Modeling of rocks hardening and softening

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Abstract

As deeper precious metals mining operations are developed, new ways of thinking about rocks deformation behavior near open pits and drill holes are required. In particular the much higher loads encountered at depth necessitate consideration of inelastic strain and shattering conditions.

The proposed correlations model rock behavior during axial compression as a functional connection between maximum shear stress and shear strain taking into account initial imperfections. Relationships are established between imperfection and damageability parameters in a kinetic equation form that follows the deformation process up until the stage of rock crushing.

A state equation has been developed using catastrophe theory methods where transition processes are presented as dissipative structures of increasing denseness. Comparison of theoretical and experiment data was performed for a number of rocks to demonstrate applicability of the methodology.

1 Introduction

Rock mass as well as Earth's crust is geomechanics inhomogeneous medium.

Surface open pit mining doesn't provide enough resources anymore, it's necessary to develop mines at the greater depth and in difficult geological conditions. With the transition of mining operations at great depths geomechanical processes in rock mass undergoes quantitative and qualitative changes. Rock under static and dynamic loads near mines and boreholes turn into limit state and collapse at the condition of heterogeneous three dimensional stress or are in dilapidated state at stress below breaking limit. This phenomenon is observed at all levels of scale, which makes it possible to study on laboratory samples.

Rocks behavior under stress can be fully characterized by experimental data as connection between stress and strain, which are obtained by testing standard samples in the laboratory on "hard" presses. As was mentioned above rocks are heterogeneous materials and are in a steady state, stable with respect to small perturbations therefore can be interpreted as a dissipative structure [1].

According to the classification of space-time dissipative structures [2], [3] rocks at greater depths can be considered as autostructures -localized by tridimensional formations, stably exciting in dissipative nonequilibrium media and independent of changes of initial and boundary conditions. That is why the process of deformation and failure of rocks can be seen as a hierarchy of instabilities caused by self-organization. In other words, self-organization is implemented relating to the terms of metabolism and energy with the environment in a system which is far from thermodynamic equilibrium. These arguments demonstrate the value of bringing synergistic representations [4] to the description of the equilibrium and stability of rocks.

2 Experimental data

Based on the foregoing, passport dependence of the maximum shear stress (T) and the relative volume change (θ) to the maximum shear strain (Γ), built on the basis of known experiments on uniaxial compression [5] submitted as the initial information.

Fig.1 shows the qualitative behavior of passport dependencies $T(\Gamma)$ (curve 1) and $\theta(\Gamma)$ (curve 2). The elastic part of the diagram $T = T(\Gamma)$ ($\Gamma \leq \Gamma_e$, $T \leq T_e$) is replaced by work-hardening range, transformed into softening region when ($\Gamma = \Gamma_0$), ($T = T_0$). After reaching the (critical) shift Γ_c destruction comes. Analysis of experimental data shows that when ($\Gamma = \Gamma_0$), ($T = T_0$) volume change does not occur in other words, the tensile strength of the material is incompressible. This establishes a relationship of volume strain (dilatancy) to shear. Elastic limit T_e is responsible for elimination of initial imperfections.

Fig.1 Fig.1b

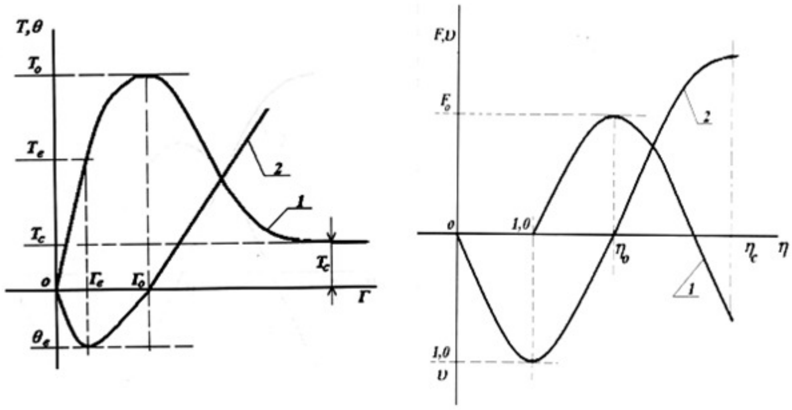

Figure 1: The qualitative behavior of passport dependencies $T(\Gamma)$ (curve 1) and $\theta(\Gamma)$ (curve 2).

