallis consists in replacement of the exponential and logarithmic functions in the relations of statistics and thermodynamics by their generalized power

$$\ln(x) \rightarrow \ln_q(x) = \frac{x^{1-q} - q}{1-q};$$

$$\exp(x) \rightarrow \exp_q(x) = (1 + (1-q)x)^{1/(1-q)} \quad (4)$$

with a parameter  $q$ . Note that at  $q \rightarrow 1$ ,  $\ln_q(x)$  and  $\exp_q(x)$  turn into the common logarithm and exponent and are inter-convertible ones. The new formula for the entropy with use of (4) has the following form:

$$S_q = -\sum_i p_i^q \ln_q(p_i) = \frac{1 - \sum_i p_i^q}{q-1}. \quad (5)$$

The essential property of the entropy introduced in such a way is its nonextensiveness. In fact, as it follows from (4), the function  $\ln_q(x)$  satisfies relationship:

$$\ln_q(xy) = \ln_q(x) + \ln_q(y) + (1-q)\ln_q(x)\ln_q(y). \quad (6)$$

Using this property, we can split out the full system into two independent subsystems A and B, then

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B) \quad (7)$$

Evidently, the parameter  $q$  here is a measure of nonextensiveness of system. The functional form of  $q$ -entropy, which proposed by Tsallis, is quite arbitrary and the value  $q$  is not determined. Nevertheless, the  $q$ -entropy maximum leads to the exponential-like functions:

$$p(\varepsilon) = \exp_q(-(\varepsilon/T)). \quad (8)$$

Let's compute the flow in the velocity space for the exponential-like function (8). The stationary states correspond to the regions in which the flow does not depend on velocity. Note that the formal generalization of the Maxwell-Gibbs equilibrium distribution function, using the function (4), to non-equilibrium states in the case of the quantum statistics becomes ambiguous one. Really, the Fermi equilibrium function can be represented as two equivalent forms:

$$f(\varepsilon) = \frac{1}{1 + \exp\left(\frac{\varepsilon - E_f}{T}\right)} = \frac{\exp\left(-\frac{\varepsilon - E_f}{2T}\right)}{\exp\left(\frac{\varepsilon - E_f}{2T}\right) + \exp\left(-\frac{\varepsilon - E_f}{2T}\right)}. \quad (9)$$

Substituting the exponent symbol in these representations for the function (9), we obtain two non-equivalent generalizations to non-equilibrium states for the Fermi function. These both functions at  $q \rightarrow 1$  are transformed into the Fermi distribution function. The distribution function

$$f_q(\varepsilon) = \frac{\exp_q\left(-\frac{\varepsilon - E_f}{2T}\right)}{\exp_q\left(\frac{\varepsilon - E_f}{2T}\right) + \exp_q\left(-\frac{\varepsilon - E_f}{2T}\right)}. \quad (10)$$

deviating from equilibrium, undergoes changes concentrated in the vicinity of the Fermi energy, whereas the second function

$$f_q(\varepsilon) = \frac{1}{1 + \exp_q\left(\frac{\varepsilon - E_f}{T}\right)} \quad (11)$$

with increasing non-equilibrium, has the exponential asymptotical form at larger energies. Joining the

obtained asymptotics we obtain the correct solution at all energies:

$$f_q(\varepsilon, E_f, T) = \frac{1}{1 + \exp_q\left(\frac{\varepsilon - E_f}{T}\right)} \theta(E_f - \varepsilon) + \frac{\exp_q\left(-\frac{\varepsilon - E_f}{T}\right)}{\exp_q\left(-\frac{\varepsilon - E_f}{T}\right) + 1} \theta(\varepsilon - E_f). \quad (12)$$

The coefficient  $q$  in above expression depends on intensity of source of particle in the system.

