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MAGNETORESITANCE RELATING TO THE INTEGER HALL EFFECT

G.J. Papadopoulos

Department of Physics, Solid State Physics Section, University of Athens, Panepistimiopolis, GR 157 84, Zografos, Athens, Greece

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Abstract. A formalism developed earlier for the magnetoresistance exhibited by a conducting material, in the form of a parallelepiped , under bias and subjected to a perpendicular magnetic field is shown to apply, with certain modification regarding the dissipative energy, in the case whereby the carriers are constrained in a way as to form a two dimensional gas. The values of the magnetic field around which the longitudinal resistance rises for high enough fields are more or less obtained on the basis of ordinary Fermi-Dirac statistics. The same applies for the magnetic field values corresponding to integer filling factors. Furthermore, the effect of diminishing resistance peaks with increasing magnetic field is also demonstrated. The evaluations were carried out at absolute zero temperature.

1. INTRODUCTION

In a previous paper [1] we developed formulae for the magnetoresistance (MR) exhibited by a material in the form of a rectangular parallelepiped (length L, width l, and thickness d), which is subjected to a constant normal magnetic field, B, along its thickness. The MR formulae refer to the experimental situations whereby a constant bias is applied along the length of the specimen or a constant current flows through along the length direction. The formulae for the MR are given for both the high and low temperature regime. Relevant endogenous parameters, in addition to the externally controlled parameters, specifying the experimental conditions, i.e. magnetic field, current or bias and temperature, enter the formulae. Furthermore, certain geometric parameters of the sample appear in the formulae. Details concerning the various parameters, in question, entering our formulae for the MR will be seen, subsequently, in the structure of these formulae, stated in the next section.

In a follow up paper [2] we dealt with MR oscillations occurring in relatively thin films. In this case it became possible to derive oscillations similar to experimental ones, utilizing in our formulae mean carrier energies in terms of the magnetic field, which, at low temperature exhibited oscillations. The derivation of the average energies, in question, was based on the total energy spectrum of a carrier in a magnetic field together with the energy associated with the motion parallel to the magnetic field. The motion along the magnetic field is restricted by the enclosure potential extending over the film thickness.

If we move to thinner and thinner enclosure potentials the ground level of the portion of energy for the motion along the magnetic field increases substantially, but what is more important the gaps between consecutive such levels increase immensely. The magnitude of their energy gaps is such that their entry in the statistics formulae leaves the result imperceptibly unaffected. Under the circum-

Corresponding author: G. J. Papadopoulos, e-mail: gpapado@phys.uoa.gr

stances the carriers form a gas whose portion of energy for the motion perpendicular to the magnetic field is fully controlled by its 2 dimensional motion perpendicular to the field. Thus, our considerations refer to a 2 dimensional carrier gas (2DCG).

In dealing with the MR of such a 2DCG the average energy obtained on the basis of the energy spectrum in conjunction with Fermi-Dirac (F-D) statistics does not suffice for obtaining the MR peaks appearing at narrow regions of the applied magnetic field at low temperatures. However, as we shall see, the spectrum in combination with the relevant F-D statistics can determine the values of the magnetic field around which the various MR excitations occur.

For the sake of making the point of view employed in this work clearer we shall consider the case of zero temperature, which provides results close to the respective low temperature ones. In what follows we sketch the procedure employed for evaluating the Fermi level for each value of the magnetic field, as well as the mean energy for the motion perpendicular to the magnetic field. The procedure begins with laying out the various energy levels due to the magnetic field in order of magnitude. As is well-known at temperature T=0K, the various energy levels participate in order of magnitude, as many as required so that the sum of the participating states reaches the number of carriers. In case the largest energy level offers states that exceed the number of carriers it participates with less states, as many so that the total number of participating states equals the number of carriers.

