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DISCRETE CRACK PROPAGATION AND COMPOSITES DELAMINATION

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Abstract. In the present work crack growth (crack tip movement) is interpreted as indentation of the influence zone into undisturbed material under the action of an end load. At the investigation, we should differentiate between two stages of fracture. The first one is the stage of the elementary cell fracture with characteristic time t_{char}, and the second one is the fracture propagation between elementary cells with characteristic time $t_* \neq t_{char}$. The delamination follows the loss of stability of the influence zone. Stability of the influence zone by indentation is investigated. From mathematical reason we approximate the shape of the influence zone not as a wedge, but as a thin equivalent plate. We can investigate the loss of stability of a rod which clutched between elastic thick foundations under the effect of end load $P(x,t)$ and additional "noise of fracture"'. According to definition, the additional perturbation (fracture of an elementary cell) is a shock one. The principal difference of the considered processes is in the fact that the shock acts not along the beam axis, and the system loses its stability not as a result of a shock load but as a result of a quasi-static load under conditions of parametrical perturbation. The non-homogeneity is represented by additional terms (change of fracture noise parameters).

The full analytical solution of the problem is received in the form of composition of exponents and generalized hypergeometric functions. The possibility of resonance modes is shown.

1. INTRODUCTION

Regrettably the classical fracture theory is very far from correct prediction of the real crack trajectory in inhomogeneous media. This theory can predict only the main regularity for brittle fracture of ideal material. By the standard way, the theory of real media fracture is built by adding some terms (representing plasticity, defect structure and other imperfection) to the linear theory. We loose the real discrete structure of the media in this way and operate with continuum description. This approximation is possible for item in specific size range but for modern technology (nanomaterials, for example) we need quantum theory of fracture [1]. It is interesting that generally fracture process in real media is realized as a time-quantized process, too: storage energy of deformation in an elementary "fracture cell"' - breaking the elementary cell - next accumulation cycle. The sketch of possible mesoscopic description was given by Novozhilov [2], Petrov, and Morozov [3]. Re-formulation of this quantized approximation was made in [4] without references to primary sources. In any case the rigorous quantum theory of fracture is far from exhaustiveness now and the mesoscopic theory looks more realistic. The main problem consists in pass by physically correct way to classical fracture model (for example, Barenblatt-Dugdale), from quantized representation. In other words, we can explain why solitary act of atomic bound breaking causes macroscopic fracture. In present article, we took into account the processes at different structural levels

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and investigated the influence of the discrete interblocks bonds breaking on the crack development.

2. METHODOLOGY

It is common knowledge that before the tip of a propagating crack exists the area of material named “influence or pre-fractured”’ zone with properties different from undisturbed material (Fig. 1). This area is moving with the crack and the shape of influence zone depends on selected model of fracture, kind of a crack (quasistatic or dynamic), and material properties. This area movement is equal to crack growth in a fixed reference frame. This movement can be interpreted as indentation of the the third body with the shape of influence zone into undisturbed material under the action of a facial load. When we investigate the sandwich composite delamination, the third body is indented along the plates boundary. If we investigate the (im-)perfect solid, the third body is indented in (in-)homogeneous continua. Since we are constructing now not exact model but the first sketch and for the sake of simplicity we approximate the shape of the influence zone not as a wedge, but as a thin equivalent beam. The edge effects are not present in the system because the plate is unlimited along *Z* axis, and the full problem reduces to a 2-D problem of beam deformation between two Winkler (elastic) foundations. In this way we can investigate the loss of stability of the beam with length *l*, thickness *h* and width *b* clutched between elastic thick foundations under the effect of facial load $P(x,t)$ and additional stochastic “noise of fracture” (Fig. 2).

In the exact theory, when building a microscopic description, this noise of fracture should be connected with the microscopic parameters of the fracture. If we believe that the structure of media is regular, the noise of fracture is correlated and vice versa for the stochastic structures [1]. It is generally clear that the transverse perturbations are caused by the fracture of inter-atomics bonds but for us the nature of the process is not important. We assume only that the frequency of the given perturbation is the generation frequency of transverse oscillations caused by breaking of a fracture cell, $\Omega = 1/t_{char}$. Transversality of the oscillation follows from the fracture local symmetry principle [1].

