Distribution of contact stresses under circular flexible plate lying on a two-layer foundation, with soft interlayer thickness and substrate stiffness taken into account

Sergey M. Aizikovich Andrey S. Vasiliev Sergey S. Volkov
saizikovich@gmail.com

Abstract

We consider axisymmetric contact problem about bending of a plate lying on inhomogeneous foundation. Foundation is modeled by elastic inhomogeneous soft interlayer and elastic homogeneous half-space. Interlayer can be stratified or continuously inhomogeneous with arbitrary varying elastic properties. Layer also can be significantly softer than an underlying half-space. Plate bends under the action of distributed load and elastic response from a foundation. Analytical solution of the problem is constructed using bilateral asymptotic method [1]. Analytical expressions for contact stresses and deflection of the plate are provided. Constructed solution is bilateral asymptotically exact both for large and small values of characteristic geometric parameter of the problem (ratio of layer thickness to plate radius). Also it is effective both for flexible and stiff plates. Numerical results demonstrates that found approximations for kernel transform of integral equation of the problem allows one to construct analytical solution that is effective in the whole range of values of inhomogeneous layer thickness and plate stiffness.

1 Introduction

Problem of plate bending on isotropic homogeneous elastic foundation was considered in [2] and developed further. Most of the known solutions are effective only for rigid plates. And very few, in particular those that described in [3] and [4], are efficient either for flexible or rigid plates, each in its own case. There are number of recent investigations in this area ([5], [6]), indicating interest in the solution of this problem. In this work we describe use of approach, based on bilateral asymptotic method for solution of dual integral equations [1], which allows one to construct analytical solution of the problem in unified form, effective for any values of geometric and mechanical properties.

2 Problem statement

Circular plate of radius R and constant thickness h lying on boundary Γ of elastic half-space Ω, consisting of inhomogeneous soft layer (coating) with thickness H and homogeneous half-space (substrate). We use cylindrical coordinate system r, φ, z, where z axis is perpendicular to plane Γ and passes through the center of the plate. Coordinate r is related to R, and z is related to H. Plate bends under axisymmetric distributed load $p(r)$ and response from the layer. Function $w(r)$ describes deflection of the plate.

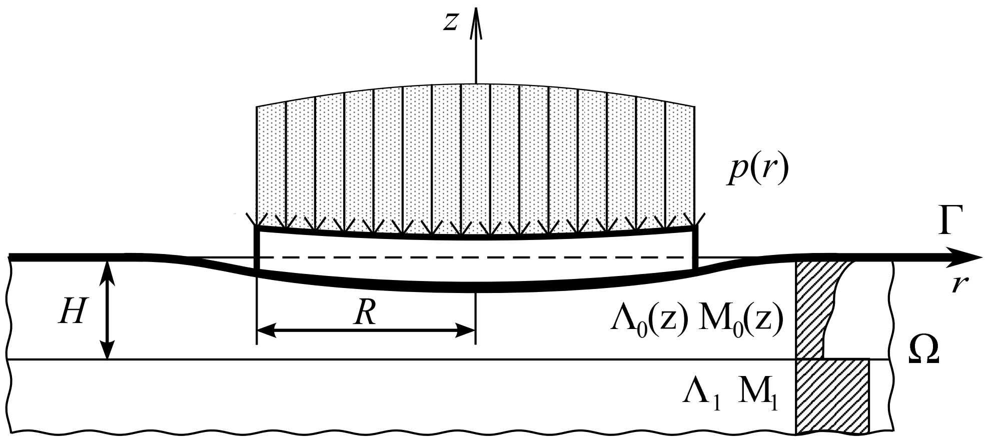

Figure 1: Problem statement.

Lame parameters $\Lambda(z)$, $\mathrm{M}(z)$ of the foundation vary with depth according to law:

$$\Lambda(z) = \begin{cases} \Lambda_0(\mathrm{z}), & -1 \le z \le 0 \\ \Lambda_1, & -\infty < z < -1 \end{cases}$$
$$\mathrm{M}(z) = \begin{cases} \mathrm{M}_0(\mathrm{z}), & -1 \le z \le 0 \\ \mathrm{M}_1, & -\infty < z < -1 \end{cases} \tag{1}$$

Young's modulus of the foundation is

$$E_1 = \beta E_0(-1).$$

Here β denotes magnitude of the leap in elastic properties on layer-substrate boundary. The larger β, the more stiff underlying half-space is.

Deflection of the plate must satisfy following boundary conditions:

$$\left.\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)\right|_{r=1} = 0, \quad \left.\frac{d}{dr}(\nabla w)\right|_{r=1} = 0 \tag{2}$$

where ν = Poisson's ration of the plate. Conditions (2) correspond to free plate edges.

