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NONLINEAR DYNAMIC MODEL OF KINETOCILIA MOTION: 2D CASE

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Abstract. The article includes a proposed approach to eukaryotic flagellums and cilium (kinetocilia) mathematical modeling. The approach is based on the total system of nonlinear elasticity theory equations, which was modified. Kinetocilia motion is examined in one plane. It makes possible to compare experimental data with results of mathematical modeling.

1. INTRODUCTION

1.1. Mobility of cells

Different types of motion play one of important roles in activity of eukaryotic cells: chromosomes move to cellular poles at the time of mitosis, vacuoles of cellular organelles and the surface of cells are moving as well. Some cells have special structures (flagellums and cilium), which allow cells to move by themselves in case of non-fixed cell (such as sperm cells or protozoa) or to move fluids around them in case of fixed cell (such as olfactory epithelium, ciliary epithelium of the trachea or oviduct epithelium). These flagellums and cilium are called kinetocilia. Flagellums and cilium have the same structure and mainly differ in pattern of motion. General molecular mechanisms are the basis of all of these numerous motor reactions. Further, any motor units' existence has to be combined and structurally connected with the existence of support, constructional or skeleton intracellular formations. Taken together, it forms the support-motor system of the cells (cytoskeleton).

1.2. Eukaryotes mobility system

The elements of cytoskeleton include filiform protein complexes (filaments). Filaments are unstable polymers, which have the ability to polymerization and depolymerization. There are three systems of filaments, which differ in its composition, metastructure and functional properties: microfilaments (mainly consists of actin), microtubules (consists of tubulin) and intermediate filaments. It is possible to divide filaments into two groups by its properties and functions: skeleton fibrils (intermediate filaments) and support-motor filaments (microfilaments and microtubules).

One of cell mobility methods is based on the fact that actin fibrils (microfilaments) and tubulin fibrils (microtubules) are guiding structures. Special motor proteins (myosin, dynein, kinesin) move within these structures. Therefore, there are two

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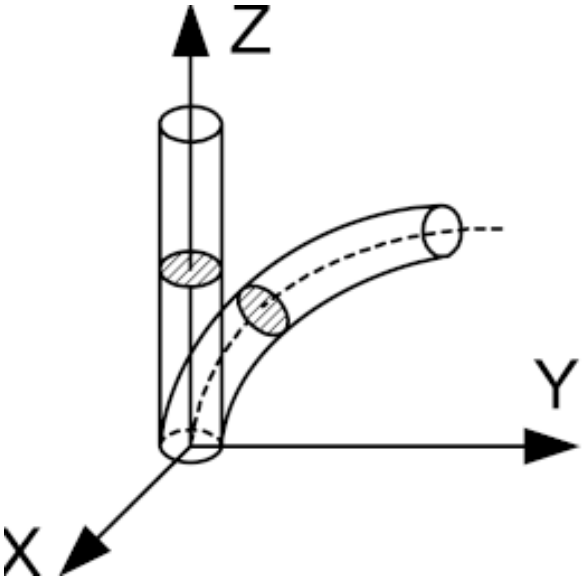

Fig. 1. Kinetocilia position in a Cartesian coordinate system.

main mobility systems: the actin-myosin system (within myocytes) and the tubulin-dynein system (within kinetocilia).

1.3. Sliding filaments model

The standard sliding filaments model is used to explain the mechanism of motion generation. Microtubules are being moved in accordance with one another due to adenosine 5'-triphosphate hydrolysis by dynein in the tubulin-dynein system. This sliding results in kinetocilia bending because of binding protein structures, which allow only limited sliding.

2. METHODOLOGY

2.1. Rod elasticity theory as a method of mobility description

On activation of motion generation mechanisms, kinetocilia performs different fluctuations. These fluctuations end when fluctuation-initiating elements are blocked. Therefore, kinetocilia can be examined as elastic bodies, which have a property of shape reconstruction after unloading. It is possible to use laws of the nonlinear theory of elasticity for kinetocilia mobility description. Speaking about nonlinearity, we mainly mean geometrical nonlinearity which is connected with large deformations and rotations (the amplitude of fluctuations of some flagellums and cilium is sufficiently large), but not physical nonlinearity. As a result, we have physical linearity and geometrical nonlinearity.