At the investigation according to microscopic understanding of real fracture, we should differentiate between two stages of fracture. The first one is the stage of the elementary cell fracture with characteristic time t_{char}, and the second one is the fracture propagation between elementary cells with characteristic time $t_* \neq t_{char}$. According to definition [5], the additional perturbation is a shock one. The principal difference of the considered processes from well known models of beam bending is in the fact that the shock acts not along the beam axis and the system loses its stability not as a result of a shock load, but as a result of a quasi-static load in the condition of parametrical perturbation. The non-homogeneity of the material in this model is represented by additional terms (change of fracture noise parameters).

In this case we should observe energy transfer of the energy of longitudinal compression into the energy of lateral oscillations [5,6]. The proposed model of parametric excitation of a previously non-bent (perfect) composite is essentially different from the well-known model of fibering of a non-perfect (previously supplied with a defect) composite [7]. In our model, there exists no limitation related to the minimal size of an initial imperfection, because the energy transfer is made in the resonance mode.

3. PROBLEM STATEMENT

The system of equations accounting shift, rotation inertia, and the influence of longitudinal oscillations on the lateral rod motion has the form [5,8]:

$$kFG(w_x - \psi)_x + EF\left[u_x\left(w_x - \psi^0_x\right)\right]_x + p(x,t) = \rho F w_u; \tag{1}$$

$$EI\psi_{xx} + kFG(w_x - \psi) = \rho I \psi_{tt}; \tag{2}$$

$$EFu_{xx} = \rho F u_{tt}. \tag{3}$$

where $u(x,t)$, $w(x,t)$ are longitudinal and lateral displacements, respectively, ψ is the angle of tangent to the curve of bend, x,t are the longitudinal coordinate and the time, E,G are elasticity and shear moduli, F,I are the area and the moment of inertia, k is the coefficient of the shape of section, ρ is the density of the material, $p(x,t)$ is the unit load which is locally orthogonal to the beam axis, and w^0 is the initial bend.

The loading is realised by load $-P$, essentially exceeding the Euler critical load, on the free undisturbed at t=0 end of the beam. Thus, the initial and boundary problem can be stated in the form of

$$w = 0;\ \ \psi_x = 0 by x = 0,\ l_0,\ t \le 0;\ \ w(x,0) = 0;$$
$$w_t(x,0) = 0;\ \ \psi(x,0) = 0;\ \ \psi_t(x,0) = 0;$$

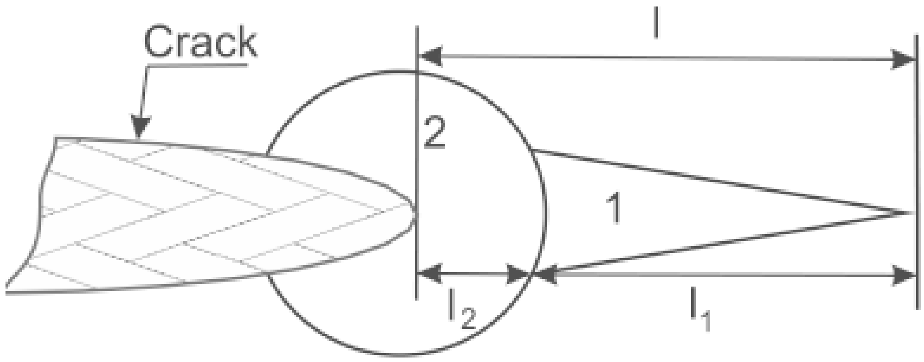

Fig. 1. Pre-fracture zone. 1 is the area of elastic deformations, 2 is the area of plastic deformations. Total length of influence zone *l* is the sum of elastic and plastic parts.

