

# Screen separation: computational model building experience

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## Abstract

The work is revealing authors' experience in developing of computational resource-effective algorithm for modeling of vibrational screen separation of granular materials. It is shown that numerical interpretation of several analytical models could lead to a new approach which is not as resource-intensive as widespread multifunctional computational methods (including DEM) are. Being based on the problem-oriented classical models the algorithm presented here appears to demand for little deployment time compared with universal approaches for it doesn't require vast calibration and adjustment procedures to precede its application. Another significant advantage of the algorithm is the fact that providing precise enough results simulations it could be performed with no extensive computational resources. Fast simulations allow performing automatic estimation of optimal screening devices design and technological parameters provided that search area and goal function are given. The algorithm allows parcelwise improvements which were illustrated with advanced openings passing and vibrational separation submodels.

## 1 Introduction

Screening is vital for the preparation of mineral resources for the dressing process. These operations directly account for 4% to 6% of the total electricity consumption for disintegration. In addition, quality screening processes directly reduce energy consumption in open and especially in closed circuit crushing, since the aggregate loading of energy-intensive crushing equipment directly depends on the accuracy of grain-size classification. Therefore, correct calculation and selection of controlled screening parameters and screen designs ensure actual energy savings in processing of all types of raw materials. Moreover, when processing non-metallic minerals and coal, they ensure high commercial quality of the final products. Until recently, the search for best combinations of design and process parameters for complex systems (without simple analytical representation) was limited to the analysis of the extensive past experience and application of the trial and error method, often directly in production. The tables turned with the emerging of computer technology and subsequent widespread application of numerical optimization methods. These methods enable identification of the best solutions for systems requiring optimization. Each system is treated as a "black box", where specific features of a model have little effect on applicability of the methods. The main requirements for an efficient original optimization model, in this case, include computational simplicity and problem parameterization in accordance with practical modeling purposes. The discrete element method [1] and other common versatile modeling approaches to the behavior of granular media used in vibrational screening studies require extremely resource-intensive computations. Detailed numerical experiments may last for months, thus eliminating the benefits of a computer simulation as compared to field experiments. Further elaboration and improvement of the

well-known analytical methods may contribute to significant simplification of new equipment development, enabling efficient selection of initial design and process parameters for further detailed numerical studies and field experiments. In combination with state-of-the-art numerical optimization methods, such as, for example, the particle swarm optimization [2], these advanced approaches will enable automatic search for optimal design and process parameters for sieve classification devices.

## 2 Basic approach

The system approach proposed by Mekhanobr and best described in study [3] was developed for modeling the screening of thick layers of lumpy granular material transported along the screening surface at a constant average speed. A thick layer, in this case, refers to a layer, all lower class particles of which cannot simultaneously be in direct contact with the screening surface. Lumpy material, for these purposes, is a material with zero or insignificant adhesive interaction between particles. The study also assumed a homogeneous flow (across the width). Figure 1 shows a longitudinal section of a material layer, the  $x$  axis is normal to the screening surface; the loading area is located on the left, and the discharge area – on the right.