Kinetocilia as elastic body has some features of structure. Kinetocilia is a thin cylindrical plasma membrane-coated outgrowth of cytoplasm. The diameter of kinetocilia is identical and equal to 200-300 nm (data from different researches). At the same time, length of different types of cells varies from 5-10 μm (for cilium) to 150 μm (for flagellums) [1]. Therefore, the ratio of kinetocilia diameter and kinetocilia length varies from 10^{-2} to 10^{-3} (taking into account the cylindrical shape of kinetocilia). Thus, it is possible to use the system of equations of the nonlinear rod elasticity theory to kinetocilia motion modeling:

$$\begin{cases} \underline{\Gamma} \equiv \underline{r}' - \underline{\underline{P}} \cdot \underline{r}'_0 \ (\underline{r}'_0 \equiv \underline{r}(s,0)) \\ \underline{\underline{P}}' = \underline{\kappa} \times \underline{\underline{P}} \\ \underline{Q}' + \underline{q} = \rho(\underline{r} + \underline{\varepsilon}) \\ \underline{M}' + \underline{r}' \times \underline{Q} + \underline{\mu} = \rho \underline{\varepsilon} \times \underline{r} + (\underline{\underline{I}} \cdot \underline{\omega})^{\cdot} \\ \underline{M} = \underline{\underline{a}} \cdot \underline{\kappa} + \underline{\underline{c}} \cdot \underline{\Gamma} \\ \underline{Q} = b \cdot \underline{\Gamma} + \underline{\kappa} \cdot \underline{\underline{c}} \end{cases} \quad (1)$$

Here s is the angular position in initial configuration; the motion of a rod in space is the time function of radius-vector $\underline{r}(s,t)$ and rotation tensor $\underline{\underline{P}}(s,t)$ for each particle;

vector $\underline{\Gamma}$ defines strain and shift,

$\underline{\kappa}$ is the vector of curvature and torsion change;

ρ is the body weight per unit of length;

$\underline{\varepsilon} = 1/m \int \underline{x}\, \mathrm{d}m$ is the eccentricity vector (it equals to zero, if a pole is a mass centre, m is the body weight);

$\underline{\underline{I}} = \int \left(|\underline{x}|^2 \underline{\underline{E}} - \underline{x} \cdot \underline{x} \right) \mathrm{d}m$ is the inertia tensor in accordance to the pole ($\underline{\underline{E}}$ is the identity tensor);

$\underline{\omega}$ is the angular velocity;

$\underline{q}$ and $\underline{\mu}$ are the intension of an external force and the moment;

$\underline{Q}$ and $\underline{M}$ define rod internal interactions; this is the force and the moment which describe the influence of one particle (with coordinate s+0) on another (with coordinate s-0). If we reverse direction of reading, $\underline{Q}$ and $\underline{M}$ will change the sign.

Prescribed initial conditions are coordinates and velocity. Typical prescribed boundary conditions on ends of the rod are forces and moments: $s = 0$: $\underline{Q} = 0; \underline{M} = 0$; $s = l$: $\underline{Q} = \underline{Q}_l; \underline{M} = \underline{M}_l$. If one end of the rod is fixed, prescribed boundary conditions usually include $\underline{r}$ and $\underline{\underline{P}}$.

2.2. Modification of the equation system for a rod subject to features of biomechanical continuum

It is possible to simplify the system of equations for a rod subject to features of kinetocilia as biomechanical continuum. The motion in space is considered to be in the fixed Cartesian coordinate system (*XYZ*). *Z*-axis corresponds with kinetocilia dominating axis (Fig. 1).

2.2.1. Inertial characteristics

The system of Eqs. (1) is the system of dynamic equations; it takes into account different inertial characteristics such as mass, eccentricity, inertia tensor. These parameters define the mass distribution of material points which organize the system. The eccentricity vector $\underline{\varepsilon}$ sets the shift of the mass center from the pole. However, it is possible to place the pole into the mass center without loss of generality. In this case $\underline{\varepsilon} \equiv 0$. Furthermore, it is considered that contribution of terms, prescribed by the tensor, is negligible because the mass center is the inertia center. Then $\underline{\underline{I}} \equiv 0$. Therefore, when we describe the mobility, we should take into account only one inertial characteristic, namely, the mass.