3 Governing equations

To describe the patterns of deformation, we introduce a normalization of coordinates, assuming

$$F = \frac{T}{T_e} - 1\,,\ \eta = \frac{\Gamma}{\Gamma_e}\,,\ \vartheta = \frac{\theta}{\theta_e}\,, \tag{1}$$

where θ_e - volume shear, corresponding to the elastic limit.

Based on (1) $T(\Gamma)$ and $\theta(\Gamma)$ diagrams can be presented in $F = F(\eta)$ and $\vartheta = \vartheta(\eta)$ and form curve 1,2 (Fig.1b), η (in catastrophe theory [6]) we shall give the status of the order parameter. Let's assume that current stress - strain state corresponds to potential function:

$$V = V(F, \eta, \beta). \tag{2}$$

And β are the parameters responsible for imperfections (defects) of the material.

Since it is assumed that β parameters are sensitive to structural changes, they can be considered as evolutionary. Introduction of β parameters can adopt function (2) as suitable for the study of dissipative systems, which include deformable geomaterials.

Let's look at representations, indicating the usefulness of methods involving catastrophe theory [6] to model the equilibrium, stability and its loss, where stability refers to deformational stability, and it's loss is loss of deformational stability (descending branch of the diagram ($F = F(\eta)$ Fig.1b). Thus, the deformable system (material) is experiencing a number of states, the change of which corresponds to the critical point. Plasticity occurs when $\eta = \eta_e$ and there is a formation of dissipative structures. Essentially synergistic effects appear in deformable environment [3]. Environmental impact slowly implements under static loading, and, therefore, the observed effects can be attributed to self-organization through the control parameters, the role of which is assigned to the parameters of imperfection β.

In fact, in the process of rock loading accumulates inelastic deformation, and material should be viewed as an imperfect (dissipative) system. In catastrophe theory [6] approach energy state function should be represented as superposition of $V_P(F, \eta)$ potential and $P = P(\eta, \beta)$ disturbance.

$$V(F, \eta, \beta) = V_P(F, \eta) + P(\eta, \beta). \tag{3}$$

For the disturbance function apply expansion of Morse form

$$P(\eta, \beta) = \beta_1 \eta + \frac{1}{2}\beta_2 \eta^2 + \frac{1}{3}\beta_3 \eta^3 + \dots. \tag{4}$$

Dependence (4) can be brought to the canonical form by a suitable nonlinear change [1]. Mathematically formal replacement is quite reliable. But this transformation is likely to lead to a complex nonlinear relationship between F and control parameters β. Therefore, in (4) discard terms higher than the second degree, and keep the quadratic term.

With that said energy state function can be written as:

$$V(F, \eta, \beta) = \beta_1 \eta + \frac{1}{2}(F_0 + \beta_2 - F)\eta^2 + \frac{1}{3}\eta^3. \tag{5}$$

Equilibrium state is determined by minimizing (5) by order parameter. We have:

$$\frac{dV(F, \eta, \beta)}{d\eta} = \beta_1 + (F_0 + \beta_2 - F)\eta + \eta^2 = 0. \tag{6}$$

Equation (6) can be viewed as two-dimensional manifold that is included in the space $\|R^3\|$ with η, F, β coordinate axes from the point of catastrophe theory. Equilibrium state will be found for each $\beta_1 = const$ meaning. Location of critical points and type of their stability can be easily determined - all the points to the left of $\eta = \eta_0 (\eta \in]1, \eta_0[)$ (Fig.1b) are locally stable critical points, all the points in $\eta \in]\eta_0, 1[$ are unstable critical points.