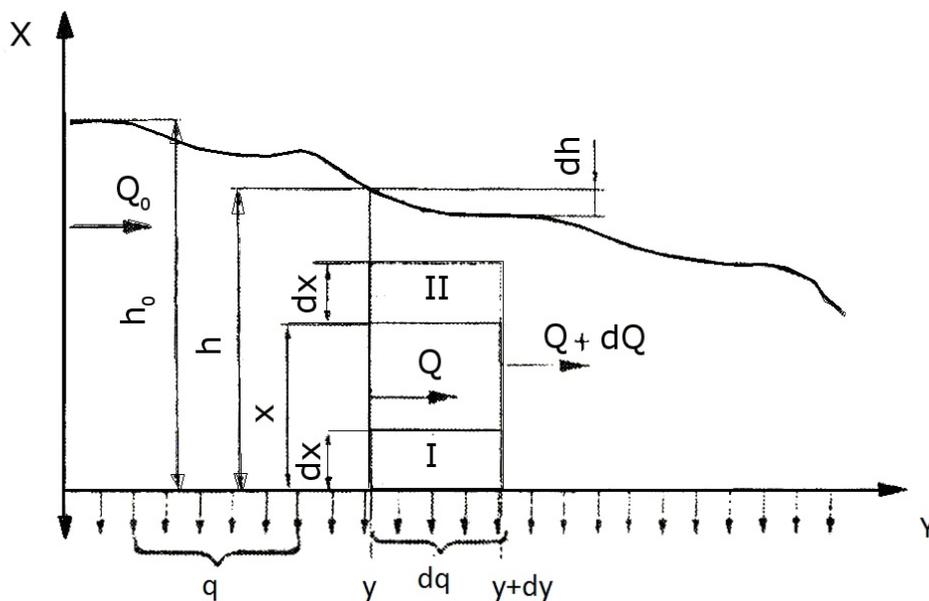


Figure 1: Material Flow Structure Along the Screening Surface

The following differential relations are made for limited grain-size class  $D$ . Assuming that only fairly small particles actually pass through the sieve, out of those in contact with the screening surface (i.e. removed by at least the value of  $dx$ ) and located directly above the sieve holes (with its probability determined by the Gaudin formula  $\varphi \left(1 - \frac{D}{d_0}\right)^2$ , where  $\varphi$  is the useful screen area, and  $d_0$  is the size its openings), the following formula was compiled for the outflow of this class of material in the screen section from its beginning up to  $y$ :

$$dq_D = \int_0^y \frac{u\varphi \left(1 - \frac{D}{d_0}\right)^2 a}{dx} dm_D \quad (1)$$

where  $u$  represents the rate of material layer flow through screen openings,  $dm_D$  is the mass of class particles in elementary material volume contacting the screening surface (see section I in Fig. 1), and  $a$  is the width of the screening surface. In this model, the  $dm_D$  differential is expressed using the content function of the grain-size classes considered along the screen length –  $f_y(D)$ , and the particle distribution function for these classes along the section thickness, as well as using material parameters and the layer shape. In this case, applicability is significantly limited by the need to know in advance the above distribution functions (that, in fact, depend on the nature of the screening process and material flow in the layer) or at least a number of properties of these functions. However, with the generality maintained, expression (1) may be integrated to obtain formulas for the recovery and total yield of undersize particles of different grain-size classes. Substitution of the above relations into the mass balance equation allowed authors of study [3] to generate a more general differential relation for the screening kinetics:

$$\frac{d\varepsilon_D}{dy} = \frac{u\varphi}{v} \left(1 - \frac{D}{d_0}\right)^2 P_{D,y}(0) (1 - \varepsilon_D), \quad (2)$$

where  $\varepsilon_D$  – undersize recovery for  $D$  narrow class particles, and  $v$  – material layer flow rate along the screen. If we consider the material in the layer evenly mixed, relation (2) transforms in a central differential equation of the classical model [4]. In case of absolute separation and linear grain-size cumulative distribution of the material, relation (2) can be integrated analytically and reduced to the following formula

$$\gamma = \frac{\beta y}{y + \frac{Q_0\beta}{u\varphi ac}} \quad (3)$$

that coincides with the formula obtained experimentally in [5]. In this equation,  $\gamma$  is the yield of undersize material,  $\beta$  is the parameter characterizing the slope of the cumulative grain-size distribution curve,  $Q_0$  is the feed capacity, and  $c$  is the bulk density of the material. Obviously, despite the flexibility and versatility of the method, its direct application in engineering entails such complications as the need to obtain additional information about the material separation process, the impossibility of direct consideration of a number of structural and process-related screen parameters, including vibration and screen angle, and the inability to deduce (except in some special cases) analytical expressions for the process. It should be emphasized, however, that by contrast to most common numerical modeling approaches to the behavior of granular media, the above method has very low resource intensity in terms of direct computer calculations.

### 3 The numerical model and some improvements of the approach

The below improved approach to modeling of the screening process for a lumpy bulk material is based on the extensive experience of Mechanobr-Tekhnika Research and Engineering Corporation in the development and manufacture of vibrational screens. A program applying the approach uses the following input parameters: grain-size composition of the material representing a mixture with a given proportion of narrow grain-size classes of

user-defined diameters ( $d_i$ ), friction coefficients for friction among material particles and with the screening surface, feeding capacity of the screen ( $Q_0$ ); screen parameters, such as length ( $l$ ), width ( $a$ ), hole diameter ( $d_0$ ) and useful screen area ( $\varphi$ ); screen vibration characteristics (with separately adjustable horizontal and vertical components in any form) or, similarly to the original model, the rate of vibrational material layer flow along the screening surface ( $v$ ), and the velocity of material particles flowing through screen openings (selectable from a special table of experimental data). The program, then, can output recovery rates for all narrow classes set, particle distribution for these classes in the material layer thickness along the entire length of the screening surface, and all derivatives, such as the efficiency of recovery for a selected class or set of classes, product yields, etc.