2.2.2. External actions

External influence on a standard rod can be realized by the external force $\underline{q}$ and the external moment $\underline{\mu}$. However, it is necessary to specify these terms when speaking about the motion in biomechanics of cell. Action of the external moment $\underline{\mu}$ leads to rod torsion. In this case, the deformation is the result of rotation of each cross-section of the rod in accordance to underlying cross-sections on a certain angle while the whole rod is straight. Generating lines of lateral surface of the rod are parallel with the rod axis. When the rod rotates, generating lines become spiral. It is possible in different constructional materials, but it is impossible in such biological continuum as cell. Rod torsion leads to plasma membrane deformation and, perhaps, to disruption. The successful performance of such cell will be impossible. Therefore, it is considered that external moments do not work in the modeling continuum, i.e. $\underline{\mu} \equiv 0$. As a result, the bending and torsion stiffness tensor $\underline{\underline{a}}$ has only three components bending stiffness in three different planes.

It is necessary to take into account viscous characteristics of the motion continuum. It can be mucus or aquatic environment. Let us suppose that the environment is the Newtonian fluid. Therefore, there is the linear dependence between shift tensions and velocities of deformation and coefficient of viscous resistance; b_* is a coefficient of proportionality. As is obvious from the foregoing, the external force $\underline{q}$ can be viewed as $\underline{q} = \underline{f} - b_*\underline{r}$, where $\underline{f}$ is the direct force of external factors (for example, gravitational field and electromagnetic field of different chemical agents); $-b_*\underline{r}$ is the resistance force of environment.

The coefficient of viscous resistance b_* can be obtained in the following way. The force of viscous friction per unit of length is calculated as

$$\frac{F}{l} = C_d \frac{1}{2} \gamma \upsilon^2 a = b_* \upsilon,$$

where υ is the velocity of motion. The magnification factor C_d can be defines as

$$C_d = \frac{C_0}{\Re},$$

where

$$\Re = \frac{\gamma a \upsilon}{\eta}$$

is the Reynolds number, $C_0 \approx 24$ [2]. Therefore $b_* = 12\eta$, where η is the dynamic viscosity of environment.

2.2.3. Stress-strain and in-plane shears of cross-connections absence

It is known that kinetocilia is a non-stretched object. Furthermore, in mechanics it is common to take into consideration in-plane shears only when we have a short thick rod; when the rod moves, in-plane shears appear. But cell continuum involved is thin and long object. That is why from the mechanical point of view we do not take into account in-plane shears, i.e. $\underline{\underline{\Gamma}} \equiv 0$. The tensor of cross-connections $\underline{\underline{c}}$ is a part of the determining equation, but it characterizes rather infrequent cases when strain-shift and bend-torsion mainly influence on each other. As a result, when we examine biological structures in general and especially kinetocilia, it is possible to consider that $\underline{\underline{c}} \equiv 0$, because its influence is negligible in comparison with main components or is absent.

2.2.4. "Cause of motion" system insertion. Prescribed internal strains

The cause of common framed structures motion in mechanics is mostly some external actions such as forces and moments. However, the motion in biological system is determined by internal factors. When we describe the motion generation mechanism from the mechanical point of view, we could say that the cause of kinetocilia motion is prescribed internal strains, which appear when microtubule duplets are sliding in accordance with one another. There is an example when deformation of different rods and plates initiates because of temperature change (in thermoelasticity theory). If one end of a rod is fixed, the change of temperature or change of polycrystal structure of the rod can lead possibly to its bending (similarly to kinetocilia). In the case of thermoelasticity it results in Hooke's law modification: additional term αT appears, where α is the coefficient of thermal expansion, T is the temperature change.

We use the vector of prescribed internal strain $\underline{\underline{\beta}}(s,t)$ to describe the cause of motion in cell biomechanics. Then there is only one determining equation to define moments, because there is no stress or strain:

$$\underline{\underline{M}} = \underline{\underline{a}} \cdot \left(\underline{\kappa} - \underline{\beta}\right),$$

External forces $\underline{f}$ can exist in the system along with prescribed internal strains. These forces directly affect on a cell as physical object, but its role is mainly modulating but not dominating.

