Fatigue strength criterion based on the energy conservation law

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Abstract

The energy conservation law applied to the fatigue problem can be formulated as follows. The total mechanical energy, expended during the cyclic loading, is consumed to accumulate the heat and internal (latent) energy. For the fracture moment the energy conservation law is expressed in limit values and it is considered as fatigue strength criterion. Assuming that the heat portion of energy is given out to the environment during the cyclic loading, so it has no effect on the fracture process, the fracture criterion is simplified. The experimental results of latent energy accumulation during cyclic loadings at the fracture moment for different construction steels are used to concretized the functions and parameters of the fatigue criterion. The theoretical curves of latent energy accumulation and fatigue curves are plotted. They are compared with the corresponding experimental results.

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Applied to the problem of cyclic deformation and fracture of a specimen, the energy consumption law can be formulated in the following manner. When an element of a system (specimen) goes from the initial state (initial loading) to the final state (fracture) a small increment of internal energy du is equal to the sum of increments of strain energy δw and the heat δq removed from the element of the system

$$du = \delta w - \delta q, \tag{1}$$

where $\delta w = \sigma_{ij} d\varepsilon_{ij}$, σ_{ij} are components of stress tensor, $d\varepsilon_{ij}$ are components of strain increment tensor.

Integrating (1) from the initial (marked 0) to the fractured state (marked *), we will receive

$$\Delta u_* = w_* - \Delta q_*, \tag{2}$$

where

$$\Delta u_* = \int\limits_{u_0}^{u_*} du = u_* - u_0, w_* = \int\limits_{0}^{w_*} \delta w, \Delta q_* = \int\limits_{q_0}^{q_*} \delta q, \tag{3}$$

Introducing notations $\Delta q_* = w_1$, $\Delta u_* = w_2$, the energy conservation law (2) can be rewritten as [1, 2]

$$w = w_1 + w_2. \tag{4}$$

So the deformation energy w is the summary of the heat energy w_1 and the energy w_2 which is consumed to increase the latent (internal) energy or stored energy. The latent energy is supplied for the generation of different types of defects which evaluate the damage state of the material. When the damage state achieves the critical value, the specimen is fractured.

In the case of cyclic loadings, the energy consumption was calculated by the procedure suggested in [3]. The loading cycle is considered starting from zero, and the assumption is accepted that the sizes of the hysteresis loop from cycle to cycle are constant. In this case, according to [3], we can obtain the summary value of the energy consumption for N loading cycles

$$w = N \int_0^{\varepsilon} \sigma d\varepsilon. \tag{5}$$

Specifying $\sigma = C \cdot \varepsilon^{1/\beta}$ (C and β are constants), we obtain from (5)

$$w = \frac{N\beta}{C^{\beta}(\beta+1)} \sigma^{\beta+1}. \tag{6}$$

Stress-strain diagrams depending on mechanical properties of materials will be defined by varying of parameters C and β. Material constants are choused as: $C = 180[MPa]$ and $\beta_1 = 3$, $\beta_2 = 5$, $\beta_3 = 10$. The corresponding diagrams are shown on Fig. 1.

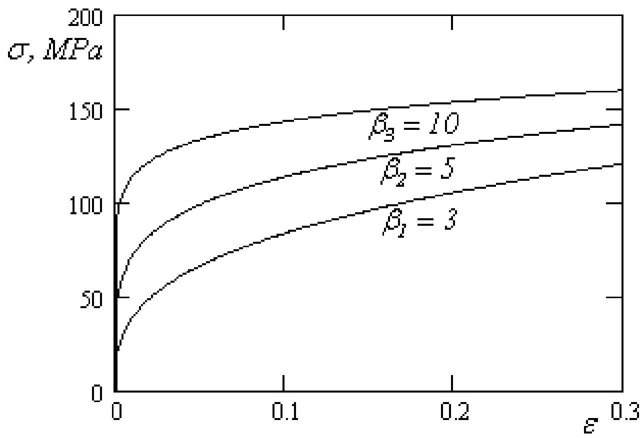

Figure 1: Stress-strain diagrams for $C = 180MPa$ and $\beta_1 = 3$, $\beta_2 = 5$, $\beta_3 = 10$.