### 3. GENERALIZED KINETIC EQUATION

Now we will demonstrate that the Fermi generalization of distribution function above-stated can be (at least roughly) represented as a generalized equation solution for the flow in the phase space. For this purpose, we use of the Jackson derivative. The Jackson derivative for the arbitrary  $f(x)$  is written in the following form:

$$D_q f(x) = \frac{f(qx) - f(x)}{qx - x}. \quad (13)$$

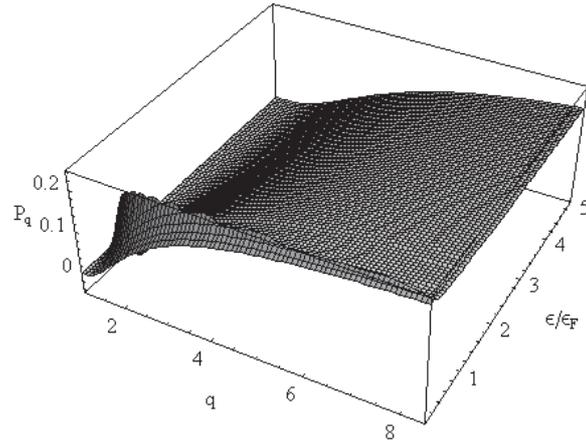
When the parameter  $q$  tends to 1 ( $q \rightarrow 1$ ), the Jackson derivative is transformed to the normal derivative  $D_q f(x) \xrightarrow{q \rightarrow 1} df(x)/dx$ . As it is known, the collision integral  $I_{st}$  can be written in the divergent form in terms of the flow in the phase space that in its ultimate case has the following form:

$$I_{st} = -\frac{1}{g(\varepsilon)} \frac{\partial}{\partial \varepsilon} \prod\{f, \varepsilon\}, \prod(\{f\}, \varepsilon, T) = v(\varepsilon) \left( T \frac{\partial f}{\partial \varepsilon} + f(\varepsilon)(1 - f(\varepsilon)) \right). \quad (14)$$

Let's substitute the normal derivative in the flow equation for the Jackson derivative with the parameter  $q$  and introduce the generalized flow  $\prod_q(\{f\}, \varepsilon, T)$  using the generalized relation:

$$\prod_q(\{f\}, \varepsilon, T) = TD_q f(\varepsilon) + f(\varepsilon)(1 + f(\varepsilon)). \quad (15)$$

By substituting the distribution function (12) in the flow equation (15), we obtain the flow as a function of the variables  $\varepsilon, q$  which is presented in Fig. 2.



**Fig. 2.** Dependence of generalized flow  $\prod_q(\{f\}, \varepsilon, T)$  in the phase space on energy and nonextensiveness parameter  $q$ .

As can be seen from the figure, there are regions of parameters where the dependence of flow on energy is small. It is confirmed by the calculation of generalization of collision integral

$$I_q = D_q \left( \prod_q(\{f\}, \varepsilon, T) \right)^q \quad (16)$$

with respect of Jackson derivative of  $q$  order from the generalized flow. The expression obtained for the analogous parameters as in Fig. 2 occurs to be equal to zero with an accuracy of  $10^{-5}$ . Thus, the quasistationary distribution function (12) precisely satisfies the generalized kinetic equation for stationary:

$$I_q(\{f\}, \varepsilon, T) = 0. \quad (17)$$

As usual, the Fermi energy values in the distribution function (12) are determined by the particle density  $n(E_f, T, q) = \int_0^{\infty} g(\varepsilon) f_q(\varepsilon, E, T) d\varepsilon$ , where  $g(\varepsilon)$  is the density of electron states in solids. Our description of generalized flow in the phase space permits to inter-relate the non-extensiveness parametric value  $q$  and the flow value. This dependence in the temperature region  $T < 0.3E_f$  can be just described by the simple analytical relation

$$\prod_q = 0.2 \ln_{q=1.677}(q). \quad (18)$$

Hence, the stationary nonequilibrium solutions of kinetic equations with sources and sinks, that

correspond to the constant flows in the phase space, can be presented as solutions of certain generalized kinetic equations without sources and sinks. The integral characteristics of sources issue from the value of nonextensivity parameter  $q$ .