However, the explicit form of the vector of prescribed internal strain $\underline{\underline{\beta}}(s,t)$ is a difficult question and it is the subject of special investigation. It is unknown how the sequence of duplets sliding in kinetocilia is regulated. The place of kinetocilia or myofibrilla strain initiation and strain events frequency are unknown as well. Meanwhile it is known that the time and the place of strain initiation do not depend on each other. As a result, it is possible to consider that the prescribed strain function is the product of deterministic functions of independent values. The general view of this function can be represented in the following way:

$$\beta(s,t) \equiv \beta_1 \cdot \beta_2(s) \cdot \beta_3(t).$$

We define the explicit form of the vector of prescribed internal strains for kinetocilia to conduct mathematical modeling, taking into account some considerations. Dynein "arms" are uniformly distributed on the sectional plane of olfactory flagellum. Therefore, it is equiprobable that either of "arms" can activate the ATP hydrolysis, because dynein has the ATP-activity. Let n is the number of dynein "arms" on the sectional plane, then $\beta_1 \equiv \beta_1(n)$.

Mitochondrial pool is mainly located under the system of microtubules. Therefore, the largest probability takes place when a molecule of ATP is hydrolyzed by first molecule of dynein (proximate to the plane of olfactory epithelium), which has the ATP-activity. It does not matter near which microtubule it would happen. When moving to the peak of olfactory flagellum, the probability of hydrolysis decreases rapidly. Thus, let the probability parameter $\chi \equiv p/l$ defines the probability of hydrolysis along an axonema, where p is the probability, l is the length of kinetocilia. Suppose that

$$\beta_2(s) \equiv \exp\left(-p\frac{s}{l}\right).$$

Molecules of ATP, which will be hydrolyzed, come into the system in arbitrary moments of time. Let us take into consideration that it is impossible to hydrolyze more that one molecule of ATP simultaneously. Generally, intervals between two successive hydrolysis events are not equal. As a result, the time dependence can de described by the periodic function with the random period T. Suppose that

$$\beta_3(t) \equiv \sin\left(\frac{2\pi}{T}t\right).$$

The prescribed strain has the following form (taking into account previously mentioned definitions of independent stochastic variables functions):

$$\beta(s,t) = \beta_1(n) \cdot \exp\left(-p\frac{s}{l}\right) \cdot \sin\left(\frac{2\pi}{T}t\right).$$

2.3. Problem definition for kinetocilia

Radius-vector can be defined as:

$$\underline{r}(s,t) = \underline{r}_0(s) + \underline{u}(s,t),$$

where

$$\underline{r}_0(s) = s\underline{k},$$
$$\underline{u}(s,t) = u_x(s,t)\underline{i} + u_y(s,t)\underline{j} + u_z(s,t)\underline{k}.$$

In this case, the vector $\underline{u}(s,t)$ characterizes the bending. As a result, taking into account the features of concerned biological continuum, the sys-

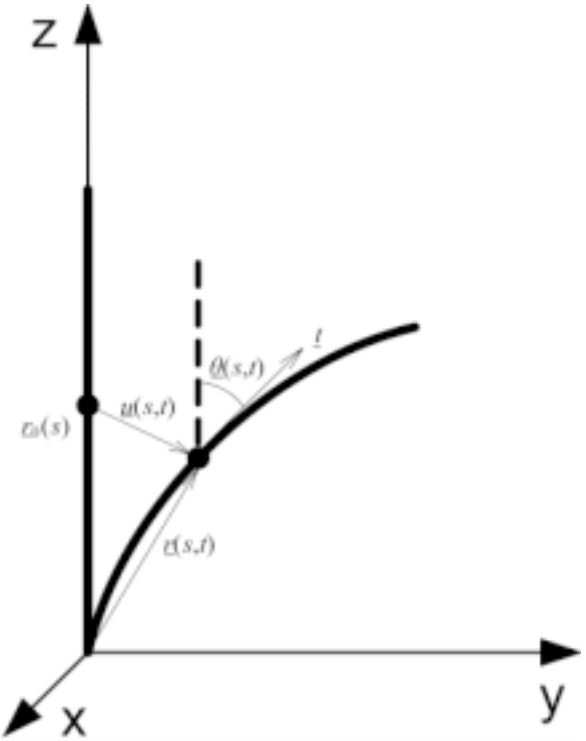

Fig. 2. Radius-vector of kinetocilia.