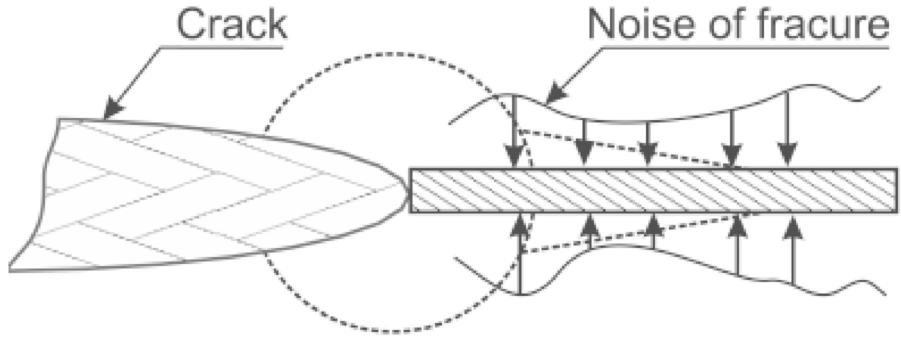

Fig. 2. Simplified model. Noise of fracture is distributed randomly in real media.

$EFu_z(0,t) = -P,\ u(l_0,t) = 0$ or $u_x(l_0,0) = 0;$
$u(x,0) = 0;\ u_t(x,0) = 0.$

The moving lateral load in the system, in each given point, is determined by a lateral wave, initiated on the free end of the beam. The physical source of this excitation is the periodical break of interatomic or interblock bonds. Since the frequency Ω of such oscillations is very high, despite the small excitation amplitude, high stress gradients are observed. In this case, the dynamical investigation (inertial part in Eq.(1)) looks justified. Since in real systems we always have excitation decay caused by dissipation, we assume that in the coordinate frame that is moving together with the crack tip, we can represent the load in the form of:

$$p(x,t) \equiv p(x) = A\exp(-\lambda x)\sin(\Omega x), \quad (4)$$

where A is the normalizing constant and λ is the logarithmic decrement of energy dissipation by the bend of an elastic-plastic beam.

In this statement of the problem, the system is considered as a 2-D system with slowly changing linear dimensions, in which oscillations are parametrically excited [6]. In this sort of system, resonance phenomena are possible. In case of intergrain boundary fracture or stripping, the excitation can localize near the interface, producing additional energy gradients and adding to the destruction caused by fracture [9].

3.1. Equation of the beam bending analysis

Set of dynamical equations (1-3) can be written in the linearized form in the usual way [10]. In the result, we obtain the equations of the bend for a beam on an elastic foundation in the form of [8]:

$$Jw,_{xxxx} + cw + Pw,_{xx} + \rho Fw,_{tt} = p(x,t) - Pw_1,_{xx}, \quad (5)$$

where TJ is the flexural beam rigidity, $T = EG/(E+G)$ is the reduced module (von Karman coefficient), E is the material Young modulus, G is the shear modulus, J is the moment of inertia of the cross-section, c is the coefficient of elastic foundation [5,8]

$$c = \frac{E_0}{1-\mu_0^2}\frac{b}{3},$$

F is the cross-section area, w_1 is the initial imperfection of the axes. Inasmuch as we investigate the forced oscillations under applier transversal force, further we assume $w_1 = 0$ for simplification.

Taking into account the restricted spread of the beam, the inertia moment of the cross section has the form

$$J = \frac{bh^3}{12\left(1-\mu_0^2\right)}.$$

Here μ is the Poisson ratio,

$$E_0 = \frac{E}{1-\mu^2}, \quad \mu_{i0} = \frac{\mu_i}{\mu_i - 1}$$

μ_i is the Poisson ratio of the foundation, indices i refer to the bottom and top plate, respectively, ρ is the density of the material.

