Shape optimization of a biaxially loaded specimen

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Abstract

In this study we investigated biaxial deformation of cruciform specimens from elastomeric materials. For this type of tests it is observed the formation of inhomogeneous displacement field in the sample central part. In order to achieve loading uniformity in the area of interest we considered different changes in the specimen geometry. The basic goal is to find an optimized shape so that the strain and stress fields are remained uniform in the larger area of a specimen under high deformation. Finite element simulations were carried out for different shapes. It was experimentally tested whether the optimal form, numerically found for specimens of hyperelastic materials, is applicable for real samples from viscoelastic materials under different types of loading on a Zwick biaxial testing machine. It is shown that the selected specimen shape is versatile enough to test both elastic and inelastic materials with complex mechanical properties.

1 Introduction

In case of biaxial stretching experiments, as opposite to the standard tests, we obtain more complete information about material properties and its behaviour. However, there is no definiteness on a specimen shape and a way of loading, which produces a uniform biaxial stress state in the area under study. There are examples of using a thin circular sheet clamped over the boundary and loaded by the pressure of injected gas or liquid [3, 5, 7]; a thin-walled tube subjected by loads combinations — internal pressure, torsional moment and axial force [3, 8]. It is also used a cross-like specimen loaded in two orthogonally related directions [1-3, 5, 6, 8]. The first type of experiments covers only a narrow range of problems about symmetrical biaxial stretching.

Investigating materials behavior under high level of deformation it was shown that a specimen form becomes highly distorted [3, 6, 8]. In a loaded cruciform sample, for instance, it is observed the formation of heterogeneous displacement field. But, as opposed to thin-walled tubes, here there is a possibility to change specimen geometry. These changes are widely presented by different authors and used for finding optimized forms [1, 2, 6, 8].

According to above mentioned, the basic goal of our research is to find an optimized specimen shape based on quantitative assessments of stress and strain fields in cruciform samples of known shapes and a new form, which we have been proposed. After this the obtained optimal shape was checked experimentally for filled rubber samples under uniaxial and biaxial stretching on the Zwick testing machine and with different loading rates.

2 Shapes of cruciform specimens

Different specimens form variations presented in the literature [1-3, 8] are generally used for materials — metals, composites and polymer composites — whose properties differ significantly from elastomers features. However, the known geometries of cruciform specimens can be considered in our case, as is done, for example, in the article [6]. We decided to analyze the impact of next modifications: 1) slots in specimen limbs, whose location and number can be varied; 2) different geometries of a corner between the arms; 3) changes in limbs forms. We also applied round holes at the one end of slots to prevent their further growth, and as a consequence, to increase the maximum allowable deformation value in the sample central part.

The quantity of possible modifications in a specimen form is large. Initially, therefore, we evaluated efficiency of applied changes in the specimen geometry based on numerical analysis for hyperelastic models. Then, these results were verified experimentally.

3 Numerical justification

3.1. Problem statement

All numerical calculations were performed for symmetrical patterns of the same size. It means that the sample central part dimensions ($l_0 = 0.03$ m) and the clamping length ($L_0 = 0.122$ m) are constant for all considered shapes (Fig.1). Also in the specimen working area were considered square regions of different sizes $k_i = \frac{l_i}{l_0}$, where l_i — the side length of the i^{th} area under consideration, in order to investigate edge effect influence.