The largest energy level entering the above procedure determines the Fermi level. It should be noted that the number of states associated with each energy level is given by the degeneracy of the level (to the nearest integer) which increases with increase in the magnetic field, which depending on the value of the magnetic field may exceed the number of carriers. In such a case the Fermi energy is given by the energy of the lowest level. Furthermore, the ratio of the degeneracy of a given level to the number of carriers, in a state of equilibrium, supplies the probability for the level in question. In case the degeneracy exceeds the number of carriers the number of states to be used equals the number of carriers, thus the associated probability is 1, and so the level in question excludes participation of any other level in the formation of the Fermi energy. Having at hand the probabilities for the various energy levels one can form the average energy for the motion perpendicular to the magnetic field. On the basis of the above, well-known facts, we shall proceed, in mathematical form, establishing quantities useful for dealing with the MR associated with the integer Hall effect. Predominant role is played by the value of the magnetic field for which the probability equals unity. Denoting by B_0 the value of the above quantity the quantities B_0/j (j = 1,2,3…) determine approximately the values of the magnetic field around which the various minima in the mean energy occur. The integers j provide the number of energy levels (minus 1) participating in the formation of the average energy corresponding to a magnetic field, B, in the range between $B_0/(j + 1)$ and B_0/j. Here B_0 has got a statistical connotation, and at the same time j forms the filling factor in the sense used by von Klitzing [3], based on flux considerations. However, our considerations are based on a more or less constant number of carriers in the sample, something that applies in the case of extremely small current flows through the sample. We shall subsequently see how one can handle the case whereby the experimental conditions are conducive to change in the number of carriers in the sample with increase in B.

From the above dynamical and statistical considerations it is not feasible to obtain directly the appropriate mean energy associated with the relevant diffusion coefficient entering the Einstein – Smoluchowski equation [1], from which the MR formulae derive, so that one can be led to the MR peaks relating to the integer Hall effect. One way, to enable derivation of these peaks it would appear necessary to restrict the dissipative energy in small regions around the values of B for which the mean energy due to the magnetic field attains maximum values. Such an effect might be attributed to additional density of carriers with accompanying energy rearrangement. When you apply magnetic field only the density of carriers in the sample does not change. However, once you employ electric field, the combination of both does induce increase in the density of the carriers, overall. There is an increase in the number of carriers, on the one hand, and on the other, the density of carriers varies along the y-axis, with increasing (or decreasing) order from one end of the sample to the other.

As the number of carriers affects the statistical distribution of energy the mean energy depends on this number. It is precisely the variation in the mean energy per carrier that requires some sort of correction.

In section 2 we cite formulae for the MR, derived earlier, appropriate for the cases of experimental conditions under constant bias and under constant current. Section 3 deals with the Fermi – energy and mean energy as functions of B. Assuming the locations of the maxima of the mean energy to fit sufficiently well the values of B around which the various MR elevations occur we proceed to modify the dissipative energy by introducing narrow Gaussians, as functions of B, centered around the B values where the various maxima of the mean energy are located. Finally in section 4 we present formulae for the mean energy maxima locations in the case whereby the number of carriers does not increase considerably with the magnetic field. Comparison with experiment follows in a few cases. Furthermore, on the basis of the modified dissipative energy we present results, obtained from the MR formulae under constant current and under constant bias for the longitudinal resistance utilizing parameters such as, surface carrier density, Landé factor, carrier mobility, and relevant collective parameters.

2. MAGNETORESISTANCE FORMULAE

We cite, initially, below formulae for the MR, developed earlier [1], one of which appropriate for the experimental condition under constant bias, which takes the form

$$MR = \frac{\xi\eta}{\sinh(\xi\eta)}\left(1+\eta^2\right)-1, \tag{2.1a}$$

while for the case of constant current

$$MR = \frac{1}{f\eta}\ln\left\{f\left(1+\eta^2\right)\eta + \sqrt{1+\left[f\left(1+\eta^2\right)\eta\right]^2}\right\}-1, \tag{2.1b}$$

where the symbols ξ, f, and η in (2.1a,b) stand for dimensionless quantities which are given by

