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ENDOCHRONIC PRESENTATION OF THE THEORY OF NONLINEAR CREEP OF RABOTNOV

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Abstract. Updating the equation of nonlinear Rabotnov's theory of creep with the help of endochronic concept is made. Analytical expression for determining functions of this transformation in case mucleus creep of Rabotnov and its nonlinear function as a power expression is received. The complex form of scale for generalized time is found. Additional endochronic generalization of Rabotnov's theory is offered that allows describe accelerated and slowed–down restoration media (hardening and softening).

1. INTRODUCTION

There exits a set of approaches for description of deformation of rheologically complex media. To the basically, most fundamental theories of nonlinear creep of hereditary (integrated) type, it is necessary to relate five of them:

- multiple-integrated concept of Volterra-Freshe [1];
- monointegrated, with a nonlinear kernel, approach of Bolzmann-Perso [2];
- theory by Rabotnov [1] which is based on the connection between deformation and stress with a linear integral equation;
- Moskvitin's concept [3], which is the development of Rabotnov's theory and is based on a similar connection between deformation and stress; and
- endochronic way with the use of generalized own internal time in monointegrated representation [4-6].

The abundance of approaches can be explained, on the one hand, by incompleteness of solutions for the problem describing nonlinear behavior of media, and, on the other hand, by nontrivial and various behavior of media which leads to the necessity of searching other different ways.

Each approach possesses the merits and demerits as well. Opportunities of approaches are not investigated up to the end.

The endochronic concept developed now allows to unify mechanical properties of media at various physical-chemical-mechanical influences: temperatures, radiation, ageing, mechanical nonlinearity etc. It gives a way of updating other theories by means of obtaining parameters of endochronic approach for them, in particular, by obtaining a scale of own time. It gives an effective way to expand the opportunities of description. This way makes it possible to carry out the unified, standartized comparison of various theories on parameters of endochronic concepts and, actually, to generalize them.

In works [6-9] standardized endochronic updatings of the equation of tenacity-elasticity by Bolzmann-Perso have been received, the equations of longevity of Zhurkov with overbarrier and

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underbarrier (tunnel) transitions, technical theories of creep of Bailey and Norton's degree functions, *etc*. In the given work endochronic updating of the equation of nonlinear creep of Rabotnov is presented.

2. RABOTNOV'S THEORY

The equation of nonlinear creep of Rabotnov looks like

$$\varphi\left[\varepsilon(t)\right] = \bar{f}\sigma = \frac{\partial(t)}{E} + \int_{0^-}^{t} f(t-\tau)\dot{\sigma}(\tau)\,\mathrm{d}\tau$$

or

$$\varphi\left[\varepsilon(t)\right] = \bar{P}\sigma = \int_{0^-}^{t} P(t-\tau)\dot{\sigma}(\tau)\,\mathrm{d}\tau,$$

$$\text{where}\quad P(t) = \frac{1}{E} + \int_{0}^{t} f(\lambda)\,\mathrm{d}\lambda. \qquad (1)$$

Then we have

$$\varepsilon(t) = \varphi^{-1}\int_{0^-}^{t} P(t-\tau)\dot{\sigma}(\tau)\,\mathrm{d}\tau = \varphi^{-1}(v). \qquad (1')$$

Here ε is the deformation; φ its nonlinear function; $\bar{f}$ and $\bar{P}$ are the nucleus and the operator of creep; $f(t)$ and $P(t)$ are linear functions of a nucleus and creep; t is the time; σ is the stress; E is the module of elasticty; φ^{-1} is the inverse function for function φ.

At constant stress $\sigma(t) = H(t)\sigma^0$, $\sigma^0 = \mathrm{const}$ (H is Heaviside function which is equal to zero at $t < 0$, and equal to unity if $t \geq 0$)

$$\varphi(\varepsilon) = P(t)\sigma \quad\text{and}\quad \varepsilon(t) = \varphi^{-1}\left[P(t)\sigma\right]. \qquad (2)$$

In case of step loading such as

$$\sigma(t) = \left[H(t) - H(t-t_1)\right]\sigma^0,$$

$$\varphi(\varepsilon) = \left[P(t) - P(t-t_1)\right]\sigma^0, \quad\text{or}$$

$$\varepsilon(t) = \varphi^{-1}\left\{\left[P(t) - P(t-t_1)\right]\sigma^0\right\}. \qquad (3)$$

According to Rabotnov's theory in elastic region, i.e. at $t = 0$, nonlinearity of type φ is kept, namely, for example, in case of $t = 0$

$$\varphi\left[\varepsilon(0)\right] = P(O)\sigma(0) \rightarrow \varepsilon(0) =$$

$$\varphi^{-1}\left[P(O)\sigma(0)\right] = \varphi^{-1}\left[\frac{\sigma(0)}{E}\right].$$

The linear area of elasticity and nonelasticity is absent. The law of nonlinearity is identical for elastic and nonelastic parts. Consequently isochronic curves $\sigma - \varepsilon$ are similar.