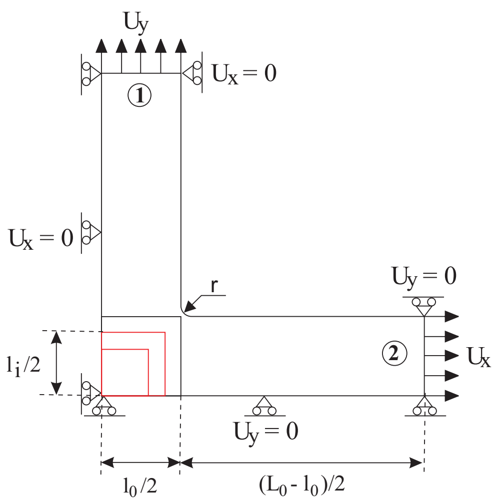

Figure 1: One quarter of a specimen with its dimensions and boundary conditions

Within the framework of the posed problem all specimens are uniformly loaded in two axis by displacements imposed on the lines of 1 and 2 (Fig.1). According to geometric symmetry, material homogeneity and equability of load application, it was considered only one quarter of a sample. There was assumed that specimen deformations at the boundaries reach 100% along each of the axis. Then, $U_x = U_y = 0.061$. Also on the lines of 1 and 2 it was restricted to move in the directions perpendicular to the loading axis, this makes possible to eliminate specimen shift in the grips.

Rubber is a viscoelastic material, but in some cases it behaves elastically. In numerical simulation it was considered the nonlinear elastic problem within the quasi-static formulation. The hyperelastic material behavior is defined by the strain energy function; it was used to compare three different hyperelastic potentials — the neo-Hookean, Mooney-Rivlin and Arruda-Boyce models. Numerical values of the material parameters in these potentials were taken from the article of Arruda and Boyce [4].

3.2. Results

In order to find an optimized specimen shape there was performed a comparison of all numerical results in uniformity of strain and stress fields of samples. The stress field in the area of interest is homogeneous, if values of the principal stresses σ_{11}, σ_{22} and the shear stress τ_{12} of each point in this area do not exceed values of the principal and shear stresses of the central point more than $\delta = 5\%$. The range of displacement values of points on the boundary also shouldn't be higher than δ in a homogeneity area. The parameter for evaluation of shape optimality is the size k_i of the area with uniform stress and strain fields. It means that for the optimal sample this parameter has the maximum value.

When designing a cross-shaped sample the first thing needed to be specified is a form of a corner between specimen arms, in order to remove high stress concentrations. According to data in the article of Abdelhay et al. [1], our simulations were carried out for samples with fillets of small radius r (Fig.1).

The presence of slots in the arms influences the specimen strain-stress state in a greater degree, this confirms results described in [6]. From the beginning it was assumed that samples of different shapes would be experimentally tested; therefore, the number of applied slots was determined on the basis of 1) given dimensions for the specimen arms; 2) values of hole radius at the slots end; and 3) distances between contours of different holes.

Regardless of the holes size it was confirmed that with an increase of the slots number in the specimen limbs the strain and stress fields become more uniform. All presented results were obtained using the Arruda-Boyce model and they differ from other models findings only in numbers. As seen in Figure 2, when limbs are slotted, the border of the initially square area ($k_1 = 0.8$) are less distorted. During loading of the sample without slots (Fig.2a) the strain value, by which the boundary deformed, reaches maximum of 61.8% and minimum of 42.2%. For the sample with 3 slots in a limb it is varied from 68.9% to 77.2%. With 9 slots per arm — 73.7%-79.7%. Thus, firstly, with larger number of slots we achieve more homogeneous strain field and, secondly, the slotted configuration allows greater deformations to occur in the working area. All these happen due to a decrease in shape distortion of limbs and fillets.

It is also shown that the range of the principal stresses in the area ($k_1 = 0.8$) decreases with increasing number of slots. In the case d (Fig.2) the specimen shape is practicaly invalid, i.e. the distance between slots is too narrow to transmit high level of load to the working area. It is presented only as a standard for comparison. From this it is seen that a significant increase in the number of cuts per arm does not lead to the achievement of the stress field homogeneity in the area under consideration. The edge effect appears significantly less, but still occurs. In spite of this, the importance of slots in arms should not be lessened. For instance, in the non-slotted sample the region of homogeneity is a square of $k_2 \approx 0.22$, for the sample with 3 slots — $k_3 \approx 0.43$, when both regions deformed by 78.1%. Hence, the size of area under equibiaxial load increased a little less than half for the slotted specimen. For the sample with 9 slots in a limb the area of homogeneity is a square of $k_4 \approx 0.48$ size. Obviously the smaller level of strain in the sample center is the wider homogeneous region. When strain value in the working area varies around of 35%,