$$\xi = \frac{qlV}{2\varepsilon_0\langle H_\perp\rangle}, \quad f = \frac{qi_0\rho_s(T)}{2\varepsilon_0 d\langle H_\perp\rangle}, \quad \eta = \frac{\mu B}{c}, \tag{2.2}$$

where q is the carrier's charge, μ its mobility ε_0 the material's dielectric constant, V the applied voltage, and $\langle H_\perp\rangle$ stands for the mean carrier energy, spin inclusive. Furthermore, i_0 denotes the current flowing through the sample, under condition of constant current, and ρ_s the sample's resistivity. In the system of units employed the speed of light, c, comes into play. It should be noted that the spin energy, when negative removes from the kinetic energy part of an amount of energy $h\omega/2$, depending on the value of the Landé factor, while when positive should similarly add an equal amount, where ω stands for the cyclotron frequency $\omega = qB/m^*c$, and m^* denotes the carrier effective mass. The pair of dimensionless quantities (ξ,η) as well as the pair (f,η) are connected via the magnetic field through the dependence of $\langle H_\perp\rangle$ on B. The temperature dependence enters the MR via the dependence of $\langle H_\perp\rangle$ and μ on temperature. It would, howevever, seem desirable to employ a pair of parameters independent of each other. To this extent we introduce the dimensionless parameter

$$\xi_0 = \frac{qlV}{2\varepsilon_0 L E_0}, \quad f_0 = \frac{qi_0\rho_s(T)}{2\varepsilon_0 d\langle H_\perp\rangle}, \quad \text{where } E_0 = \langle H_\perp(B=0)\rangle. \tag{2.2a}$$

The parameters ξ_0 and f_0 are free of B and can be used to from ξ and f as

$$\xi = \xi_0\frac{E_0}{\langle H_\perp\rangle}, \quad f = f_0\frac{E_0}{\langle H_\perp\rangle}. \tag{2.2b}$$

Given the mean energy per carrier, $\langle H_\perp\rangle$, the independent pair of parameters (ξ_0,η) obtained from (2.2) and (2.2a) can provide via (2.2b) the MR obtained from (2.1a), appropriate for the experimental condition under constant bias. It should be noted here that the value of ξ_0 is controlled by the external parameter V. In the case of constant current similar remarks apply for the independent pair of parameters (f_0,η). The value of f_0 is now controlled by the flowing current, i_0.

As pointed out earlier the formulae for the MR hold well, utilizing $\langle H_\perp\rangle$ deriving via F-D statistics, as long as the carriers' channel is 3D. Once we reach the case of a 2DCG the mean energy per carrier entering the MR formulae requires modification. Subsequently, we shall proceed obtaining $\langle H_\perp(B)\rangle$ and furthermore carry out appropriate modification leading to a dissipative energy, $E_d(B)$, which upon replacing $\langle H_\perp\rangle$ in (2.1) can produce the desired pattern of MR for a 2DCG.

3. DISSIPATIVE ENERGY

In the case of 2DCG experiments it is found that depending on the surface charge density the longitudinal resistance, R_{xx}, of the channel at high enough magnetic fields is almost zero apart from narrow regions whereby the resistance elevates substantially. As pointed out earlier, the elevations occur in between consecutive values of the magnetic field, B, given by B_0/j (j = 1,2,3…) or more accurately between consecutive solutions determined by Eq. (3.4), whenever the surface density $n_s(B)$ changes substantially with B. The heights of the elevations, in question, are more or less proportional to the square of B around which the elevation takes place. The values of B at which the elevations occur are determined by the values of B for which the mean carrier energy, obtained via Fermi-Dirac (F-D) statistics, acquires maximum values. In what follows we shall present modifications to the mean energy which when fed into the MR formulae lead more or less to the desired resistance elevations.

Let us now proceed to obtain the Fermi-energy, E_F, in terms of the magnetic field. The carrier's spectrum in a magnetic field, B, perpendicular to the carrier channel is given by

$$\varepsilon_{ns} = (n + 1/2)h\omega + \frac{s}{2} g \frac{h\omega}{2}, \quad (n = 1, 2, ...), \quad (s = \pm 1), \tag{3.1}$$

where g is the Landé factor.

