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MAGNETORESISTANCE OSCILLATIONS

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Abstract. Formulae for the magnetoresistance, developed earlier, are employed in the case of thin films. With the magnetic field perpendicular to the film, the energies for both the motions, along and perpendicular to the magnetic field are quantized. This, in conjunction with Fermi-Dirac statistics at low temperature leads to oscillatory mean energy, with the magnetic field for the perpendicular motion. As a result the formulae, in question, under combinations of values for the carrier mobility and a relevant collective parameter magnetoresistance oscillations become manifest in terms of the magnetic field. Evaluations of the magnetoresistance are presented for films with thickness of 1 μm and different mobilities in combination with choices of relevant collective parameters. The collective parameter determines whether the magnetoresistance proceeds initially, for small fields with positive or negative values, while its combination with the mobility value the size of the oscillations.

1. INTRODUCTION

In a previous paper [1] we developed formulae for the magnetoresistance (*MR*) exhibited by a material in the form of a rectangular parallelepiped (length *L*, width *l*, and thickness *d*), which is subjected to a constant normal magnetic field, *B*, along its thickness. The *MR* formulae refer to the experimental situations whereby a constant bias is applied along the length of the specimen or a constant current flows through along the length direction. The formulae for the *MR* are given for both the high and low temperature regime. Relevant endogenous parameters, in addition to the externally controlled parameters, specifying the experimental conditions, i.e. magnetic field, current or bias and temperature, enter the formulae. Furthermore, certain geometric parameters of the sample appear in the formulae. Details concerning the various parameters, in question, entering our formulae for the *MR* will be seen, subsequently, in the structure of these formulae, stated in the next section.

At this point we should like to point out that there exists extensive experimental literature concerning oscillatory *MR*, e.g. [2-6], to mention a few. However, the set of parameters given do not suffice for a complete feeding of our formulae, thus detailed comparison with experiment is somewhat hard to deal with. To be more specific, according to our formulae, stated in Section 2, the collective parameter f_0 is crucially controlled by the value of the current flowing through the sample and correspondingly ξ_0 by the applied bias. In general, one can hardly find data concerning the current or bias in expositions dealing with *MR* oscillations. Depending on the values of the collective parameters the

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MR starts with positive or negative values for small *B*. As *B* increases there may be a change in sign in the *MR*. Although, in certain experimental expositions one is faced with a situation whereby the *MR* starts with negative values and with increasing *B* enters the positive regime, and vice versa, lack of relevant data prevents quantitative comparison. Under the circumstances, we have opted to provide evaluations exhibiting *MR* oscillations, as well as the phenomenon of switch from negative to positive *MR* without regard to particular experimental data.

In this work we present cases of oscillatory *MR*, pointing at the same time the conditions under which the phenomenon occurs. In Section 2 we cite the *MR* formulae for the cases of constant bias and constant current together with relevant parameters entering the respective formulae. Section 3 provides a scheme for obtaining the mean energy for the motion perpendicular to the magnetic field as a function of *B*. Finally, Section 4 deals with evaluations of the *MR* for thin films having thickness 1 μm, for cases of low temperature and different combinations of carrier mobility and appropriate collective parameter entering the *MR* formulae.

2. MAGNETORESISTANCE FORMULAE

We cite below two formulae for the MR, developed earlier [1], one appropriate for the experimental condition under constant bias, and another corresponding to the case under constant current. The MR under constant bias takes the form

$$MR = \frac{\xi\eta}{\sinh(\xi\eta)}(1+\eta^2) - 1, \tag{1}$$

where the quantities ξ and η in Eq. (1) are dimensionless and are given by

$$\xi = \frac{qV}{2\varepsilon_0 < H_\perp >}, \qquad \eta = \frac{\mu B}{c}, \tag{2}$$

where q is the carrier's charge, ε_0, the material's dielectric constant, $<H_\perp>$ stands for the mean carrier kinetic energy. In the system of units employed the speed of light, c, comes into play. It should be noted that the spin energy, when negative, removes from the kinetic energy approximately an amount of energy $\hbar\omega/2$, while when positive adds in the same way $\hbar\omega/2$, where ω stands for the cyclotron frequency, $\omega=qB/m^*c$, and m^* denotes the carrier effective mass.

The dimensionless parameters ξ, η are connected via the magnetic field through the dependence of $<H_\perp>$ on *B*. The temperature dependence enters the *MR* through the dependence of $<H_\perp>$ and μ on the temperature.