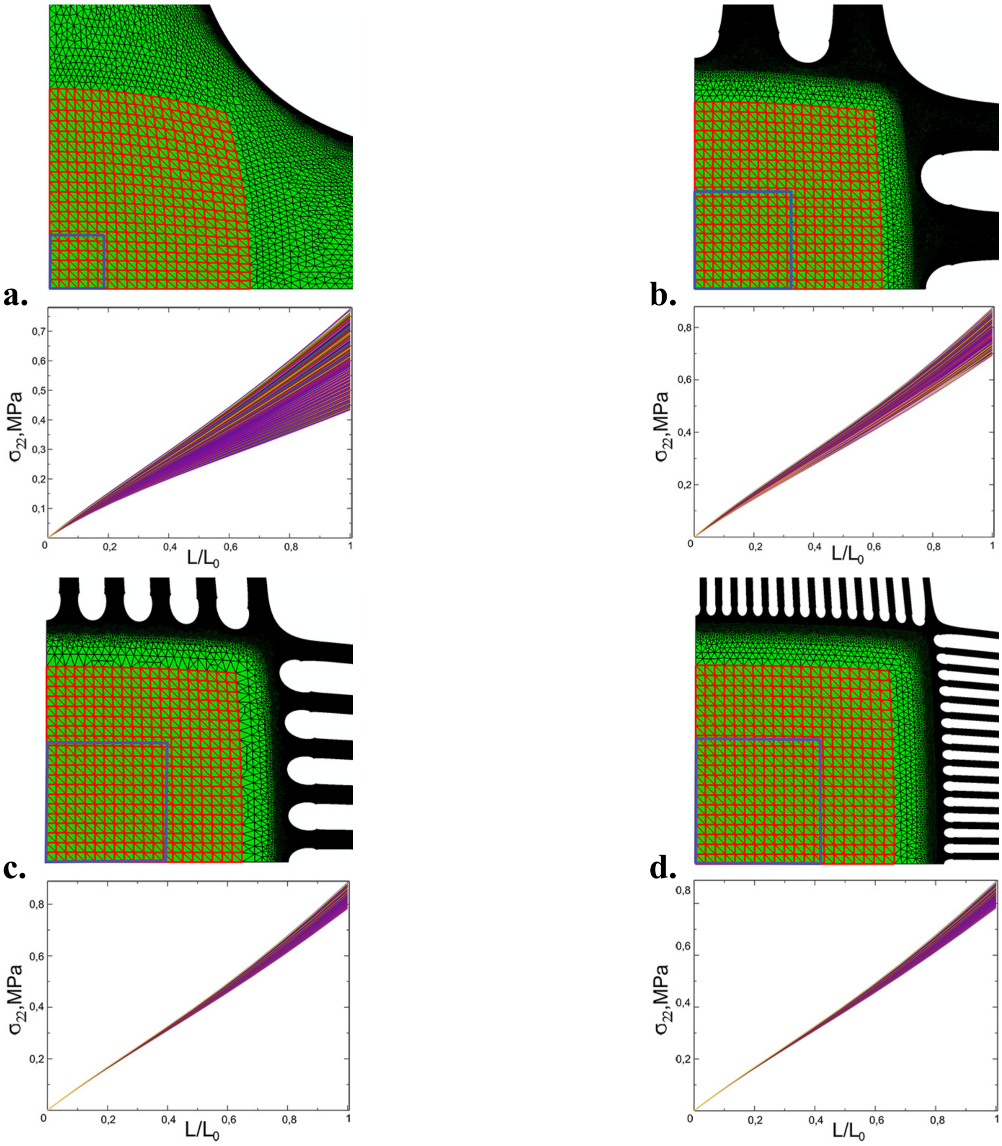

Figure 2: Geometries of deformed specimens: a) non-slotted; with b) 3; c) 9 and d) 30 slots per arm and families of σ_{22} stress curves in the area ($k_1 = 0.8$) specified with red mesh. Blue squares are homogeneity areas for these specimens.

the homogeneity area for the sample with 9 slots is $k_5 \approx 0.65$.