Arranging the various energy eigenvalues in order of magnitude for a given B we have $E_1 = \varepsilon_{0,-1}$, $E_2 = \varepsilon_{0,1}$, $E_3 = \varepsilon_{1,-1}$, $E_4 = \varepsilon_{1,1}$, $E_5 = \varepsilon_{2,-1}$,… In general

$$E_j = \varepsilon_{(j-1)/2,\,1} \quad \text{for} \quad j \quad \text{odd}, \qquad E_j = \varepsilon_{(j-2)/2,\,1} \quad \text{for} \quad j \quad \text{even} \quad (j = 1, 2, 3, ...). \tag{3.1a}$$

As is well known each energy level is highly degenerate with degeneracy

$$G_\perp = \frac{m^* \omega L l}{2\pi h}, \tag{3.2}$$

which is proportional to B taking account of the expression for the cyclotron frequency $\omega = qB/m^*c$. Clearly, for sufficiently large B it is possible for the degeneracy to reach the total number of carriers, $N = n_s L l$ in the channel, n_s being the surface charge density. Let us now evaluate B_0, mentioned earlier, by equating the degeneracy, given by (3.2) to the number of carriers and obtain

$$B_0 = \frac{2\pi h n_s c}{q} \tag{3.3}$$

B_0, essentially, is the value of the magnetic field associated with filling factor 1, as given in reference [3].

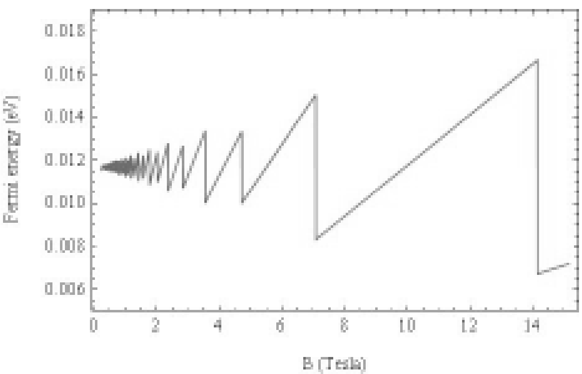

Fig. 1. Fermi energy for the motion perpendicular to the magnetic field, as predicted by F-D statistics at T=0K, in the case of a 2DCG with carrier surface density n_s=3.42·10^{11} cm^{-2}, effective mass m^*=0.07m_e and Landé factor g = 0.85. The values of B for which there occur abrupt changes in the Fermi energy equal B_0/j (j=1,2,3,….), B_0=14.1539 Tesla.

Prior to proceeding to statistical evaluations we shall point out how one can proceed more accurately for obtaining the values of B just below which we have an additional entry into the set of participating levels for the mean energy formation. As stated, earlier, these values are adequately approximated by the quantities B_0/j for small values of the flowing current through the sample. However, whenever the number of carriers in the channel increases substantially with increase in the magnetic field the values of the field, in question, are determined via the corresponding probabilities, which are obtained by solving the equation $G_\perp(B)/N(B) = 1/j$ (j = 1,2,3,…), where $N(B)$ stands for the number of carriers in the sample as a function of the applied magnetic field, B In the case of a 2DCG the above equation takes the form

$$qB / 2\pi h c n_s(B) = 1/j, \tag{3.4}$$

where $n_s(B)$ denotes the mean surface carrier density at magnetic field B In what follows we shall deal with the case whereby the various B_0/j approximate the more exact values obtained from the above equation.