Deformation of some materials is well described by Rabotnov equation. On the basis of comparison of experimental data and calculations at step loadings of various polymers [10], it was shown, that they are media with the accelerated response to influence. In [1] examples of deformation description for various metals are given at increased temperatures, as well as for graphite and fibreglass. As the Rabotnov equation for creep is received by the author [1] from multiple-integrated representation of Volterra-Freshe, it, as is noted in [11], possesses the property of accelerated response, peculiar to multiple- integrated theory.

Other materials can show linear elasticity and, at long influence by a low stress, linear creep. The third media show different nonlinear properties in elastic and nonelastic areas and have a region of linear creep.

Let's consider a variant of Rabotnov's theory at following assumptions:

1) We assume that at small stresses $\sigma \leq \sigma_l$ where σ_l is the limit of linear creep, linear creep takes place, when at a constant stress

$$\varepsilon(t) = P(t)\sigma = \frac{\sigma}{E} + P_c(t)\sigma.$$

Here P_c is a creep component.

2) At $\sigma > \sigma_l$ expression (1) is valid, at $\sigma = \mathrm{const}$ – expression (2).

Thus, we accept the Rabotnov function of the following type

$$\eta^{-1}\left[\varepsilon(t)\right] = \left[H(\sigma) - H(\sigma-\sigma_l)\right]\cdot$$

$$\left[\frac{\sigma(t)}{E} + \int_{0^+}^{t} P_c(t-\tau)\sigma(\tau)\,\mathrm{d}\tau\right] + \qquad (4)$$

$$H(\sigma-\sigma_l)\varphi^{-1}(v),$$

Here linear and nonlinear parts are joined by means of Heaviside functions. This specified kind of Rabotnov's function corresponds to the form of isochronic curves of creep in materials tested in [1]. Up to a certain value of stress, the creep is linear, so isochronic lines are straight.

3. ENDOCHRONIC THEORY

On the basis of the endochronic concept, nonlinear creep is described by the equations

$$\varepsilon(t)=\overline{D}^{\xi}\sigma=\int_{0^-}^{t}D\left(\xi^{\sigma}-\zeta\right)\dot{\sigma}(\tau)\,\mathrm{d}\tau, \quad (5)$$

$$\xi^{\sigma}-\zeta=\int_{\tau^+}^{\tau^-}G^{\sigma}\left[t-\rho,\sigma(\rho)\right]\mathrm{d}\rho=$$

$$\int_{\tau^+}^{\tau^-}\left\{g^{\sigma}\left[t-\rho,\sigma(\rho)\right]-\frac{\partial g^{\sigma}\left[t-\rho,\sigma(\rho)\right]}{\partial\rho}\right\}\mathrm{d}\rho. \quad (6)$$

Here $\overline{D}^{\xi}$ is the creep operator, D is its function; ξ^{σ} is the internal time, and G^{σ}, g^{σ} are its scales (measures); at that $g^{\sigma}(t,\sigma) = 1$ if $t = 0$ or $\sigma < \sigma_l$. In the case σ = const,

$$\xi^{\sigma}-\zeta=g^{\sigma}(t-\tau,\sigma)(t-\tau),$$

since the expression under the sign of integral becomes a total differential.

For the description of the accelerated or slowed down response, we shall use instead of the scale-function $G^{\sigma}(t,\sigma)$ the scale-functional $\overline{g}^{\sigma}_{\cup}(\dot{\sigma})$, representing multiplication of scale $G^{\sigma}(t,\sigma)$ and scale-functional $\overline{g}^{\sigma}_{\cup}(\dot{\sigma})$, thus correcting G^{σ}:

$$\overline{G}^{\sigma}(t,\sigma,\dot{\sigma})=G^{\sigma}(t,\sigma)\cdot\overline{g}^{\sigma}_{\cup}(\dot{\sigma}),$$

$$\overline{g}^{\sigma}_{\cup}(\dot{\sigma})=1+\int_{0}^{\rho}q^{\sigma}(\lambda)\dot{\sigma}(\lambda)\,\mathrm{d}\lambda;\;\; q^{\sigma}(0)=0. \quad (7)$$

In regions where $\dot{\sigma} = 0$, we have $\overline{G}^{\sigma} = g^{\sigma}$. The function $q^{\sigma}(t)$ can depend on a sign of $\dot{\sigma}$, i.e. it looks like $q^{\sigma} = q^{\sigma}[t,\mathrm{sing}(\dot{\sigma})]$. For the description of accelerated response, the size of the scale-functional $\overline{g}^{\sigma}_{\cup}(\dot{\sigma})$ should be more than unity, and for slowed down less than unity. In [6] the variant of the media with nonlinear function q^{σ}, which does not depend on time is considered.