When slots are applied, a specimen arm is transformed into multiple strips which drift apart under biaxial loading. And this leads to more uniform load application to the working area and less distortion. However, the limbs width in the clamp region does not change; hence, there appears a peculiarity that the central strips, which are closer to the axis of symmetry, transmit larger loading to the working area than the side strips.

Another possible parameter for variation is the radius of holes used at the slots end. We considered samples with the same number of slots, but with holes of small radius and radius value, which is 2.5 times larger. It appeared that under the same level of deformation in the specimen central part the region of homogeneity increases insignificantly in the latter case, but near the boundary of considered area ($k_1 = 0.8$) percentage of stress inhomogeneity increases more. Therefore, it is more effective in testing specimens to set a small radius value, since, firstly, there increases the possible number of slots applied (the more number of them the more uniform stress and strain fields in the central part) and, secondly, reduces edge effect influence.

Analyzing the obtained results, it was assumed that the subsequent modifications are needed to be performed for the limbs form. Previously, it was shown that under biaxial stretching the sample central part increases in size while the arm dimension in the clamp region remains constant. As a result, through the side strips load is transmitted at some angle and, therefore, the part of working area closed to them is deformed less. To solve this problem, it was decided to separate strips by some distances from each other so that they become perpendicular to the central deformed part. In order to not preload a sample these strips are originally cut at some angles to each other, so that a limb looks like a fan (Fig.3).

When considering this type of samples it is necessary to introduce the concept of an opening angle α, i.e. the angle at which a side strip inclined to the loading axis (Fig.3). It is clear that this angle value affects the sample stress-strain state in some way. Figure 4 shows the stress fields in the ($k_1 = 0.8$) area for the fan-like specimens with 10 strips and different opening angles — 5°, 13°, 20°. Even in the case when we define a small opening angle of 5°, the sample strain-stress state in the central part becomes more uniform in comparison with results obtained for a conventional specimen with 9 slots in a limb. Here, the biaxial area increased by 20% and reached the area of size $k_6 = 0.58$. Although for the sample with the opening angle of 13° the homogeneity region is about 73% of the total area of the specimen central part, for the sample with larger opening angle, this region is not more than $k_7 = 0.65$. This occurs due to the fact that the side strips length is more than the length of strips close to the axis of symmetry.

4 Experimental conformation of the selected geometry

Subsequent studies were carried out using the Zwick biaxial testing machine. Performed experiments were uniaxial stretching of cross-shaped specimen with following unloading. When induced anisotropy had been formed in a sample, it was symmetrically loaded along two axes. We were interested to find out whether a new specimen geometry is appropriate for studying behavior of material with inhomogeneous structure and under different types of loading.

Testing cross-like samples there was detected the displacement field by a video extensometer videoXtens Array. In the central part were plotted straight lines, whose distortion

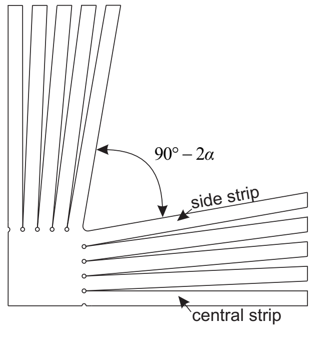

Figure 3: Geometry of a fan-like specimen with 10 strips per limb and the opening angle of α.