Now, for $B \geq B_0$ the Fermi energy is $E_F = E_1$, while for B such that $B_0/2 \leq B < B_0$ the allowed levels for occupation are two, namely, E_1 and E_2, of which the largest is E_2. Thus, for the above region of B $E_F = E_2$. Proceeding in the same way for the region $B_0/3 \leq B < B_0/2$, $E_F = E_3$ and so on. In general

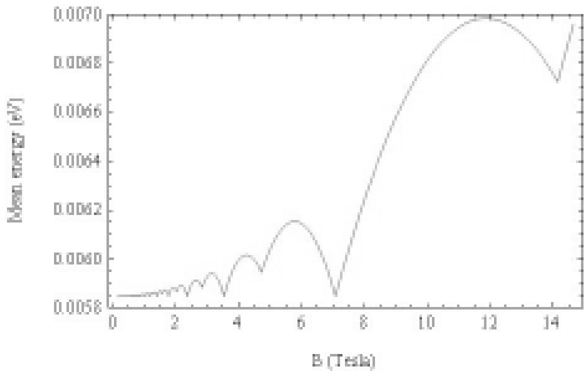

Fig. 2. Mean energy for the motion perpendicular to the magnetic field, as predicted by F-D statistics at T = 0K, in the case of a 2DCG, with data as per Fig. 1. The values of B for which the various sharp minima occur equal B_0/j (j = 1,2,3,….).

we can express E_F, utilizing step functions defined as, $\Theta(B - B') = 0$ for $B < B'$ and 1 for $B \geq B'$, as

$$E_F = \Theta(B - B_0)E_1 + \sum_{j=1}^{N'}\left[\Theta\left(B - \frac{B_0}{j+1}\right) - \Theta\left(B - \frac{B_0}{j}\right)\right]E_{j+1}. \tag{3.5}$$

The upper limit in the summation, basically, reaches the number of carriers, as determined so that the degeneracy acquires value 1, but for the purpose of calculations quite a finite integer would do, as long as B_0/N' is much smaller than unity. An example of E_F as a function of B is shown in Fig. 1.

Let us now proceed to obtain the mean energy for a given B associated with the energy spectrum (3.1) and a carrier surface density n_s, in accordance with F-D statistics. We act in a similar fashion as in the case for obtaining E_F, but this time we need the probability for the various energy levels. In the region $B \geq B_0$ only one level comes into play, the smallest, namely, E_1. Thus the probability for this level in the above region equals unity. Next, for the region $B_0/2 \leq B < B_0$ the available energy levels are E_1 and E_2. The smaller of the two, E_1, enters with all its states given by its degeneracy $G_\perp(B)$, while E_2 with the remaining states which complete the number of carriers, N, namely, with $N - G_\perp(B)$. Thus, the probability associated with the level E_1 becomes $G_\perp(B)/N = B/B_0$ while the probability for the level E_2 equals $[N - G_\perp(B)]/N = 1 - B/B_0$. So, the mean energy for the above region becomes $(B/B_0)E_1 + (1 - B/B_0)E_2$. In the same way we find for the region $B_0/3 \leq B < B_0/2$ the expression for the mean energy as: $(B/B_0)(E_1 + E_2) + (1 - 2B/B_0)E_3$ and in general the expression for the mean energy takes the form

$$\langle E(B)\rangle = \Theta(B - B_0)E_1 + \sum_{j-1}^{N'}\left[\Theta\left(B - \frac{B_0}{j+1}\right) - \Theta\left(B - \frac{B_0}{j}\right)\right]\bar{E}_j(B), \tag{3.6}$$

where

$$\bar{E}_j(B) = \sum_{i=1}^{j}\left(\frac{B}{B_0}E_i\right) + \left(1 - j\frac{B}{B_0}\right)E_{j+1}. \tag{3.6a}$$

An example of mean energy, as a function of B, based on the spectrum of a charged carrier in a 2DCG on which a constant magnetic field is applied perpendicularly is shown in Fig. 2.

Unfortunately the pattern of such a mean energy when fed into the MR formulae fails to reproduce the experimentally observed results for the longitudinal resistance. This is in accord with Ando's premise [4] quoting that assumption based on a phenomenological relaxation time is insufficient to provide the MR oscillations occurring in the case of a 2DCG. However, the locations of the maxima deriving from the above scheme determine the positions where the longitudinal resistance elevations take place. Moreover, the magnitudes of these elevations are more or less proportional to B^2 as determined by the experiment. In what follows we shall see the sort of modification which when applied to the above procedure for obtaining mean energies which can approach the sort of resistance pattern obtained in experimental situations.