tem of nonlinear elasticity theory equations for a rod can be modified into the system of equations which defines 3D biomechanical continuum motion. The general formulation of 3D problem with initial and border conditions:

$$\begin{cases} \underline{u}'+\underline{k} = \underline{\underline{P}} \cdot \underline{k} \\ \underline{\underline{P}}' = \underline{\kappa} \times \underline{\underline{P}} \\ \underline{Q}'+\underline{f} = \rho\ddot{\underline{u}} + b_*\dot{\underline{u}} \\ \underline{\underline{M}}'+(\underline{u}'+\underline{k}) \times \underline{Q} = 0 \\ \underline{\underline{M}} = \underline{\underline{a}} \cdot (\underline{\kappa} - \underline{\beta}) \end{cases}$$

$$t = 0 \quad \begin{matrix} \underline{u}(s,t) = 0 & \underline{\underline{P}}(s,t) = 0 \\ \underline{u}(s,t) = 0 & \underline{\underline{P}}(s,t) = 0 \end{matrix}; \qquad (2)$$

$$s = 0 \quad \begin{matrix} \underline{u}(s,t) = 0 & \underline{\underline{P}}(s,t) = 0 \\ \underline{u}'(s,t) = 0 & \underline{\underline{P}}'(s,t) = 0 \end{matrix}.$$

This formulation can be used to define the motion activity of kinetocilia.

2.4. Variation method of 3D problem solution

The variation method is the formalistic-pure one to transfer 3D problem into 1D problem. The variation principle for rod-similar biomechanical continuum has the following view, taking into account the principle of rod's virtual work, the similar problem definitions in thermoelasticity [3] and biological features of the continuum:

$$\int_0^l \left\{ \left[\underline{f} - (\rho\underline{u} + b_*\underline{u}) \right] \cdot \delta\underline{u} + \delta A^{(i)} \right\} ds = 0.$$

Here $\delta A^{(i)} = -\delta\Pi$ is the virtual work of internal forces. As a result, $\delta\Pi = \underline{M}\delta\underline{\kappa}$. However, for biomechanical continuum $\delta\Pi$ is the variation of free energy (Helmholtz energy), which is similar to corresponding principle in thermomechanics, but not potential energy. Therefore, the general formulation of 1D problem can be written in the following way:

$$\int_0^l \left\{ \left[\underline{f} - (\rho\underline{u} + b_*\underline{u}) \right] \cdot \delta\underline{u} + \delta A^{(i)} \right\} ds = 0.$$

where

$$\begin{cases} \underline{u}'+\underline{k} = \underline{\underline{P}} \cdot \underline{k} \\ \underline{\underline{P}}' = \underline{\kappa} \times \underline{\underline{P}} \end{cases}.$$

3. RESULTS

Let us examine the kinetocilia motion in OYZ plane, i.e. $\underline{r}(s,t) = \underline{r}_0(s) + \underline{u}(s,t)$, where: $\underline{r}_0(s) = s\underline{k}$, $\underline{u}(s,t) = u_y(s,t)\underline{j} + u_z(s,t)\underline{k}$. Then

$$\begin{cases} y = u_y(s,t) \\ z = u_z(s,t) + s \end{cases}.$$

Because we examine only one plane, the bending stiffness tensor has the view $\underline{\underline{a}} = a$.

Let $\underline{t}$ is the ort of a tangent (Fig. 2). The definition of the tangent is

$$\underline{t} = \underline{r}' = u_y'\underline{j} + (u_z'+1)\underline{k}.$$

On the other hand from the geometry the plane of a curve can be described as

$$\underline{t} = \underline{j}\sin\theta + \underline{k}\cos\theta.$$

Therefore, it is possible to identify the relation between $\underline{u}$ and $\underline{\theta}$ as

$$\begin{cases} u_y' = \sin\theta \\ u_z' = -1 + \cos\theta \end{cases}.$$

It is necessary to know the form of the function $\theta(s,t)$ to identify u_y and u_z. Suppose that

$$\theta(s,t) = \sum_{i=1}^{3} \psi_i(s) q_i(t).$$

Therefore, the problem definition has the following view:

$$\int_0^l \left\{ a\left(\theta^i - \beta\right) \frac{\partial \theta'}{\partial q_i} + \frac{\partial u_y}{\partial q_i} \sum_{i=1}^{3} \left(\frac{\partial u_y}{\partial q_i} \left(\rho \ddot{q}_i + b_* \dot{q}_i\right) + \frac{\partial^2 u_y}{\partial q_i^2} \right) + \frac{\partial u_z}{\partial q_i} \sum_{i=1}^{3} \left(\frac{\partial u_z}{\partial q_i} \left(\rho \ddot{q}_i + b_* \dot{q}_i\right) + \frac{\partial^2 u_z}{\partial q_i^2} \rho q_i^2 \right) - f_y \frac{\partial u_y}{\partial q_i} - f_z \frac{\partial u_z}{\partial q_i} \right\} \mathrm{d}s = 0,$$