#### 4. DISPERSIVE PROPERTIES OF A COVERING.

Large deformations of distribution function of electrons in a material of covering lead to essential changes of material permittivity, which describes the material behavior in the external magnetic field. It follows from the determination of permittivity:

$$\begin{aligned} \varepsilon^\ell(\omega, \vec{k}) &= 1 + \delta\varepsilon^\ell, \\ \delta\varepsilon^\ell &= \frac{4\pi e^2}{k^2} \int d\vec{v} \frac{1}{\omega - \vec{k}\vec{v}} \vec{k} \frac{\partial f}{\partial \vec{v}}. \end{aligned} \quad (19)$$

By integration (19) on angles and taking into account that  $1/(x+i0) = \text{p.v.}/x - i\pi\delta(x)$  we obtain the simple estimation:

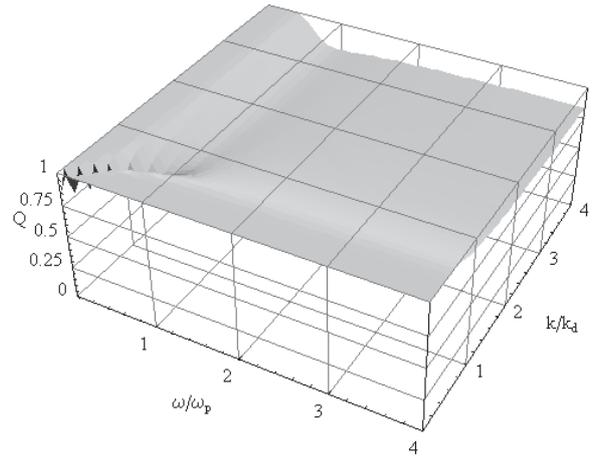
$$\begin{aligned} \text{Re } \delta\varepsilon &= -\frac{16\pi^2 e^2}{k^2} \int dv \frac{v^2 f(v)}{\frac{\omega^2}{k^2} - v^2}, \\ \text{Im } \delta\varepsilon &= -\frac{8\pi^3 m^2 e^2}{\omega^2} \left(\frac{\omega}{k}\right)^3 f\left(\frac{\omega}{k}\right). \end{aligned} \quad (20)$$

From Eq. (20) one can see that the increase of distribution function in the region of Fermi energies lead to the drastic wave decreasing. The radius of Debye  $r_d$ , essential to study the electromagnetic properties of plasmas, is related to the distribution function by the equation:

$$r_d = \left| \frac{4\pi e^2}{m} \int \left( \vec{k} \frac{\partial f}{\partial \vec{v}} \right) \frac{d\vec{v}}{k\vec{v}} \right|^{-\frac{1}{2}}. \quad (21)$$

Hence, for  $r_d$  in the nonequilibrium states of electrons in solids with distribution function (12), it follows:

$$\begin{aligned} k_{dq}^2 = r_d^{-2} &= \left| \frac{4\pi m v^3}{3T} \left( \frac{T}{E_i(q-1) + T} \right)^{\frac{q}{q-1}} \right. \\ &\left. {}_2F_1 \left[ \frac{3}{2}, \frac{q}{q-1}, \frac{5}{2}, \frac{m(q-1)v^2}{2E_i(q-1) - 2T} \right] \right|. \end{aligned} \quad (22)$$



**Fig. 3.** Coefficient of electromagnetic wave absorption in nonequilibrium composite covering subject to the wave vector of incident wave and its frequency.

The coefficient of wave absorption calculated for the stationary distribution function (12) subject to the wave vector of incident wave and its frequency is presented in Fig. 3.

The decreasing of waves over the wide range of parameters of damping wave becomes, as shown in Fig. 3, to be very extensive and full energy of wave can be absorbed by relative layer of covering.

#### 5. CONCLUSIONS

The proposed method can permit to construct the effective thermodynamical model for physical systems with sources and sinks which does not depend on the specific interaction potential in a system. We hope that the macroscopical characteristics of stationary nonequilibrium states possess the universality properties and the study of generalized kinetic equations for systems with sources and sinks are of fundamental importance to interpret the nonequilibrium statistical thermodynamics in open systems which are far away from the local equilibrium.

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