What, essentially, we aim at is to exclude contribution to dissipative energy by the various states, extended states, apart from those located in a narrow region around the positions where the maxima in the mean energy occur, localized states. One way to proceed to this effect in mathematical form is to modify the formula for the mean energy (3.6a) utilizing a narrow Gaussian function of B centered at the value of B, between $B_0/(j+1)$ and B_0/j, for which the above mean energy acquires maximum value. Denoting this value of B by B_j the function in question takes the form

$$S_j = \exp\left[-\frac{1}{2(\sigma B_0)^2}(B - B_j)^2\right], \tag{3.7}$$

where σ is a small dimensionless number.

The modified dissipative energy, denoted by E_d, for obtaining the MR in the region of B between approximately zero and B_0 takes the form

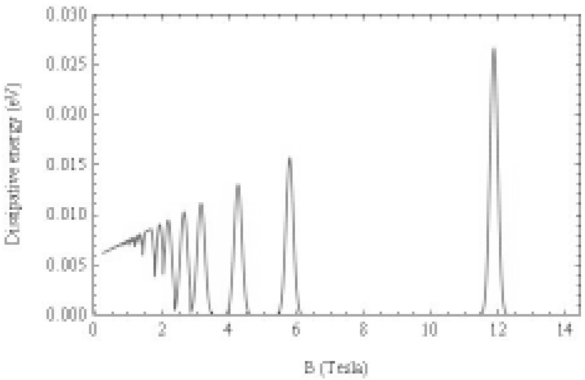

Fig. 3. Dissipative energy for the motion perpendicular to the magnetic field, as modified for yielding the longitudinal resistance peaks in the case of a 2DCG. Data as per Fig. 1 and, furthermore, $\sigma = 0.008$ determining the variance σB_0 of the Gaussian peaks. The centers of the peaks B_j are obtained from the values of B for which the mean energy in Fig. 2 acquires maximum values.

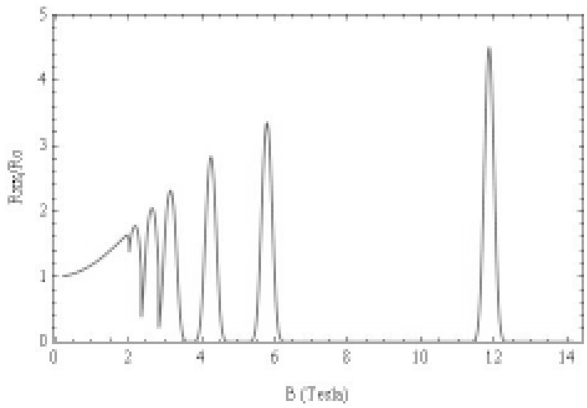

Fig. 4. Reduced longitudinal resistance exhibited by a 2DCG subjected to a perpendicular magnetic field under condition of constant current along its length. Data as per Fig. 1 and furthermore, $f_0 = \sqrt{6} - 1.7$ and mobility $\mu = 4.5 \cdot 10^3$ V s cm^{-2}.

$$E_d(B) = \sum_{j=1}^{N'} \left[\Theta\left(B - \frac{B_0}{j+1} \right) - \Theta\left(B - \frac{B_0}{j} \right) \right] \times \left(\bar{E}_j(B) + h\omega \right) S_j(B). \tag{3.8}$$

An example of such a dissipative energy as a function of B is shown in Fig. 3. With the aid of (3.8) we are able to obtain formulae for the longitudinal resistance, as will be seen in the next section. It should be noted that the above considerations bear a similarity to models, involving extended and localized states, employed for the same purpose in the broad sense that certain states contribute to current formation with little obstruction, while others inhibit [5]. Effectively, our approach is based on a phenomenological description providing a form of dissipative energy which leads to the desired MR elevations.