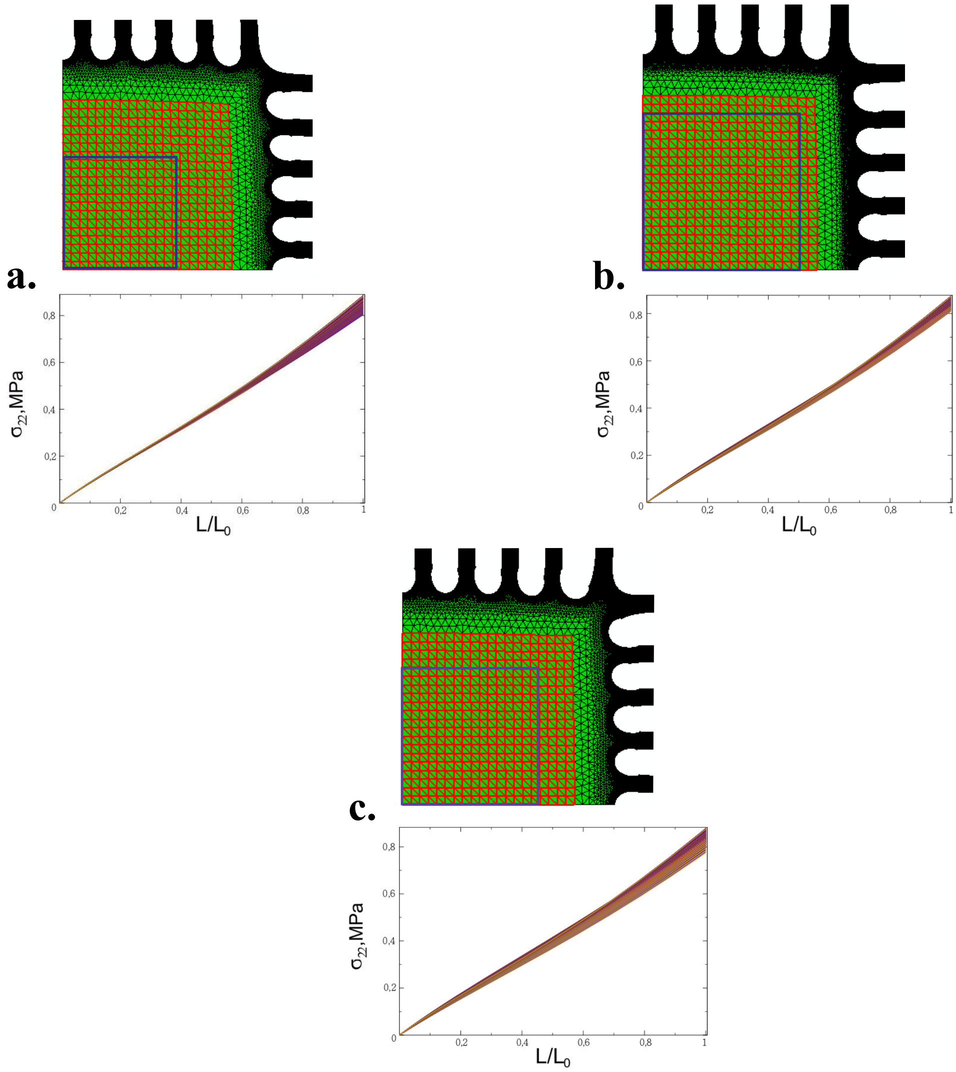

Figure 4: Geometries of deformed fan-shaped specimens with the opening angle of a)5°; b)13°; and c)20° and families of σ_{22} stress curves in the area specified with red mesh. Blue squares are homogeneity areas.

was monitored during sample loading.

Experimental studies confirmed the applicability of the FEM results. Investigating displacement fields in specimens with different geometries it was obtained that found optimal form is effective for samples from inelastic materials with complex mechanical properties.

5 Conclusion

— By calculation there is shown that the more slots number in the specimen limbs the more uniform strain and stress fields in the sample center. Based on numerical results and practical considerations in the sample with given dimensions we propose to use no more than 9 slots per arm.

— In order to obtain more homogeneous stress and strain fields in the working area, it was proposed to locate sample strips in the clamp region at some angles relative to the elongation axis. By calculations there was determined that the optimal angle of the side strip deflection from the figure axis is 13°. Other strips are uniformly spaced between two side strips of a sample limb.

— It is shown that for this specimen form the homogeneity area covers 73% of the total area of the working region.

— Experimentally there was shown that the numerically found optimized shape for hyperelastic materials is also effective for inelastic materials with complex mechanical behavior at the high level of deformation. This refers to the filled rubber with pronounced viscoelastic properties and the effect of material softening.

— It is shown that in the sample of proposed form the strain field heterogeneity at the boundaries of the area does not exceed 5%.

Acknowledgements

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