Considering the alternative experimental set up whereby the current is kept fixed at a given value, say, i_0 the *MR* is expressed via

$$MR = \frac{\ln\left\{f(1+\eta^2)\eta + \sqrt{1+\left[(1+\eta^2)\eta\right]^2}\right\}}{f\eta} - 1, \tag{3}$$

where

$$f = \frac{qi_0\rho_s}{2\varepsilon_0 d < H_\perp >}. \tag{4}$$

ρ_s in (4) stands for the resistivity of the sample.

The parameter *f* for the case of fixed current corresponds to the parameter ξ in the case of fixed bias. For a given temperature the parameters ξ and *f* for a given sample can be fixed to a given value by appropriate choice of the value of the applied bias, *V*, for ξ, and correspondingly the current, i_0, for *f*. As is seen from the corresponding *MR* formulae the two pairs of parameters (ξ,η) or (*f*,η) suffice for describing the *MR* behaviour for a given range of *B*.

As pointed out earlier, the collective parameters ξ and η or *f* and η are dependent on *B*, particularly in the low temperature regime. It would, however, be desirable to make use of pairs of parameters whose members are disengaged from each other. This is simply done [1] utilising the limit values of ξ and *f*, as *B* tends to zero. The limits in question read as follows

$$\xi_0 = \frac{qV}{2\varepsilon_0 L E_0}, \qquad f_0 = \frac{q\rho_s i_0}{2\varepsilon_0 d E_0}, \tag{5}$$

where $E_0=<H_\perp(B=0)>$.

With the aid of the parameters ξ_0, f_0, which are free of *B*, the parameters ξ and *f* take the form

$$\xi = \xi_0 \frac{E_0}{< H_\perp >}, \qquad f = f_0 \frac{E_0}{< H_\perp >}. \tag{6}$$

For a given temperature, the parameters ξ_0 and f_0, employed, depending on the experimental condition, can be fixed to a given value by appropriate choice of the applied bias, V, in the case of ξ_0 and correspondingly by the current, i_0, for f_0. As pointed out earlier the two pairs of parameters (ξ,η) or (f,η) suffice for describing the *MR* behaviour. As pointed out previously [1] $\sqrt{6}$ constitutes a critical value for both collective parameters ξ_0 or f_0 in the sense that if e.g. $f_0<\sqrt{6}$ then the corresponding *MR* starts with positive values, while if $f_0>\sqrt{6}$, the corresponding *MR* begins with negative values. The same remarks apply for the parameter ξ_0.

In what follows we shall provide a scheme for computing the mean carrier energy for the motion perpendicular to the magnetic field. $<H_\perp>$, needed for obtaining the *MR*.

3. MEAN ENERGY FOR THE MOTION PERPENDICULAR TO THE MAGNETIC FIELD

In this section we shall cite a procedure leading to the mean energy for the motion perpendicular to the magnetic field, $<H_\perp>$, needed for the *MR* evaluation. To this extent we require the total energy spectrum under the influence of the magnetic field

$$\varepsilon_{nsj} = \varepsilon_j + \varepsilon_{ns}\,, \qquad (7)$$

where ε_i refers to the spectrum for the motion parallel to the magnetic field, and is given by

$$\varepsilon_j = \left(\frac{j\pi\hbar}{d}\right)^2 / 2m^* \qquad (j = 1,2,\ldots), \qquad (8)$$

while the spectrum for the motion perpendicular to the magnetic field reads as

$$\varepsilon_{ns} = \left(n+\frac{1}{2}\right)\hbar\omega + sg\frac{\hbar\omega}{4} \qquad (n = 0,1,2,\ldots),\ (s = \pm 1) \qquad (9)$$

g being the Landé factor. Furthermore, the energy levels ε_{ns} are highly degenerate with degeneracy $G_\perp = m^*\omega Ll/2\pi\hbar$.

Following the rules of Fermi-Dirac statistics, the probability of finding a carrier at the energy level ε_{nsj} is expressed as

$$P_{nsj} = \frac{m^*\omega}{2\pi n_0 d\hbar}\frac{1}{\exp\left[\beta\left(\varepsilon_{nsj}-\zeta\right)\right]+1}, \qquad (10)$$

where $\beta=1/kT$, ζ the chemical potential and n_0 the carrier number density. The required chemical potential for a given B and temperature T is obtained, as per usual, by solving the equation

$$\sum_{n=0}^{\infty}\sum_{s=\mp 1}\sum_{j=1}^{\infty} P_{nsj} = 1. \qquad (11)$$

Having at hand P_{nsj}, we can find the required average energy for the motion perpendicular to the magnetic field as

$$< H_\perp > = \sum_{n=0}^{\infty}\sum_{s=\mp 1}\sum_{j=1}^{\infty} P_{nsj}\varepsilon_{ns}\,. \qquad (12)$$

The numerical evaluation of $<H_\perp>$ in conjunction with the values of the collective parameters (ξ_0,η) or (f_0,η) can provide for a given B the *MR* under constant current or constant bias respectively.