If we adopt the model of broadened density of states [3-6], and proceed with the evaluation of the mean energies for the various values of the magnetic field, B, the result will be essentially almost the same as the one obtained in accordance with the spectrum provided by the Landau levels (3.1). The Fermi energy is shifted half way to the next level. However, the model of broadened density of states goes further pointing to the existence of localized and extended states. The energies associated with the extended states are located around the Landau discrete levels, whereas the corresponding ones for the localized in between successive levels.

It is well known that the localized states are associated with increased current obstruction, while the extended ones with minor inhibition. It is precisely the feature of increased obstruction together with that of minimized obstruction that we simulate utilizing a mathematical form, as given in (3.7). The procedure with dissipative energies, in the way they have been formulated, constitutes a first step towards obtaining a more appropriate mean energy in terms of B, directly from energy spectrum with the aid of a probability scheme pertaining to the situation. Nevertheless, as things stand, since the dissipative energy, employed, enables obtaining patterns of longitudinal resistance, which approximate observation to a good extend, its form may serve as a guiding goal to be sought on the basis of an appropriate field of force in addition to the magnetic field.

4. MAGNETORESISTANCE ELEVATIONS

In this section we shall apply considerations drawn in the previous section with regard to the dissipative energy for obtaining MR patterns similar to experimental results. As far as the locations of the MR peaks on the magnetic field axis are concerned, as mentioned earlier, can be determined via the corresponding values of the magnetic field for which the carrier mean energy, as determined by F-D statistics, acquires maxima.

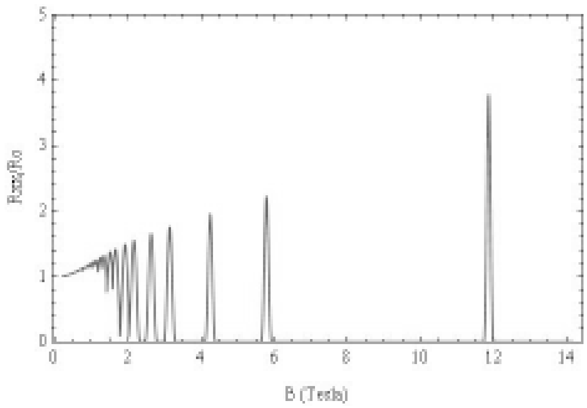

Fig. 5. Reduced longitudinal resistance for a 2DCG under condition of constant bias, with collective parameter $\xi_0 = \sqrt{6} - 0.1$ and mobility $\mu = 1\cdot10^4$ V s cm^{-2}. Rest of data as per Figs. 1,2.

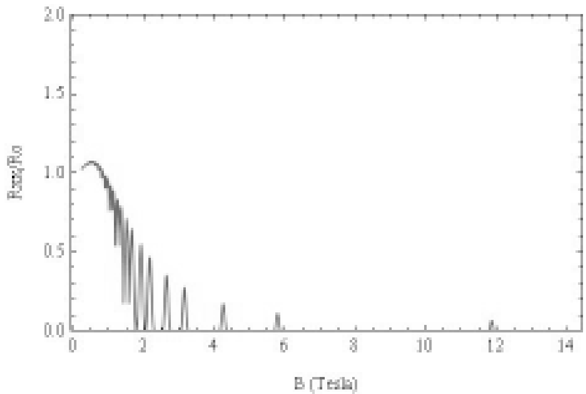

Fig. 6. Shows the effect of a particular combination of the collective parameter ξ_0 and mobility on the pattern of the longitudinal resistance in the 2DCG case under condition of constant bias. Data as per Fig. 5 apart from the mobility, which now is $\mu = 2\cdot10^4$ V s cm^{-2}.