where

$$\beta(s,t) = \beta_1 \beta_2(s) \beta_3(t);$$
$$u_y = \int \sin\theta \, \mathrm{d}s;$$
$$u_z = \int \left(-1 + \cos\theta\right) \mathrm{d}s.$$

We have the similar initial and boundary conditions as in (2). If there is no external influences, then

$$\underline{f} \equiv 0 \Rightarrow f_y \equiv 0; \; f_z \equiv 0.$$

The pure view of gradient angle of the tangent ($\theta(s,t)$) for motion with different numbers of degrees of freedom can be defined as $\theta = q\sin$ in case of one degree of freedom, $\theta = q_1 s + q_2 s^2$ in case of two degrees of freedom and $\theta = q_1 s + q_2 s^2 + q_3 s^3$ in case of three degrees of freedom.

Example: cilium motion in a hit phase without external influences

In the case of middle amplitude fluctuations of kinetocilia (the fluctuations vary from 00 to 900) functions $\cos\theta$ and $\sin\theta$ can be expanded in the Taylor series, and we can take into consideration only some first members of the series. Then:

$$u_y = \int \left(\theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \ldots \right) \mathrm{d}s;$$
$$u_z = \int \left(-\frac{\theta^2}{2} + \frac{\theta^4}{24} - \ldots \right) \mathrm{d}s.$$

In this case angles of bending are not large, but not small. Therefore, in the Taylor series we can use linear and quadratic members. Then

$$u_y = \int \theta \, \mathrm{d}s; \; u_z = -\int \frac{\theta^2}{2} \mathrm{d}s.$$

This motion has one degree of freedom, so we can use the approximation $\theta = qs$. In this case

$$\frac{l^5}{20} \left(1 + q^2 l^2 \frac{20}{63} \right) \left(\rho \ddot{q} + b_* \dot{q}\right) + alq + \frac{l^7}{63} \rho \dot{q}^2 q = a\beta_1\beta_3(t) \int_0^l \beta_2(s) \mathrm{d}s + \frac{1}{2} \int_0^l s^2 f_y \, \mathrm{d}s - \frac{q}{3} \int_0^l s^3 f_z \, \mathrm{d}s;$$

$$\frac{l^5}{20} \left(\rho \ddot{q} + b_* \dot{q}\right) + \frac{l^7}{63} \rho \dot{q}^2 q \frac{1}{1 + q^2 l^2 \frac{20}{63}} + alq \frac{1}{1 + q^2 l^2 \frac{20}{63}} = \left(a\beta_1\beta_3(t) \int_0^l \beta_2(s) \mathrm{d}s + \frac{1}{2} \int_0^l s^2 f_y \, \mathrm{d}s - \frac{q}{3} \int_0^l s^3 f_z \, \mathrm{d}s \right) \frac{1}{1 + q^2 l^2 \frac{20}{63}}. \quad (3)$$

The equation can be solved numerically in different software. However, it can be analyzed analytically.

Suppose that external influences are absent, i.e. $f_y \equiv 0$; $f_z \equiv 0$. Then the equation can be solved by asymptotic method.

Let $q(t)$ is a small magnitude, then $20l^2q^2/63$ is a small magnitude as well. Let us use the following equation: $1/(1 + \alpha) = 1 - \alpha + \alpha^2 - \alpha^3 + \ldots$, where a is α small magnitude. We take into consideration linear and quadratic members of the Taylor series to examine the nonlinear case. Then the differential equation will have the following view:

$$\frac{l^5}{20} \left(\rho \ddot{q} + b_* \dot{q}\right) + alq = a\beta_1\beta_3(t) \int_0^l \beta_2(s) \mathrm{d}s - q^2 l^2 \frac{20}{63} a\beta_1\beta_3(t) \int_0^l \beta_2(s) \mathrm{d}s.$$