In the next section we shall make use of the machinery cited, so far, for obtaining oscillatory *MR* as a function of B, pointing out circumstances leading to the effect. We shall restrict the discussion to thin conducting films.

4. MAGNETORESISTANCE OSCILLATIONS

In this section we shall proceed considering thin films made of conducting material for which we shall mainly obtain the *MR* in the case of constant current for different values of the collective parameter f_0, given in Eq. (5) and the mobility, μ. As pointed out earlier the parameter f_0 for a given sample can be varied by changing the current flowing through the sample. Change in the current does not affect the mean energy associated with the motion perpendicular to the magnetic field. Of course, for a given current one can be led to a given value for f_0 selecting a sample with appropriate thickness, d, a parameter which affects the mean energy, $<H_\perp>$, whose oscillatory behaviour causes the *MR* oscillations. Similar remarks apply in the case whereby the *MR* is obtained under constant bias, in which the corresponding collective parameter is ξ_0.

Restricting our attention to the case of *MR* on condition of constant current after replacing the resistivity in terms of mobility, via $\rho_s=1/n_0q\mu$, the expression for f_0 takes the form $f_0=i_0/2\varepsilon_0 n_0\mu E_0 d$. This shows that if we have a sample for which we obtain the *MR* employing a fixed current, i_0, and then proceed annealing it there will occur increase in the mobility, and certain other of its endogenous

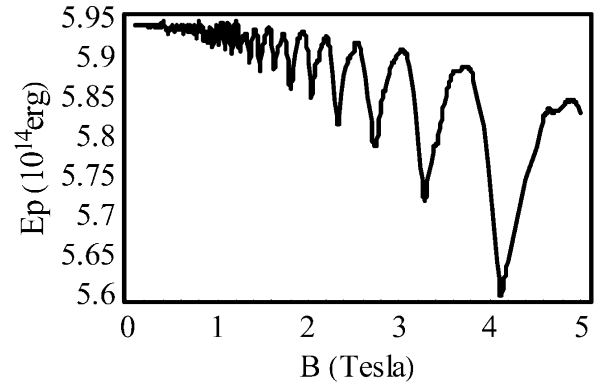

Fig. 1. Oscillatory mean energy for the motion perpendicular to the magnetic field in the case of a thin film, thickness d=1μm, at T=2K, with carrier density n_0=3.6·10^{17}/cm^3, effective mass m^*=0.02 m_e, and Landé factor g=2.

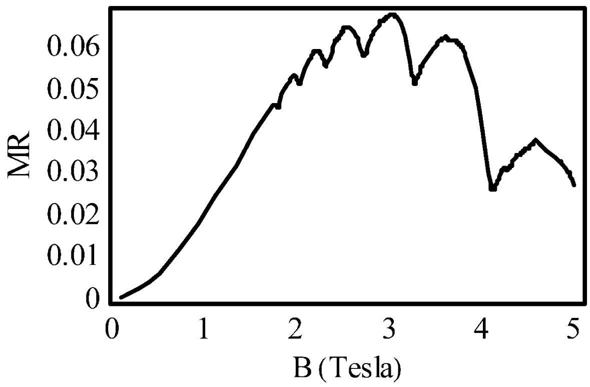

Fig. 2. Shows MR oscillations as a function of the applied magnetic field for a thin conducting film with data as per Fig. 1 and, furthermore, mobility μ=2.5·10^3 cm^2/V·s while the collective parameter $f_0=\sqrt{6}$ -0.5≈1.94949, smaller than the critical value $\sqrt{6}$, thus the MR starts with positive values.

parameters may undergo change. Upon performing the evaluation using the same value for the current, the value of the collective parameter f_0 in this instance will be altered. Incidentally, in the case under constant bias the corresponding parameter ξ_0 will not be affected, as far as changes in the mobility are concerned. One reason for the above discussion lies in the fact that several experimental expositions leave aside parameters that would enable comparison with experiment. Owing to this difficulty our evaluations are just based on a choice of parameters of our own, but lying within the existing range of experimental work. In what follows we shall confine ourselves to presenting applications of formula (3) for the *MR* on condition of constant current.