At $\sigma(t) = H(t)\sigma^0$ we obtain $\varepsilon(t) = D(\xi^{\sigma})\sigma^0$, and then

$$\xi^{\sigma}=g^{\sigma}(t,\sigma)\cdot t. \quad (8)$$

In the case of step loading

$$\sigma(t)=\left[H(t)-H(t-t_1)\right]\sigma^0$$

$$\varepsilon(t)=\left\{D\left[g^{\sigma}(t,\sigma^0)t\right]-D\left[g^{\sigma}(t-t_i,\sigma^0)\cdot\left(1+q^{\sigma}(t_i)\sigma^0\right)(t-t_i)\right]\sigma^0\right\}. \quad (9)$$

4. ENDOCHRONIC UPDATING OF RABOTNOV EQUATION

Equating Eqs. (8) and (2) in case of $t > 0$ at a constant stress, it is possible to find the scale g^{σ} in a nonlinear area. Then, equating (9) and (3) at step loading, the function q^{σ}. In the nonlinear area at a constant stress, we shall have the equation

$$D\left(\xi^{\sigma}\right)=P\left(\xi^{\sigma}\right)=P\left(g^{\sigma}t\right)=\varphi^{-1}\left[P(t)\right].$$

It allows to determine scale g^{σ} if the functions P and φ^{-1} are known. In the case of step loading, the equation for finding the function q^{σ} has the form

$$\left\{P\left[g^{\sigma}(t,\sigma^0)t\right]-P\left[g^{\sigma}(t-t_i,\sigma^0)\left(1+q^{\sigma}(t_i)\sigma^0\right)\cdot(t-t_i)\right]\sigma^0\right\}=\varphi^{-1}\left\{\left[P(t)-P(t-t_i)\right]\sigma^0\right\}.$$

For Rabotnov function of type (4), using power functions, traditionally applied to different materials, we have

$$\varphi^{-1}(v)=\mathrm{d}v^{m},\;\; P(t)=\frac{1}{E}+bt^{m},\;\; t\ge t_{0^+}, \quad (10)$$

$$D\left(\xi^{\sigma}\right)=P\left(\xi^{\sigma}\right)=\frac{1}{E}+b\left(\xi^{\sigma}\right)^{m}=$$

$$\frac{1}{E}+b\left(g^{\sigma}t\right)^{m},\;\; g^{\sigma}=1 \;\text{ at }\; \sigma\le\sigma_1 \quad (11)$$

and $q^{\sigma}=ct^{s}$.

The required determining functions of endochronic variant for Rabotnov equation are equal, the scale is

$$g^{\sigma}(t,\sigma)=b^{-1/m}t^{-1}\left[a\left(\frac{1}{E}+bt^{m}\right)^{n}\cdot\sigma^{n-1}-\frac{1}{E}\right]^{1/m}, \quad (12)$$

but the function $q^{\sigma}(t)$ (coefficients c and s), corresponding to the scale-functional $\overline{g}^{\sigma}_{\cup}(\dot{\sigma})$ is found from the solution of the system of equations (i = 1,2)

$$\left\{P\left[g^{\sigma}(t,\sigma^0)t\right]-P\left[g^{\sigma}(t-t_i,\sigma^0)\left(1+q^{\sigma}(t_i)\sigma^0\right)\cdot(t-t_i)\right]\sigma^0\right\}=\varphi^{-1}\left\{\left[P(t)-P(t-t_i)\right]\sigma^0\right\}.$$

which can be received at use of equations (3) and (9) for step loading.

As for chosen functions, it is necessary to note that in monography by Rabotnov [1] the type of power function $P(t)$ is given; and it is shown that for some materials (metals) this function, in essence Bailey's function [12], corresponds to Abel's kernel with the index of degree "-0.7". Function $\varphi^{-1}(v)$ is similar to Norton's nonlinear power function [12]. In the directory [13] the values of power for the Norton function are given for various metals.

Thus, the equation of nonlinear creep by Rabotnov (1) with power functions (10), (11) is transformed into the endochronic form (5) – (7). We obtained the determining parameters for this updating. The scale $q^{\sigma}(t,\sigma)$ is complex, and depends on two parameters, contrary to "simple" conformity of the modified technical theory (e.g. a creep function with separable variables in theory by Bolzman-Perso) [8]. For the description of nonmonotonic loading, the scale-functional $\bar{g}^{\sigma}_{\smile}(\dot{\sigma})$ is also introduced and determined, giving an opportunity to describe accelerated and slowed–down response. Updating can be carried out and for other type of initial functions, as well as when the functions are given in a tabulated form, according to experiments. The received equation of creep has quasy-linear type in a scale of internal time. Nonlinearity is taken into account due to internal time. Generalized through the endochronic concept, Rabotnov's theory allows to carry out considerably wider description of nonlinear properties of materials due to application of internal time; and one of its scales in the form of integrated functionals, whereas in the initial equation there is only one nonlinear function.

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