If we investigate an exfoliation crack in laminated composites, the real influence zone includes the areas located in both materials, and the beam is two-layered. In the microscopic scale, the same mechanism is realized by the grain boundary cracking. Since we consider a beam clutched between two elastic foundations, Eq. (5) with account of (4) can be modified as follows:

$$\begin{aligned} &Jw,_{xxxx} + \left(c_1 + c_2\right)\exp\left(-\lambda x - Vt\right)\cdot \\ &\sin\left(\Omega x - Vt\right)w + Pw,_{xx} = \\ &\left(\rho_1 F_1 + \rho_2 F_2\right)w,_{tt} = 0, \end{aligned} \qquad (6)$$

where c_1, c_2 are the foundation coefficients of the bottom and the top elastic foundations, respectively, ρ_i,; F_i are the density and the cross-section area of the first and the second materials of the beam, respectively, V is the velocity of crack propagation (crack growth). In the quasi-static case $V = 0$. Generally, at derivation of (6), we assume that the bend is small [5]. In case of high gradients of beam deformation, we can take into account the nonlinear terms in the series of expansion of the bend [11], and (6) is only the first approximation.

3.2. Solution analysis

We shall seek the solution of Eq. (6) in the following form:

$$w(x,t) = a(x)\exp\left(-i\left(kx - \phi t\right)\right), \qquad (7)$$

where $a(x)$ is the amplitude, generally complex, k is the wave number, and ϕ is the frequency. Substituting (7) into (6) and separating the real and imaginary parts, we obtain the set of equations:

$$J\frac{\mathrm{d}^3}{\mathrm{d}x^3}c(x) - \left(\frac{1}{2}P - TJk^2\right)\frac{\mathrm{d}}{\mathrm{d}x}c(x) = 0, \qquad (8)$$

$$\begin{aligned} &J\frac{\mathrm{d}^4}{\mathrm{d}x^4}b(x) - Q\frac{\mathrm{d}^2}{\mathrm{d}x^2}b(x) + \\ &b(x)\left[R + \exp(Zx)\right] = 0. \end{aligned} \qquad (9)$$

where $a(x)b(x)$ are the imaginary and real parts of the amplitude, respectively,

$$Z = \left(-\lambda + \Omega - 2Vt\right), \quad Q = P - 6TJk^2,$$
$$R = -Pk^2 + TJk^4 - \left(\rho_1 F_1 + \rho_2 F_2\right)\phi^2.$$

Strictly speaking, we need an additional condition for dispersion relations but in the first approximation it is possible to study the medium without dispersion [6]. The formal solution of the first Eq. (8) has the form:

$$\begin{aligned} &c = C_1 + C_2\exp\left(\frac{\sqrt{-P + 2TJk^2}\,x}{\sqrt{2TJ}}\right) + \\ &C_3\exp\left(= \frac{\sqrt{-P + 2TJk^2}\,x}{\sqrt{2TJ}}\right), \end{aligned} \qquad (10)$$

where C_1, C_2, C_3 are integration constants defined from the initial conditions. The general behaviour of solution (10) depends on the sign under the root and we can obtain both periodical oscillations and the exponential increase. The formal solution of the second equation of the system can be found with the help of the Maple package in the form:

$$\begin{aligned} &b(x) = C_4 e^{-\frac{\delta x}{2Z}}{}_0F_3\left([\,],\{1\},-\frac{e^{(Zx)}}{TJZ^4}\right) + \\ &C_5 e^{-\frac{\delta x}{2Z}}{}_0F_3\left([\,],\{2\},-\frac{e^{(Zx)}}{TJZ^4}\right) + \\ &C_6 e^{-\frac{\delta x}{2Z}}{}_0F_3\left([\,],\{3\},-\frac{e^{(Zx)}}{TJZ^4}\right) + \\ &C_7 e^{-\frac{\delta x}{2Z}}{}_0F_3\left([\,],\{4\},-\frac{e^{(Zx)}}{TJZ^4}\right), \end{aligned} \qquad (11)$$

where C_4, C_5, C_6 are the integration constants, defined from the initial conditions, $d = -Q^2+4TJR$, $\beta_1 = Z^2Q$,

$$\beta_2 = \sqrt{-\alpha^4 d},\ \ \delta_1 = -2\beta_1 - 2\beta_2,\ \ \delta = \sqrt{\frac{\delta_1}{TJ}},\ \ \delta_2 = \sqrt{\frac{(4\beta_2+\delta_1)}{TJ}}, \tag{12}$$

and expression ${}_0F_3(0;\{i\};-z)$ is the generalized hypergeometric function of the power (0,3) (This function is also known as Barnes's extended hypergeometric function).

