Rev.Adv.Mater.Sci. 8 (2004) 170-175

CONDUCTIVITY IN QUANTUM WIRES IN A HOMOGENEOUS MAGNETIC FIELD

E.P. Sineavsky[1], R.A. Khamidullin[1], T.E. Huber[2], A.A.Nikolaeva[1,3] and L. A. Konopko[1, 3]

[1]Institute of Applied Physics, Academy of Sciences of Moldova, MD-2028, Academy str. 5, Chisinau, MD-2028, Moldova
[2]Department of Chemistry, Howard University, DC 20059, 525 College St. N.W. Washington, USA
[3] International Laboratory of High Magnetic Fields and Low Temperatures, Wroclaw, Poland

Received: November 24, 2004

Abstract. The electroconductivity of quantum wires in homogeneous magnetic fields directed along the quantum wire and perpendicular to it is calculated in the model of parabolic potentials that takes into account the anisotropy of the effective mass of current carriers.
The temperature and field dependences of the conductivity are calculated for nondegenerate and degenerate quantum quasi one-dimensional systems; the results are compared with the experimental data. The features of the electroconductivity arising in 1-D dimension quantum systems are discussed in detail. In particular, it is shown that the conductivity in 1-D dimensional systems for a perpendicular magnetic field in the quantum limit $\omega_c\tau$ >>1 (ω_c- cyclotron frequency, $1/\tau$ is the probability of carriers scattering on acoustic vibrations) can be significantly higher than in bulk semiconductor systems. We assume that this is the probable reason for the sharp increase in electroconductivity observed in quantum Bi wires 50-100 nm thick in the perpendicular magnetic field.

1. INTRODUCTION

To-date the investigation of transport phenomena in low dimensional systems is a very actual problem. Such systems are perspective for applications in quantum electronics because they possess unique properties arising from the size quantization of the carriers energy spectrum.

The development of the technology for preparation of quantum wires stimulated the interest to the kinetic phenomena in low dimensional systems. Studies of the transport phenomena in Bi semimetal nanowires are undoubtedly interesting because of the fact that the free path length in these 1-D systems at low temperatures may be significantly larger than the wire radius. On the other hand, the very small effective masses of electrons facilitate the manifestations of the size quantization effect. Experimental investigations of the electric conductivity in quantized Bi wires were carried out in [1,2]; the resistance peculiarities of the single Bi nanowires in homogeneous magnetic field of various orientation were studied in a wide temperature range (1.4K $\le T \le$ 300K) [1-6].

Previous theoretical investigations were mainly devoted to the calculation of the relaxation time with account of various scattering mechanisms [7-9]. In the present paper, the electric conductivity of quantized wires in homogeneous magnetic field (|| and ⊥) is investigated in the framework of the Kubo formalism taking in consideration the carrier scattering on long-wave acoustic vibrations. The theoretical results are compared with experimental data [7-9].

Corresponding author: A.A.Nikolaeva, e-mail: A.Nikolaeva@mail.ru

© 2004 Advanced Study Center Co. Ltd.

2. RESULTS

The conductivity in quantum wires has been studied taking into account the interaction of carriers with acoustic vibrations. The conductivity tensor was calculated with the aid of the Kubo formula using the cumulant averaging over the vibrational subsystem [10]. In the plane perpendicular to the axis of the size-confined system the quantum wire potential may be described by an elliptical paraboloid:

$$U(y,x)=\frac{m_z\omega^2}{2}z^2+\frac{m_y\omega_1^2}{2}y^2. \quad (1)$$

In the transverse magnetic field the conductivity tensor is defined by the relation:

$$\sigma_{xx}=\frac{e^2\hbar^2\beta}{Vm^{*2}}\sum_\alpha k_x^2 n_\alpha(1-n_\alpha)\tau_\alpha, \quad (2)$$

where α are quantum numbers describing a definite electron state, $1/\tau_\alpha$ is the probability of the carrier scattering on phonons per unit of time, n_α is the equilibrium distribution function of a charged particle in a system with the size quantization in presence of the magnetic field.