The results of experiments on latent energy carried out in recent years [4-6] show that the relation of the latent energy or the stored energy is expressed in the form of logistics function. Our experiments [7] conducted on cyclic bending of specimens made of construction materials show that the relations of different physical and mechanical characteristics from number of cycles has two precisely expressed regions. Similarly to the logistics function these relations have the precisely expressed point of inflection. These results are the additional confirmation for the introduction of the logistics function when formulating the fatigue strength criterion. We assume that metals cyclic strength is defined by the value of latent energy, when it is reached the critical value.

It is well known that the logistics function describes the different processes which lead to saturation [8], and it is the solution of the following kinetic equation

$$\frac{dw_2}{dN} = Aw_2(w_* - w_2). \tag{7}$$

where A and w_* are constants. In the common case A and w_* can be considered as functions of stress $A = A(\sigma)$, $w_* = w_*(\sigma)$.

Solving equation (7) with the initial conditions $N = 0$, $w = w_0$, we will have the following relation, expressed in the form of logistic function

$$w_2 = \frac{w_*}{1 + \left(\frac{w_*}{w_0} - 1\right) \cdot e^{-ANw_*}}. \tag{8}$$

Taking into account the relations (6) and (8), the energy conservation law (4) can be written as

$$\frac{N\beta}{C^\beta(\beta+1)}\sigma^{\beta+1} = w_1 + \frac{w_*}{1 + \left(\frac{w_*}{w_0} - 1\right) \cdot e^{-ANw_*}}. \tag{9}$$

Assuming that the heat energy w_1 is given out during the cyclic experiment, so it has no effect on the fracture process, the relation (9) is simplified in the form

$$\frac{N\beta}{C^\beta(\beta+1)}\sigma^{\beta+1} = \frac{w_*}{1 + \left(\frac{w_*}{w_0} - 1\right) \cdot e^{-ANw_*}}. \tag{10}$$

Transforming (10), we will receive the fatigue fracture criterion

$$\sigma = \left[\frac{C^\beta(\beta+1)}{N\beta}\frac{w_*}{1 + \left(\frac{w_*}{w_0} - 1\right) \cdot e^{-ANw_*}}\right]^{\frac{1}{\beta+1}}. \tag{11}$$

To describe the evolution of latent energy another kinetic equation is used

$$\frac{dw_2}{dN} = kw_*e^{-kN}. \tag{12}$$

Solving equation (12) with the initial conditions $N = 0$, $w_2 = 0$, we will have relation, expressed in the form of exponent function, which also lead to the energy saturation

$$w_2 = w_*\left(1 - e^{-kN}\right). \tag{13}$$

Comparing (6) and (13) and assuming that the heat energy w_1 is given out during the cyclic experiment, we will receive the fatigue fracture criterion in the form

$$\sigma = \left[\frac{C^\beta(\beta+1)}{N\beta}w_*\left(1 - e^{-kN}\right)\right]^{\frac{1}{\beta+1}}. \tag{14}$$

Theoretical fatigue curves, according to the fatigue fracture criterions (11) (curve 1) and (14) (curve 2), are shown on Fig. 2.

The following values of constants were accepted: $A = 10^{-20} \cdot [m]^3 \cdot [J \cdot cycles]^{-1}$, $C = 180[MPa]$, $w_0 = 1000 \cdot 10^6 \cdot [J] \cdot [m]^{-3}$, $w_* = 7353 \cdot 10^6 \cdot [J] \cdot [m]^{-3}$, $k = 10^{-6}$, $\beta = 5$. The values of initial and saturation energy for different steels were defined experimentally in [9].

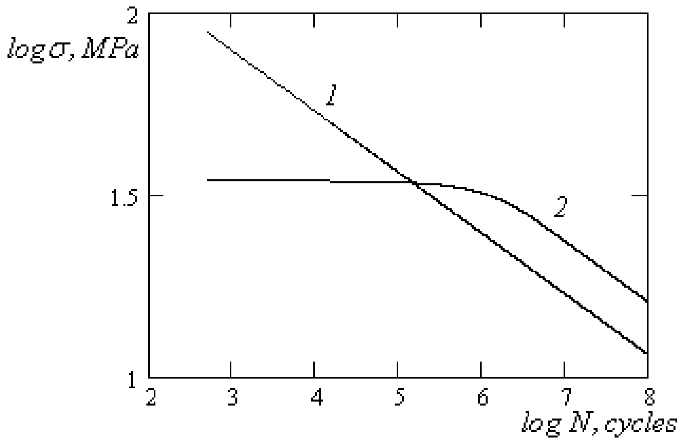

Figure 2: The fatigue curves according to the criterions (11) (curve 1) and (14) (curve 2).

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