Since differential equation (2) of the basic model is generally not analytically integrable, the improved approach was originally formulated so as to enable computer-aided calculations. The material layer on the screening surface was represented as a set of spatial cells (see Fig. 2).

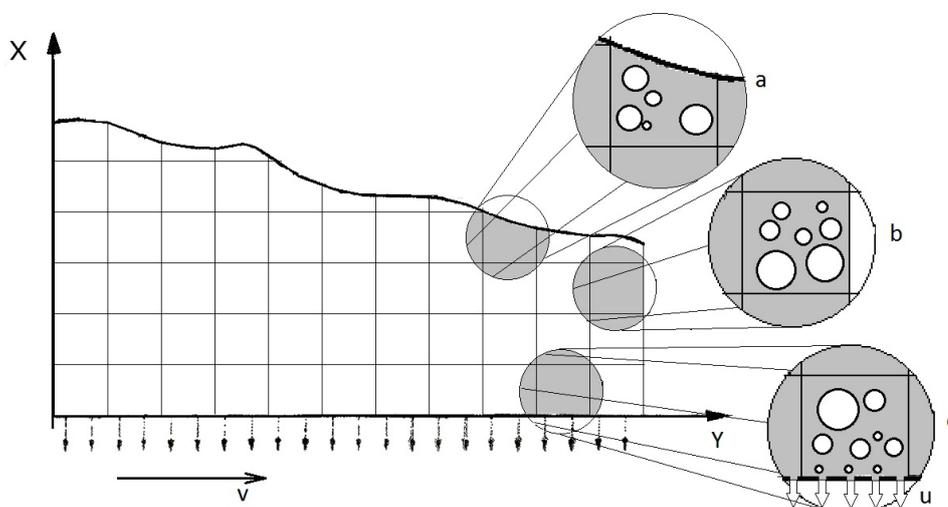


Figure 2: Longitudinal section of a material layer on the screen, cell distribution a) surface cell, b) material package cell, c) cell of the contact area between the material and the screen

For each cell (numbered  $j$  upwards from the bottom of column  $k$ , starting from the feeder), the following parameters are set:  $F_{k,j}(d_i)$  – mass fraction of  $d_i$  narrow class in the cell;  $M_{k,j}$  – gross weight of all the material in the cell.

This material breakdown into cells, as required for convenient numerical operations, preconditioned the modular architecture of the program. The approach is based on three relatively independent sub-models: displacement, screening and redistribution of particles within the material layer. Depending on the goals and objectives of modeling, the user may change each individual model and related subprograms to achieve a reasonable balance between resource-intensity and calculation accuracy, or even replace it with a new subprogram created for another model, without changing the rest of the code.

Vibrational displacement was represented in the basic model (see (2)) only by parameter  $v$  – material layer flow rate along the screening surface. This means that only indirect consideration was given to the relation between vibration parameters and the nature of material slide along the screening surface, which, in some cases, may be inconvenient for engineering calculations. For general analysis of the problem, a subprogram was created

that simulates vibrational displacement of a rigid body on a vibrating inclined rough plane. The program was based on the methods described in [6]. The average displacement rate was assumed to be equal to the material layer flow rate in similar circumstances.

According to the basic model, the mass outflow through screen segment  $k$  for each class may be calculated by the formula:

$$q_k(d_i) = P(d_i) u F_{k,0}(d_i) \Delta y a, \quad (4)$$

where  $P(d_i)$  – coefficient that specifies the proportion of particles with diameter  $d_i$  on the screening surface that would pass through the screen openings (conventionally called – narrow class particle passage probability);  $u$ ,  $F_{k,0}(d_i)$ ,  $\Delta y$  and  $a$  correspond to the material passage rate through the screen, mass fractions of different classes in cells of the granular media, the horizontal cell size and the width of the screening surface, respectively.