$$m^*=m_x(1-\Delta);$$

$$\Delta=\frac{m_y}{m_x}\left(\frac{\omega_y}{\omega_1}\right)^2,$$

$$\omega_y=\frac{eH}{m_yc},$$

$$\beta=\frac{1}{k_0T}.$$

In the case of elastic scattering of electrons on long-wave acoustic vibrations, the conductivity of the non-degenerate electron gas is described by the following relation

$$\sigma_{xx}=\frac{4e^2n_e\hbar^3p\nu^2\sqrt{\beta2\pi}}{E_1^2m_x\sqrt{m_xm_ym_z\omega\omega_1}}(1+\Delta)^{-7/4}J(\delta_0),$$

$$J(\delta_0)=\int_0^\infty\frac{xdxe^{-x}}{1+e^{-\delta_0x}},\quad \delta_0=\frac{4}{\hbar\omega_1\beta}\times\frac{\Delta}{(1+\Delta)^{1/2}}, \quad (3)$$

here E_1 is the deformation potential constant, n_e is the electron concentration, ρ is the density of the quantum system under examination, v is the sound velocity. In accordance with Eq. (3), the relation between the magnetoresistance $R(H)$ and $R(0)$ (where $R(0)$ is the magnetoresistance in the absence of the magnetic field) can be written as:

$$\frac{R(H)}{R(O)}=\frac{(1+\Delta)^{7/4}}{2J(\delta_0)}. \quad (4)$$

For a degenerate electron gas

$$\frac{R(H)}{R(O)}=\frac{1}{2}(1+\Delta)^{9/4}(1+e^{-\delta}),$$

$$\delta=\frac{4\xi_0}{\hbar\omega_1}\times\frac{\Delta}{(1+\Delta)^{3/2}}, \quad (5)$$

where ξ_0 is the chemical potential in the absence of the magnetic field.

As it follows directly from Eq. (3), the conductivity and the carrier mobility decrease with temperature increase (σ_{xx}~$T^{-1/2}$) in the absence of magnetic field (Δ =0), (see the definition of Δ above introduced). This effect was observed in Bi nanowires at T > 50K [3]. When Δ << 1 and the size quantization frequency ω_1 is much bigger than the cyclotron frequency ω_y, the dependence of $R(H)/R(0)$ on the magnetic field is described by a non-monotonic function (Fig. 1). Such a dependence was also observed in Bi nanowires [11]. At the same time, this type of $R(H)/R(0)$ behavior appears only in the transverse magnetic fields as a result of the wave function asymmetric character relative to the direction of the wave vector κ_x, i.e. $|\psi_{\kappa x}(r)|^2\neq|\psi_{-\kappa x}(r)|^2$. If the elastic scattering of carriers occurs on long-wave vibrations (i.e. for isotropic effective masses and ω_1=ω) and the magnetic field is directed along the quantum wire axis, the reverse relaxation time is defined as

$$\frac{1}{\tau_\alpha}=\gamma\frac{1}{|k_x|},$$

$$\gamma=\frac{2E_1^2\mu L_E}{\beta\rho\upsilon^2\hbar^3}\int d\vec{r}\left|\psi_{NK_x}(r)\right|^4, \quad (6)$$

where $\psi_{NKx}(r)$ is the wave function of a carrier in the longitudinal magnetic field. This function was derived in [12] for the cylindrical quantum wire modeled by an infinite potential well and expressed via Kummer functions $W(a,b,t)$. If the potential of a quantum-sized system is described by relation (1) (m_z=m_y=μ, ω=ω_1), then the according to Eq. (6) the expression for γ in the lowest magnetic - sized state E_0 can be written in the form:

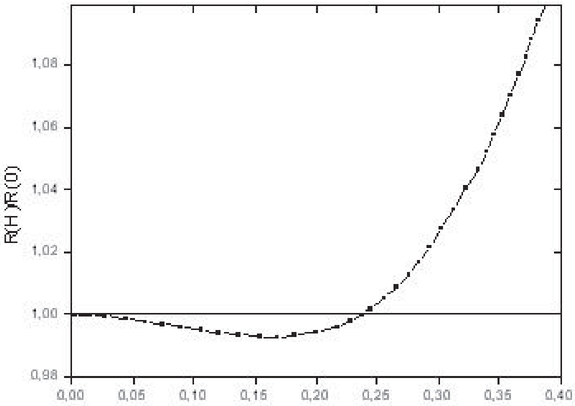

Fig. 1. $R(H)/R(0)$ as a function of $x=(\omega_y/\omega_1)$ according (4) calculations were made for $(m_y/m_x)=2$, $(kT/\hbar\omega_1)\approx 0.25$.

$$\gamma = \frac{\mu E_1^2 R^2 A^4}{\beta\rho\upsilon^2\pi\hbar^3\delta}\int_0^q d\tau e^{-2\tau} W^4(a,1,\tau),$$
$$a = \frac{1}{2} - \frac{1}{\delta\hbar\omega_c}\left[E - \frac{\hbar^2 k_x^2}{2\mu}\right],$$
$$q = \frac{1}{2}\delta\left(\frac{R_0}{R}\right)^2,$$
$$R = \frac{c\hbar}{eH}, \tag{7}$$
$$\delta^2 = 1 + \left(\frac{2\omega}{\omega_c}\right)^2,$$