We consider the case of a thin film with thickness d=1 μm, and carriers having effective m^*=0.02 m_e, while the carrier density is taken n_0=3.6·10^{17}/cm^3. Clearly, d and m^* suffice for fixing the energy spectrum, Eq. (1), for the motion parallel to the magnetic field. Now, for a given value of the collective parameter f_0 we, further, require to specify the mobility, μ, for completing the evaluation of the *MR* as a function of *B*.

It should be noted, however, that the collective parameter f_0 depends on the mobility through its connection with the resistivity, ρ_s. Once fixing f_0 for a given μ, if one wishes to keep the same current for another evaluation with a different mobility, say, μ_1, f_0 has to be replaced by $f_0\mu/\mu_1$.

There follow results concerning the mean kinetic energy $\langle H_\perp \rangle$ for the motion perpendicular to the magnetic field, as well as for related cases of *MR*.

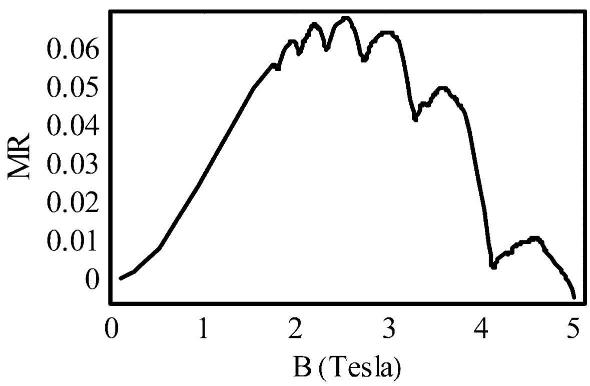

Fig. 3. Shows the pattern of MR oscillations obtained from a thin film characterised by the parameters stated in Fig. 2, apart from the mobility, which now is taken μ=2.5·10^3 cm^2/V·s. Care has been taken so that the value f_0 incorporates the change in mobility, but retains the same current i_0 as in Fig. 2. The appropriate value, in question, is f_0≈1.62457, still such that the starts with positive values.

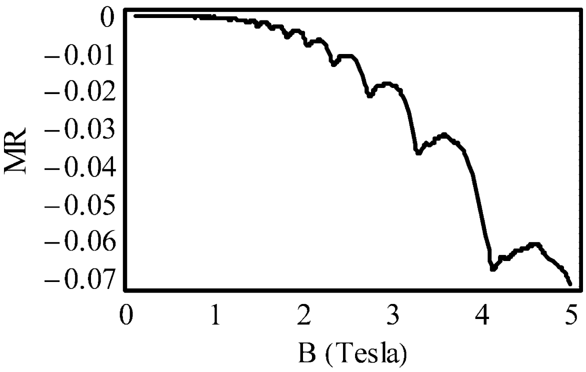

Fig. 4. Shows negative oscillatory MR for a sample with data as per Fig. 1 and same current as that determining f_0 in Fig. 2, but with sufficiently lower mobility, namely, $\mu=2\cdot10^3$ cm^2/V·s so as to increase f_0 for this case above the critical value $\sqrt{6}$, which now becomes $f_0\approx2.43486$.

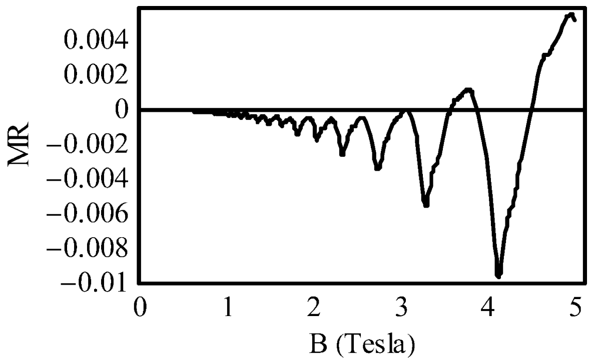

Fig. 5. Shows start of *MR* with negative values and the possibility of attaining positive values with increasing *B*. Data as per Fig. 1 in addition to which $f_0=\sqrt{6}+0.03\approx2.47949$ and $\mu=1.1\cdot10^3$ cm^2/V·s.

The data used for obtaining the results in question are listed in the corresponding figure captions, see Figs. 1 and 2.