The original model, similarly to most of the earlier analytical approaches, was based on the Gaudin's passage probability. However, the classical Gaudin formula does not take into account the possibility of simultaneous passage of particles of one or more grain-size classes through the opening on the screening surface, which may result in somewhat understated screening intensity values for small classes. The model presented in this article was corrected through the application of the improved formula by A.E. Pelevin. [7]. If the Gaudin's probability for particle diameter  $d_i$  is

$$P^{d_0}(d_i) = \varphi \left( 1 - \frac{D}{d_0} \right)^2, \quad (5)$$

the corrected Pelevin probability (in a shorter representation) shall be as follows:

$$P(d_i) = P^{d_0}(d_i) + \sum_{j:d_j+d_i \leq d_0} F_{k,0}(d_j) P^{d_0}(d_j) P^{d_0-d_j}(d_i). \quad (6)$$

In the equation, all terms under the summation symbol represent conditional passage probability for a particle with a diameter of  $d_i$ , provided passage of particles with a diameter of  $d_j$  in the remaining gap of  $d_0 - d_j$  for all  $j$  values for which  $d_j + d_i \leq d_0$  is true.

All processes occurring inside the material layer during its vibrational transportation may significantly influence the sieve classification process, as they determine which particles come into contact with the screening surface. Various methods to describe and account for these processes in vibration screening simulations are mentioned in numerous studies [4, 8, 9]. Specific model construction features presented in this paper enable flexible application of almost any ideas for modeling the behavior of particles in a material layer. As shown in [3], in some cases, material may be considered as completely separated along the entire length of the screen, as the passage rate of fine particles through the gaps between large particles enables rapid saturation of the layer in contact with the screening surface with easy-passage classes. However, in order to achieve high program flexibility, the original version of the model treated the separation process as a uniform flow, with application of compulsory averaging at a given class distribution function proportionality along the section height with the function corresponding to fully classified distribution. An experimental check of the algorithm demonstrated that in screening simulations of sufficiently thick material layers, assumption of vibrational segregation being a non-instantaneous process increases the accuracy of numerical experiments.

The above vibration segregation consideration method has, however, a distinct drawback: in such a simulation, essentially inhomogeneous materials are sorted at the same rate as almost homogeneous materials. For this reason, the algorithm was completed with an

improved model representing separation as part of the material exchange between computational cells (Fig. 3). Such an exchange may be due to vibrational convection or otherwise generated random movements of particles in the granular media; when shaken, small class particles tend to pass between large particles blocking them. That is, small class particles are screened through a layer of larger particles, as if through a sieve.

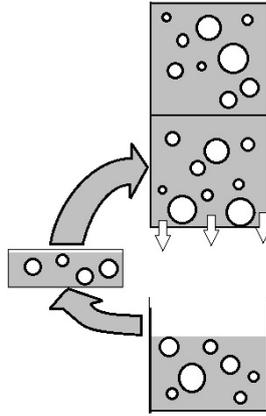


Figure 3: Material Flow between Cells in Separation Simulation

In improved separation modeling, the lower boundary of each cell represents a conventional screening surface with an opening diameter equal to the gap size between packed particles of an average cell radius. If the average particle radius in a cell is  $\tilde{R}$ , the opening diameter of a conventional screening surface with dense packing is

$$\tilde{r} = \left( \frac{2\sqrt{3}}{3} - 1 \right) \tilde{R},$$

and its useful area is

$$\tilde{\varphi} = \frac{(2 - \sqrt{3})^2}{3\sqrt{3}} \pi$$

These formulas were also amended with regard to the fact that shaking produces additional gaps between material particles. If average particles are removed from the dense package by the distance of  $\Delta r$ , the amended formulas shall be as follows:

$$\begin{aligned} \tilde{r} &= \tilde{r} + \frac{2\sqrt{3}}{3} \Delta r, \\ \tilde{\varphi} &= \tilde{\varphi} + \frac{(4 - 2\sqrt{3}) \tilde{R} + (4 - \sqrt{3}) \Delta r}{3 (\tilde{R} + \Delta r)^2} \pi \Delta r \end{aligned}$$

Therefore, small classes get an advantage in terms of access to the screening surface, which enables more realistic recovery curves. Table (Fig. 4) shows comparative results of modeling and field experiments for three versions: original, improved screening and improved separation versions. The recovery curves shown in Fig. 5 for field and numerical simulations using the original and improved methods correspond to the first experiment in the table.