$\hbar\omega_c = \frac{eH\hbar}{\mu c}$, R_0 is the nanowire radius,

$$A^{-2} = \left(\frac{R^2}{\delta}\right)^{-1}\int_0^q e^{-\tau} W^2(a,1,\tau)d\tau,$$

As a result, the conductivity in a longitudinal magnetic field in the lowest zone of the quantized wire has the form:

$$\sigma_{xx} = \frac{4e^2}{\pi^2 R_0^2 \beta\hbar^2\gamma} - \ln[1+e^{-\xi}] \tag{8}$$

where $\xi=\beta(\varepsilon_0-\xi)$, ξ is the chemical potential.

According to relationship (8) for a non-degenerate electron gas, the following relation takes place

$$\left(e^{-\vec{\xi}} = n_e(\pi R_0)^2\left(\frac{\hbar\beta}{2\pi\mu}\right)^{1/2} << 1\right)$$
$$\sigma_{xx} = \frac{2e^2 n_e}{\hbar y}\left[\frac{2}{\pi\beta\mu}\right]^{1/2}. \tag{9}$$

For a degenerated electron gas ($e^{-\vec{\xi}} >> 1$)

$$\sigma_{xx} = \frac{4e^2}{\hbar^2\pi S\beta y}(-\vec{\xi}),$$
$$S = \pi R_0^2. \tag{10}$$

Analytical calculation of γ (7) can be performed for particular cases. When $q >> 1$, for the case of an infinite potential well ($\delta=1$), $R_0 >> \frac{70}{\sqrt{H}}(nm)$ (H is measured in Tesla) we find

$$\gamma = \frac{E_1^2\mu}{2\pi\beta\rho\upsilon^2\hbar^3 R^2}. \tag{11}$$

Consequently,

$$\sigma_{xx} = \sigma_0^{(bulk)} 2\left(\frac{R}{R_0}\right)^2,$$
$$\sigma_0^{(bulk)} = \frac{2e\hbar\rho\nu^2}{\pi\mu E_1^2}\ln\left[1+e^{-\vec{\xi}}\right], \tag{12}$$

$\sigma^{(bulk)}$ is the conductivity of a bulk semiconductor material in the magnetic field.

As directly follows from Eq. (12), the conductivity in quantum wires is smaller than in bulk materials because of the stronger carrier localization. In this case σ_{xx} decreases when the wire radius increases and, consequently, the resistance increases. The latter phenomenon was repeatedly observed in Bi nanowires [4, 11].

For weak magnetic fields q<<1, the Kummer functions $W(a,1,\tau)$ for τ <<1 can be expanded over the Bessel functions [13]. In particular, for $H = 0$ we arrive at the relation

$$\gamma = \frac{4E_1^2\mu}{\pi R_0^2\beta\rho\nu^2\hbar^3}.$$
$$\frac{1}{J_1^4(\beta_0)}\int_0^1 d\tau\tau J_0^4(\beta_0\tau), \tag{13}$$

where $J_n(Z)$ is the Bessel function, β_0= 2.4, b is the 1st root of Bessel function $J_n(z)$ (for the lowest en-

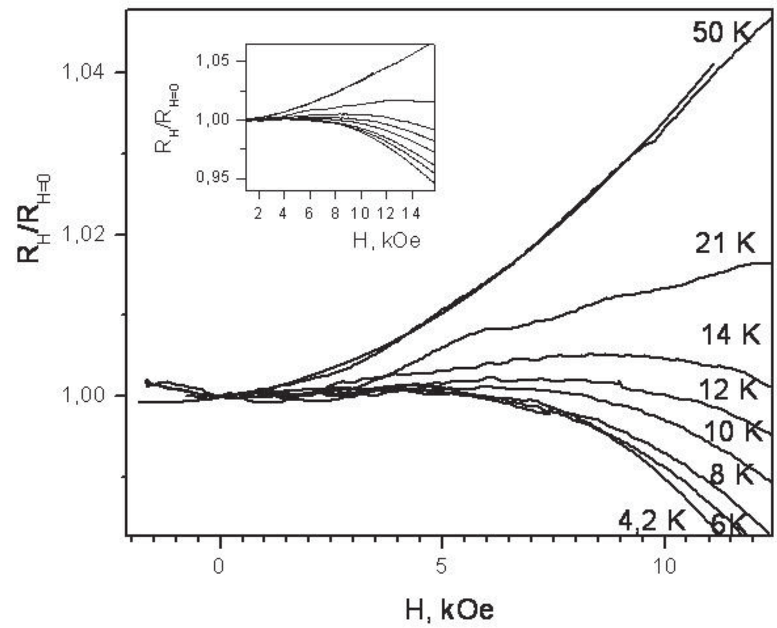

Fig. 2. Longitudinal magnetoresistance (LMR) of the H II I Bi-wire with *d*=75nm at different temperatures. Insert reflects the initial sector of the curves.