Additional formal small argument l can be inserted into the equation:

$$\ddot{q} + \frac{20a}{\rho l^4} q = \lambda \left(-\frac{b_*}{\rho} q + \frac{20}{\rho l^5} a\beta_1\beta_3(t) \int_0^l \beta_2(s) \mathrm{d}s - q^2 \frac{400}{63\rho l^3} a\beta_1\beta_3(t) \int_0^l \beta_2(s) \mathrm{d}s \right).$$

We will try to find the solution as:

$q(t) = q_0(t) + \lambda q_1(t) + \lambda^2 q_2(t) + \ldots$

Suppose that the right part of equation is the periodic function, i.e. $\beta_3(t) = \sin(2\pi t/T)$. Then we will define only periodical solutions. We will examine resonance oscillations, i.e. $2\pi / T = \sqrt{20a / \rho l^4} = \omega$. Here we use the method of Poincaré:

$$\ddot{q}_0 + \omega^2 q_0 = 0 \Rightarrow q_0 = A\cos\omega t + B\sin\omega t,$$

$$\begin{cases} \frac{1}{\pi}\int_0^{2\pi}\left\{-\frac{b_*\omega}{\rho}(-A\sin\psi + B\cos\psi) + \frac{20}{\rho l^5}\alpha\beta_1\int_0^l \beta_2(s)\,ds\sin\psi - \right. \\ \left. \frac{400}{63\rho l^3}\alpha\beta_1\int_0^l \beta_2(s)\,ds\sin\psi\left(A^2\cos^2\psi + B^2\sin^2\psi + 2AB\cos\psi\sin\psi\right)\right\}\sin\psi\,d\psi = 0 \\ \frac{1}{\pi}\int_0^{2\pi}\left\{-\frac{b_*\omega}{\rho}(-A\sin\psi + B\cos\psi) + \frac{20}{\rho l^5}\alpha\beta_1\int_0^l \beta_2(s)\,ds\sin\psi - \right. \\ \left. \frac{400}{63\rho l^3}\alpha\beta_1\int_0^l \beta_2(s)\,ds\sin\psi\left(A^2\cos^2\psi + B^2\sin^2\psi + 2AB\cos\psi\sin\psi\right)\right\}\cos\psi\,d\psi = 0 \end{cases},$$

$$\begin{cases} b_*\omega A + \left(\frac{1}{l^2} - \frac{5}{63}A^2 - \frac{5}{21}B^2\right)\frac{20}{l^5}a\beta_1\int_0^l \beta_2(s)\,ds = 0 \\ b_*\omega B + AB\frac{200}{63l^3}a\beta_1\int_0^l \beta_2(s)\,ds = 0 \end{cases}.$$

The second equation of this system has two possible solutions. We choose the event, when B=0, because in this case $q(t) = A\cos\omega t$. It is corresponded with the qualitative representation of the motion when the resonance phase shift will is equal to $\pi/2$. Then

$$q(t) = 63\frac{b_*\omega l^2 \pm \sqrt{b_*^2\omega^2 l^4 + \frac{400}{63l^3}a^2\left(\beta_1\int_0^l \beta_2(s)\,ds\right)^2}}{200a\beta_1\int_0^l \beta_2(s)\,ds}\cos\omega t \qquad (4)$$

As a result, the motion path of cilia can be defined by the following parametric system:

$$\begin{cases} y = \frac{q(t)}{2}s^2 \\ z = s - q(t)^2\frac{s^3}{3} \end{cases},$$

where $s \in [0;l]$, $q(t)$ is the solution of Eq. (3) or, in case of resonance periodic oscillations, of (4).

4. DISCUSSION AND CONCLUSIONS

In previous researches [4] the similar approach was used in building of linear models of kinetocilia mobility. However, flagellums and cilium fluctuations are not small. That is why it is necessary to complicate the model by taking into consideration the geometrical nonlinearity. It has been done in this article. When we make the model of motion more complicated and more realistic, it is more difficult to solve the equations. We have to use the more complicated and modern software and hardware as well. That is why we have examined the simple example, which sometimes can be solved analytically. Also it is necessary to make some experimental investigations to define the explicit view of the vector of prescribed strains.

Thus, we can draw some conclusions:

1. The proposed approach is based on the system of equations of the nonlinear rod dynamic. It can be used in mathematical modeling of kinetocilia mobility.
2. The proposed vector of prescribed strains defines the internal condition of cell as a biomechanical continuum.

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