	Feed material grain-size classes (mm)	Grain-size class content (%)	Mass screening capacity (t/h)	Sieve cell size (mm)	Recovery rates			
					Experimental data	Original model	Improved screening	Improved separation
1	-0.071+0	1.6721	1.7612	0.6300	0.8333	0.9737	0.9626	0.9840
	-0.18+0.071	11.4054			0.9091	0.9720	0.9604	0.9509
	-0.315+0.18	8.5752			0.8750	0.9669	0.9540	0.8610
	-0.63+0.315	16.7392			0.5319	0.8448	0.8196	0.6064
	+0.63	61.6080			0.0000	0.0000	0.0000	0.0000
2	-0.071+0	1.2631	0.6263	0.6300	1.0000	0.9784	0.9690	0.9910
	-0.18+0.071	11.1155			1.0000	0.9779	0.9684	0.9688
	-0.315+0.18	8.8419			1.0000	0.9766	0.9666	0.8995
	-0.63+0.315	17.1158			0.6923	0.9504	0.9348	0.7241
	+0.63	61.6637			0.0000	0.0000	0.0000	0.0000
3	-0.071+0	1.2405	0.5374	0.3150	1.0000	0.9659	0.9515	0.9629
	-0.18+0.071	11.1700			0.9885	0.9637	0.9487	0.9028
	-0.315+0.18	10.3020			0.7419	0.9251	0.9030	0.7469
	-0.63+0.315	16.5616			0.0000	0.0000	0.0000	0.0000
	+0.63	60.7259			0.0000	0.0000	0.0000	0.0000
4	-0.071+0	1.5283	1.7250	0.3150	0.8889	0.9594	0.9430	0.9523
	-0.18+0.071	11.9831			0.8657	0.9509	0.9325	0.8779
	-0.315+0.18	8.5949			0.6129	0.7682	0.7376	0.6118
	-0.63+0.315	17.0231			0.0000	0.0000	0.0000	0.0000
	+0.63	60.8706			0.0000	0.0000	0.0000	0.0000

Figure 4: Table 1.

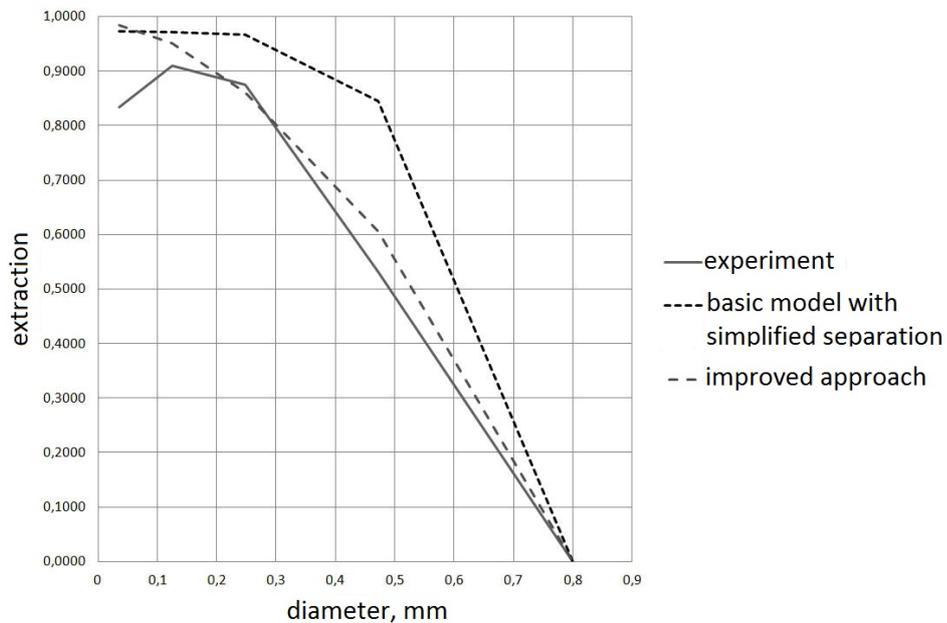


Figure 5: Recovery Curves for Various Grain-Size Classes

## 4 Conclusion

The vibrational screening modeling approach presented in this paper has several advantages as compared to both advanced numerical and most common analytical methods. It enables high accuracy modeling without a time-consuming setup, calibration and preliminary experimental preparations, and may be applied for a wide range of devices. This approach was used as the basis for a computer program with a friendly user interface that enables designers to evaluate future operation of the device designed based on its input parameters. Low resource-intensity of calculations allows using the program in combination with stochastic optimization methods for preliminary assessment of optimal parameters for the screens designed.

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