ergy state). Hence, for a non-degenerate electron gas

$$\sigma_{xx}^{(ng)} \cong \frac{e^2\rho\nu^2\hbar^2}{\mu E_1^2}\sqrt{\frac{\beta}{2\pi\mu}}N_e\,, \tag{14}$$

here $N_e=N/L_x$ is the linear concentration of carriers.

For the degenerate electron gas we derive

$$\sigma_{xx}^{(dg)} = \frac{e^2\hbar\rho\upsilon^2}{2\pi E_1^2\mu}\beta(\xi-\varepsilon_0). \tag{15}$$

If one takes into account the lowest over the magnetic field magnitude terms, then we obtain

$$\sigma_{xx} = \sigma_{xx}^{(0)}\left\{1-0.01\left(\frac{R_0}{R}\right)^4\right\}, \tag{16}$$

here $\sigma_{xx}^{(0)}$ is the quantum wire conductivity in the absence of magnetic field .

According to Eq. (16)

$$\frac{\Delta R(H)}{R(0)} = 0.01\left(\frac{R_0}{R}\right)^4 \sim H^2. \tag{17}$$

Just the same dependence of the relative resistance change on the magnetic field was observed in Bi nanowires arrays in a wide temperature range [6], and in Bi- nanowires in glass cover at *T*>20K (Fig. 2).

3. EXPERIMENT

The single Bi nanowires (*d* ~ 50-200 nm) in glass cover were obtained utilizing the Ulitovsky method by liquid phase casting [5]. Bi-wires were single crystals and had the same orientation: the angle between the wire axis and the bissector axis C_1 in the bissector-trigonal plane was ~20^0 . One of the binary axes C_2 was perpendicular to the wire axis and the trigonal axis C_3 makes up an angle of ~70^0 with the wire axis. This was established by the aid of the X-ray diffraction experiments, Shubnikov de Haas (SdH) oscillations at (*H* II *I*), and the angular diagrams rotations of the transverse magnetoresistance (TM) [5].

According to the SdH oscillations for wires with diameters up to 80 nm, the transition semimetal-semiconductor (TSS) arising from the quantum-mechanical nature of the carriers motion was not observed. Such a transition was previously predicted theoretically in [14].

It is known that the TM in bulk Bi- samples and microwires strongly increases in both weak and strong magnetic fields, especially at low temperatures (4.2K). TM was shown to decrease in the case when the wire diameter *d* is lower than100 nm at both 300K and 4.2K. The region of quadratic growth of the resistance decreased with *d* decreasing at all directions of the magnetic field (*H*⊥*I*). As it is seen from Fig. 3, the TM *R*(*H*) curve contains several spe-

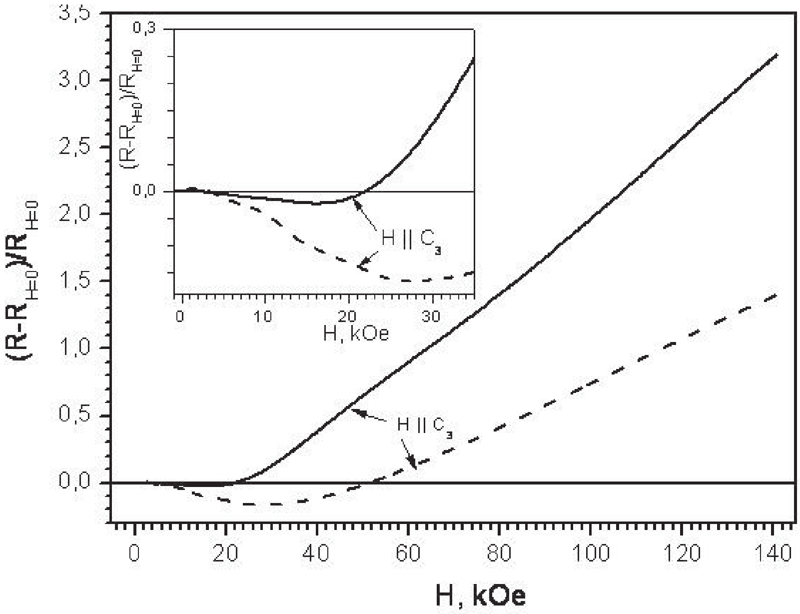

Fig. 3. The field dependences of the TMR of nanowires Bi. 1. d=50nm, T =1.5K; 2. d =75nm, T =4.2K. Insert reflects the initial sector of the curves.