Denoting by B_j the value of the magnetic field for which the mean energy acquires maximum value between $B_0/(j+1)$ and B_0/j we can find the required expression for B_j determining B for which $\bar{E}_j(B)$, an analytic function of B, attains maximum, and is given by

$$B_j = \frac{B_0}{2(j+1)}\left(1+\frac{j+1}{g+j-1}\right) \text{ for } j \text{ odd},$$
$$B_j = \frac{B_0}{2j}\left(1+\frac{j}{j+2-g}\right) \text{ for } j \text{ even}. \qquad (4.1)$$

Support to such coincidences can be found e.g. in Fig. 1c of reference Takano et al. [7]. In this work $n_s = 4.6\cdot10^{11}$ cm^{-2} which, assuming minute current flow through the sample, leads to B_0 = 19.04 Tesla. Utilizing, now, (4.1) with *Lande* factor g=0.95 we find for the first few peak locations lying below $B_0/3$ centered approximately at the values: 5.61, 4.26, 3.51, 2.93, … Tesla, fitting sufficiently well the corresponding experimental data. A further reference invoked to the same extent appears in the work of Chita et al. [8]. Fig. 2 whereby $n_s = 2.25\cdot10^{12}$ cm^{-2} yielding B_0 = 93.12 Tesla. In this case taking g=0.9 we are led to MR elevation centers lying below $B_0/4$, which corresponds to filling factor 4, approximately located at magnetic field values as: 20.77, 17.26, 14.31, 12.5, (10.94, 9.88), 8.85, (8.15), 7.43 Tesla. It should be noted that the locations in brackets, above, do not appear in the corresponding figure as separate peak locations, but nevertheless those associated with elevation correspond to the appropriate filling factor, provided in the reference. Analogous fittings have been performed in the case of Fig. 1 of reference Mathews et al. [9]. Here $n_s = 5.2\cdot10^{11}$ cm^{-2} yielding B_0 = 21.52 Tesla and peak centers at magnetic field values smaller than $B_0/2$ as follows: 8.69, 6.6, 4.74, 4.06, 3.28, 2.94, 2.5, 2.3, 2.03 Tesla.

However, the above coincidences do not form the rule. There are cases whereby deviations from the outcome of formula (4.1) do occur. As pointed out earlier, this sort of departure may be attributed to increase in the number of carriers in the sample with increase in the magnetic field. Such an effect follows whenever the collective parameters ξ_0 or f_0, depending on the experimental condition of constant bias or constant current, are sufficiently high so that increase in the number of carriers in the sample leads to increase in the values of the solutions for B, obtained via Eq. (3.4). These solutions determine the values of B which correspond to the middle points of the Hall plateaus. Indeed Buth et al. [10] noticed cases of movement in the Hall plateaus with increase in the field, with B_0/j suffering the larger shift toward higher values. However, in our treatment, presently, we shall not proceed to improve our calculations considering the increase in carriers, as this improvement does not affect essentially the qualitative points we wish to accentuate.

In quite a lot of experimental references certain data such as n_s, g, μ and in particular parameters contributing to the formation of the collective parameters ξ_0 and f_0 are not provided. In what follows we shall produce patterns of longitudinal resistance $R_{xx} = (MR + 1)R_0$, R_0 being the sample's resistance at B=0, for a 2DCG utilizing appropriate choice of relevant required parameters, nevertheless within range of experimental situations. Furthermore, by varying certain of the above parameters we shall be able to point out the influence of such parameters in the manifestation of certain modulations of the R_{xx} patterns, e.g. the diminishing of particular peaks. Fig. 4 provides a pattern of R_{xx} peaks utilizing relevant formulae developed earlier [1] as well as in the present work under condition of constant current. Although our evaluations refer to absolute zero temperature it would appear that within small differences are valid for low temperature.

Hitherto, we have dealt with the case of a 2DCG under condition of constant current. A case dealing with the reduced longitudinal resistance under constant bias is shown in Fig. 5. The case in question provides a similar pattern to the one obtained under condition of constant current, namely increase in B results in increase in the height of the resistance peaks. However, upon considering a situation with higher mobility, while keeping the same collective parameter ξ_0, we reach an instance whereby with increasing magnetic field the resistance elevations diminish substantially, to a point of almost disappearance. Experimental evidence to this effect can be found in Fig. 3 of the work by Ebert et al. [11]. Fig. 6 in our work provides such a case, utilizing an appropriate combination of the parameters ξ_0 and μ.

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