Due to these conditions, solution of the problem is reduced to following system of equations:

$$\mathbf{D}w(r) = p(r) - q(r), \ \ 0 \le r \le 1 \tag{3}$$

$$\begin{cases} \int\limits_0^\infty Q(\alpha)L(\alpha\lambda)J_0(\alpha r)d\alpha = sw(r), \ \ 0 \le r \le 1 \\ \int\limits_0^\infty Q(\alpha)J_0(\alpha r)d\alpha = 0, \ \ r > 1 \end{cases} \tag{4}$$

where $p(r)$ = applied load; $q(r)$ = contact stresses under the plate; $w(r)$ = deflection function; $L(\alpha\lambda)$ – kernel transform; $\lambda = H/R$; s is the bending stiffness of the plate:

$$s = \Theta R^3 D^{-1} \tag{5}$$

where D = cylindrical stiffness of the plate; Θ is given by

$$\Theta = 2\mathrm{M}_0 \left(\Lambda_0 + \mathrm{M}_0\right) \left(\Lambda_0 + 2\mathrm{M}_0\right)^{-1}.$$

Dual integral equation (4) defines relation between contact stresses and plate deflection. Relations between $Q(\alpha)$ and contact stresses $q(r)$ are:

$$\begin{aligned} Q(\alpha) &= \int_0^1 q(\rho) J_0(\alpha\rho)\rho d\rho, \\ q(r) &= \int_0^\infty Q(\alpha) J_0(\alpha r)\alpha d\alpha. \end{aligned} \tag{6}$$

3 Constructing a solution

It is shown [7] that the solution of described problem can be written out as:

$$\begin{aligned} M_r &= \left(\tfrac{D}{R^2}\right) \textstyle\sum_{m=0}^{M} w_m A_m k_m^2 \left[\tfrac{v-1}{k_m r} V_1\left(k_m r\right) + V_0\left(k_m r\right)\right], \\ M_\varphi &= \left(\tfrac{D}{R^2}\right) \textstyle\sum_{m=0}^{M} w_m A_m k_m^2 \left[\tfrac{v-1}{k_m r} V_1\left(k_m r\right) + v V_0\left(k_m r\right)\right]. \end{aligned}$$

where

$$V_i\left(k_m r\right) = J_i\left(k_m r\right) + B_m I_i\left(k_m r\right), \quad i = 0,\ 1.$$

However, obtaining accurate solution for average values of λ requires high accuracy of approximation used for kernel transform. Below, we show that, using recent results in approximation [8] it is possible to obtain a solution, which is effective for all possible values of characteristic parameters of the problem.

4 Numerical results

4.1 Kernel transforms

Let's consider case of soft elastic homogeneous layer lying on harder elastic foundation. Elastic layer is considered to be softer 2, 100 or 1000 times than a substrate. We also consider that layer is tightly coupled to substrate (we can consider in similar way that layer lie freely on substrate).

For estimating error of approximation we use following expression:

$$\Delta_L(u) = \left| \frac{L_N(u) - L(u)}{L(u)} \right| \cdot 100\% \tag{7}$$

The maximum error of approximation shown at Figure 2 is less than 3.5%. In case of $\beta =$ error estimate of 3.5% can be achieved at $N = 1$ (1.4% at $N = 8$), though in the case of $\beta = 1000$ to reduce error to 3.1% we need to take $N = 20$.

It is shown [9] that with growth of β kernel transforms are getting close to limiting case, which is corresponds to non-deformable foundation. Below we show that values of contact stresses for $\beta = 1000$ and non-deformable foundation are close to each other.