cific points – (1) the point corresponding to the minimal value of the negative magnetoresistance and 2) a point where TM=0. When the wire diameter d decreases from 75 nm to 50 nm, these points shift to the region of strong magnetic fields and the minimal value of the magnetoresistance increases in magnitude.

We should emphasize that the intensity of the magnetic field corresponding to the above TM minimum agrees with the value of the field at which the following equality is realized:

$$d=R_c=cP_F/eH,$$

where R_c is the cyclotron radius, P_F is the Fermi pulse of holes of the corresponding direction at $H \perp I$. In future we plan to carry out an examination of the second specific point ($R(H_\perp)$=0).

Fig. 2 shows the field dependences of the LMR $R(H)$ of the Bi-wire with d=75nm at different temperatures. At temperatures above 20K at weak magnetic fields, as mentioned above, $R(H)$~H^2. This behavior agrees well with both experimental nanowire arrays [6] and theoretical ideas described above.

4. SUMMARY

The mechanism of the elastic scattering of the carriers on long-wave acoustic vibrations allows us to understand qualitatively the dependence of the electric conductivity of quantized Bi wires on temperature as well as on the magnitude of the homogeneous magnetic field oriented ⊥ and II to the wire axis. The model suggested qualitatively explains the experimental data on the conductivity in Bi nanowires. In order to compare quantitatively the experimental data and the theory (particularly for the case of R_0<R and low temperatures (T<10K)), the scattering of carriers on the surface of the quantum-sized system should be taken into account. We intend to carry out these investigations in future.

ACKNOWLEDGEMENTS

The work was supported by the Civilian Research Development Foundation, CGP- grant # MO-E1-2603-SI-04.

REFERENCES

[1] Yu-Ming Lin, Stephen B. Cronin, Jakie Y. Ying, M.S. Dresselhaus and Joseph P. Heremans // *Appl. Phys. Letters* **76** (2000) 3944.

[2] Yu-Ming Lin and M.S. Dresselhaus // *Appl. Phys. Letters* **83** (2003) 3567.

[3] Zhibo Zhang, Xiangzhong Sun, M.S. Dresselhaus, Jackie Y.Ying and J. Heremans // *Phys. Rev. B* **61** (2000) 4850.

[4] J. Heremans, C.M. Thrush, Yu-Ming Lin, S. Cronin, Z. Zang, M.S. Dresselhaus an J.F. Mansfield // *Phys. Rev. B* **61** (2000) 2921.

[5] N.B. Brand, D.V. Gitsu, A.A. Nikolaeva and Ya.G. Ponomarev // *Sov. Phys. JETP* **45** (1977); *Zh. Eksp. Teor. Fiz.* **72** (1977) 2332.
[6] J. Heremans, and C.M. Thrush, Z. Zang, X.Sun, M.S. Dresselhaus, J.Y.Ying and D.T.Morelli // *Phys. Rev. B* **58** (1998) R10091.
[7] Henrik Bruns, Karsten Flensberg and Henrik Smith // *Phys. Rev. B* **48** (1993) 11144.
[8] A.Gold and A. Ghazali // *Phys. Rev. B.* **41** (1990) 7626.
[9] P.J.M. Peters, P. Scheuzger, M.J. Lea, Yu.P. Monarkha, P.K.H. Sommerfeld and R.W. van der Heijden // *Phys. Rev. B* **50** (1994) 11570.
[10] E. Sinyavskii and R.A. Khamidullin // *Semiconductors* **36** (2002) 924, Russ. Phys. FTP **36** (2002) 989.
[11] A.A. Nikolaeva, D.V. Gitsu, T.E. Huber and L.A. Konopko // *Physica B* **346-347** (2004) 282.
[12] N.C. Constantinou, M. Massale and D.R. Tilley // *J. Phys: Condens. Matter* **4** (1992) 4499.
[13] *Hundbook of mathematical functions with formulas graphs and mathematical tables*, ed. by Milton Abramowitz and Irene A. Stegun. (1964).
[14] Yu-Ming Lin, Xiangzhong Sun and M.S. Dresselhaus // *Phys. Rev.* **62** (2000) 4610.