The change in mobility in Fig. 3 was such that, while retaining the same value for the current, the new value for f_0 was still below the critical value and so the *MR*, thus obtained, started with positive values. If, however, the mobility used were sufficiently low the value for f_0 would become larger than the critical value and the *MR* would fall into the negative regime. This situation is shown in Fig. 4.

As pointed out in the introduction, the possibility whereby the *MR* starts with negative values and can attain positive values with increasing *B* exists [6]. This is shown in Fig. 5 for the case of constant current through the sample. Similar result, however, applies in the case on condition of constant bias.

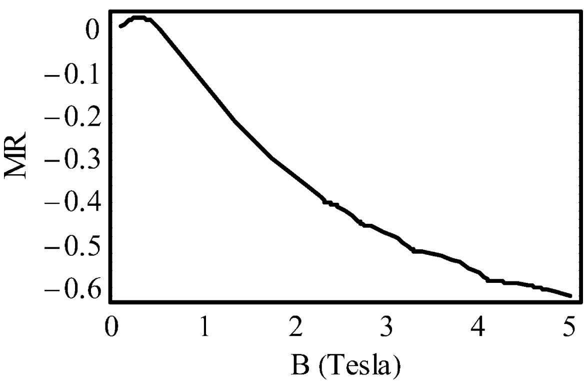

Fig. 6. Shows almost disappearance of the MR oscillations in the case of constant current. Data as per Fig. 1 in addition to which $f_0=\sqrt{6}-0.3\approx2.14949$ and $\mu=2\cdot10^4$ cm^2/V·s.

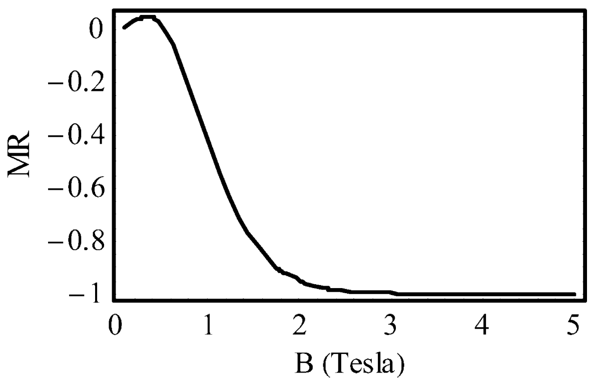

Fig. 7. Shows disappearance of the MR oscillations in the case of constant bias. Data as per Fig. 1 in addition to which $\xi_0=\sqrt{6}-0.3\approx2.14949$ and $\mu=2\cdot10^4$ cm^2/V·s.

The *MR* oscillations result from the mean energy, $<H_{\perp}>$, oscillations, which in turn derive from the energy quantization along the magnetic field together with the quantization of the corresponding energy associated with the motion perpendicular to the magnetic field in conjunction with the associated Fermi-Dirac statistics. As pointed out earlier, the *MR* oscillations become manifest with appropriate choice of the values for f_0 and μ. It should, however, be noted that in the *MR* oscillations appearance predominant role is played by the mobility. Rise in the mobility value results in diminished oscillations. Figs. 6 and 7 serve to show a case of disappearance of oscillations for a sample for which the data in Fig. 1 apply.

Finally, we proceed to show the result presented in Fig. 6 in the case whereby the sample is subjected to a constant bias.

It should be noted that in the cases exhibited in Figs. 6 and 7, the *MR* starts with positive values, on account of the value for both collective parameters f_0 and ξ_0 is smaller than the critical value $\sqrt{6}$. In the case of constant bias, Fig. 7, the sample's resistance approaches zero with increasing *B*.

REFERENCES

[1] G. J. Papadopoulos // *J. Non-Cryst. Solids* **352** (2006) 4206.

[2] F. Y. Yang, Kai Liu, Kimin Hong, D. H. Reich, P. C. Pearson and C. L. Chien // *Science* **284** (1999) 1335.

[3] S. V. Krachenko, A. A. Shashkin, D. A. Bloore and T. M. Klapijk // *Solid State Commun.* **116** (2000) 495.

[4] G. Stöger, G. Brunthaler, G. Bauer, K. Ismail, B. S. Meyerson, J. Lutz and F. Kuchat // *Semicond. Sci. Technol.* **9** (1994) 765.

[5] Ken-ischi Suga, Coichi Kindo, Shuichi Ishida, Atsushi Okamoto and Ichiro Shibsaki // *Physica B: Condens. Matter* **346-347** (2004) 470.

[6] Shuichi Ishida, Keiki Takeda, Atsushi Okamoto and Ichiro Shibasaki // *Physica E* **18** (2003) 159.