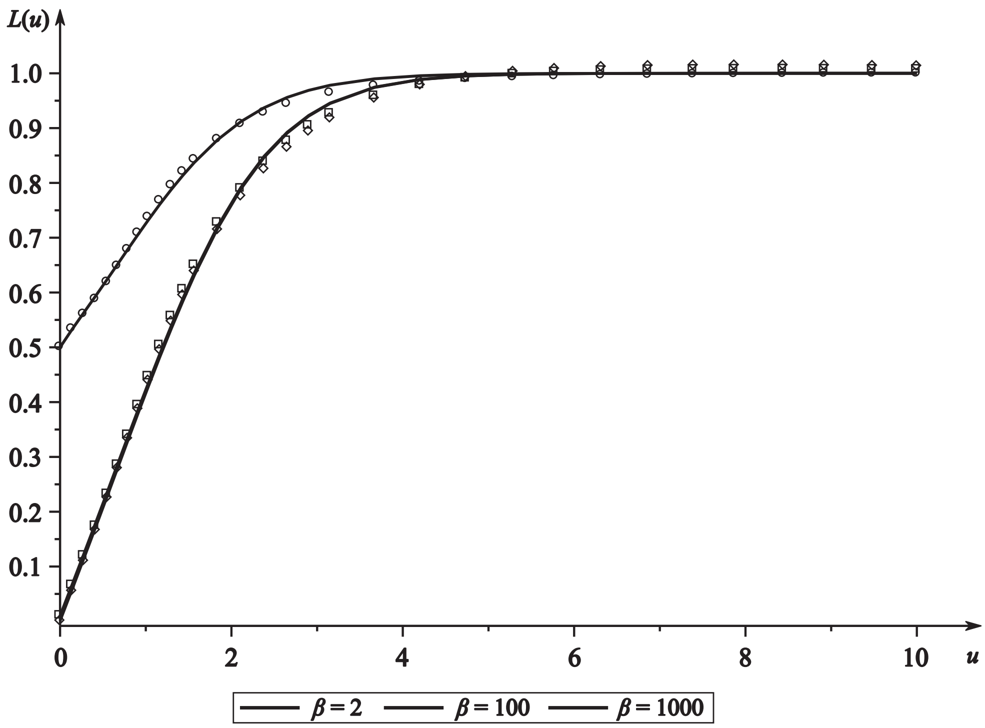

Figure 2: Kernel transforms of integral equation for $\beta = 2, 100, 1000$; lines – exact values, dots – approximation.

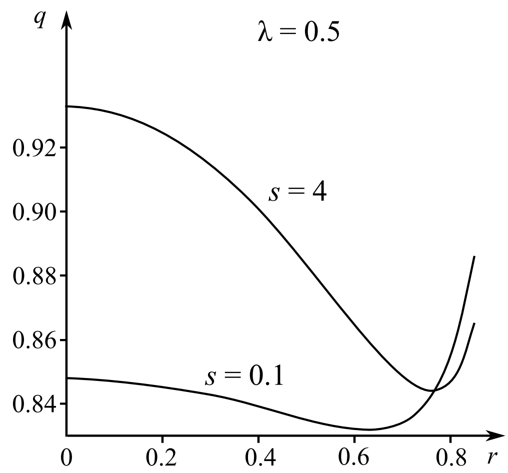

Figure 3: Contact stresses for homogeneous layer on elastic foundation, $\beta = 1000$, $\lambda = 0.5$.

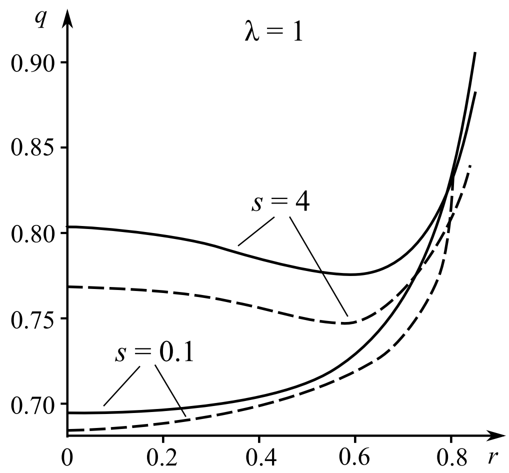

Figure 4: Contact stresses for homogeneous layer, $\lambda = 1$; solid lines – elastic foundation, $\beta = 1000$; dashed lines – non-deformable foundation (from [10], given for comparison.

4.2 Contact stresses

Let's consider case when plate is subjected to uniform load. Elastic layer is considered homogeneous, and softer 1000 times than a substrate.

On Figures 3-4 solid lines designate contact stresses for both flexible and stiff plates

($s = 0.1$ and $s = 4$, respectively), which are calculated with approximated kernel transform for $\beta = 1000$ (shown on Figure 2). On Figure 4 dash line designates results obtained by orthogonal polynomials method in [10]. Analysis of Figure 4 shows that difference between these results is less than 6.5%.

5 Conclusions

Expressions for determining contact stresses, which are plotted at Figures 3-4, are constructed using bilateral asymptotic method, which is proven to be convergent both for large and small values of characteristic geometric parameter λ of the problem. Comparison performed above shows that method is also effective for average values of λ. So it can be concluded that using sufficiently accurate approximation for kernel transform for integral equation (4) with bilateral asymptotic method it is possible to obtain solution of the problem which is effective for all possible values of λ, and both for flexible and stiff plates.

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Sergey M. Aizikovich, Research Institute of Mechanics and Applied Mathematics, Southern Federal University, Stachki prospekt 200/1, Rostov-on-Don 344090, Russia

Andrey S. Vasiliev, Research Institute of Mechanics and Applied Mathematics, Southern Federal University, Stachki prospekt 200/1, Rostov-on-Don 344090, Russia

Sergey S. Volkov, Don State Technical University, Gagarin sq. 1, Rostov-on-Don 344000, Russia