In general, the hypergeometric function is given by equation

$$F(a,b,c;z) = \sum_{z=0}^{\infty}\left(\frac{a(a+1)\ldots(a+s-1)b(b+1)\ldots(b+s-1)}{c(c+1)\ldots(c+s-1)}\frac{z^s}{s!}\right) \tag{13}$$

by the additional limitation $c \neq 0$ or c not equal to negative integer number. The hypergeometric function is convergent series by the $|z| < 1$.

The generalized hypergeometric functions of the power (0,3) has the form [12]

$$F(a_0;c_1,c_2,c_3;z) = \sum_{s=0}^{\infty}\frac{(a_0)_s}{(c_1)_s(c_2)_s(c_3)_s}\frac{z^s}{s!}, \tag{14}$$

where a_i is the Pochhammer symbol which defined for the positive integer s and the complex number a as

$$a_s = a(a+1)\ldots(a+s-1). \tag{15}$$

The parameters of the hypergeometric functions {1}, {2}, {3}, {4} are the functions of material properties and conditions of loading.

$$\begin{aligned}
\{1\} &= \left[-\frac{-Z^2+\delta}{Z^2},\ -\frac{1}{2}\frac{-2Z^2+\delta+\delta_2}{Z^2},\ -\frac{1}{2}\frac{-2Z^2+\delta-\delta_2}{Z^2}\right],\\
\{2\} &= \left[\frac{Z^2+\delta}{Z^2},\ \frac{1}{2}\frac{2Z^2+\delta-\delta_2}{Z^2},\ \frac{1}{2}\frac{2Z^2+\delta+\delta_2}{Z^2}\right],\\
\{3\} &= \left[\frac{Z^2+\delta}{Z^2},\ -\frac{1}{2}\frac{-2Z^2+\delta-\delta_2}{Z^2},\ \frac{1}{2}\frac{2Z^2+\delta+\delta_2}{Z^2}\right],\\
\{4\} &= \left[\frac{Z^2-\delta}{Z^2},\ -\frac{1}{2}\frac{-2Z^2+\delta+\delta_2}{Z^2},\ \frac{1}{2}\frac{2Z^2+\delta-\delta_2}{Z^2}\right].
\end{aligned} \tag{16}$$

Additional conditions on parameters result from the necessity to have physical solutions (for example, the Mandelstam radiation principle) and mathematical properties of generalized hypergeometric functions [12]. Since for the order of the hypergeometric function, the condition 0 < 3 is satisfied, the function is an entire one and is converging at all x.

In the general case, for the calculation we can use the asymptotic representation of the generalised hypergeometric function [13]:

$${}_pF_q(z) = \sum_{n=0}^{\infty}\frac{f(n)}{\Gamma(n+1)}z^n \tag{17}$$

where $f(n)$ is the pre-determined function.

4. CONCLUSION AND DISCUSSION

Thus the general solution of Eq. (11) is the sum of exponents with some coefficients. The behaviour of the solution depends on functions Z,R and in principle both periodical oscillations and the resonance regime with exponential increment [14] by $Z < 0$ are possible. This behaviour is consistent with behaviour of the beam at the dynamical columnar deflection [10]. For the rod it is also shown that the exponential growth of deflection is present with a superimposed fast sinusoidal component.

Since a physically unlimited growth of oscillations is impossible, the system should reach its bifurcation point, which switches the system into a qualitatively different state. Such bifurcation is the start of fracture, after the onset of which the model stops to be valid. From the viewpoint of the multilevel hierarchical system theory [1], the system changes the hierarchical level. It means the state of the system with an existing crack is essentially different from undamage system and description of this system demands to use additional hierarchical variables (slaving principles). This variables belong to the higher hierarchical level and can be not expressed in terms of the lower level.

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