Change of stability type occurs when $\eta = \eta_0$. Material behavior depending on the load connected with β parameter. At $\beta_1 < 0$ each value there are two critical points in the allocated area $\beta_1 > 0$ in which there are no critical points. This can be seen when

$$-\beta_1 + \left(\frac{F_0 + \beta_2 - F}{2}\right)^2 < 0. \tag{7}$$

Thus material sensitivity to system's imperfection weakly depends on β_2 and allows to discard all disturbances from equilibrium, except linear.

Considering analysis we find that state equation corresponds to equilibrium condition (6):

$$F = \frac{\beta}{\eta} + \eta + F_0, \tag{8}$$

where $\beta = \beta_1 < 0$; $\eta \in]\eta_0, 1[$.

Thus, an equation of the relationship between the normalized maximum shear stress F and shear strain, which in terms of the theory of phase transitions, serves as the order parameter. In addition, in equation (8) is the control parameter β, called the parameter of imperfection. It is responsible for the current structure changes of deformed geomaterial sample.

4 Damage parameter

In [7] first introduced the notion of damage of material during the deformation and its characteristics - damage parameter ω. The above description is considered an internal state parameter, evolution of which is determined by corresponding kinetic equation.

For mathematical modeling of deformation process, we assume that the quality dependence of the damageability on deformation has the form shown in Fig.2. This result was assured on the range of axial compression testworks for different types of geomaterials by recording their acoustic emission of crack formation in Physics and Rock mechanics institution of NAN KR. In this approach, the evolution of damage parameter can be considered as a multi-stage blurred irreversible phase transition, and ω as the degree of completeness [8]. Let's approach the connection between imperfection parameter β and damage param-

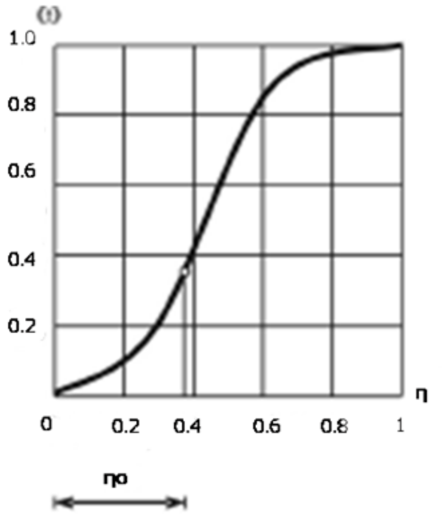

Figure 2: The quality dependence of the damageability on deformation.

eter ω. Note that in the elastic deformations region ω parameter slightly increases. During the transition to the plastic state ω begins to grow rapidly. This is explained by the nature of the initial residual deformations of heterogeneous materials, which, as noted above, are rocks. Here, in addition to purely shear processes, a significant contribution to the deformation gives loosening. Thus in created structure new more sophisticated substructures appear, the transition to which is performed by changing the type of stability. New state appears where with growth of deformation stress decreases [9]. Damage parameter increases in out of limit zone and with the appearance of the main crack $\omega \longrightarrow 1$.

Evolution of damage parameter is defined by stochastic nature, Fokker-Planck equation is taken as kinetic equation for defining its static form [3]:

$$\lambda \frac{\partial \omega}{\partial \eta} = R(\eta)\omega. \tag{9}$$

Here $R = R(\eta)$ - is function responsible for internal glide and loosening of material. λ - is diffusion coefficient that's assumed as constant.

Solution of (9), following [3] is found in the form:

$$\omega(\eta) = A \cdot \exp\left(\frac{-2 \cdot \Phi(\eta)}{\lambda}\right), \tag{10}$$

where

$$\Phi(\eta) = -\int_0^\eta R(\eta) d\eta. \tag{11}$$

Taken as potential, and normalized factor A is found from condition

$$\int_{-\infty}^{\infty} \omega(\eta) d\eta = 1. \tag{12}$$

Representation (10)...(12) due to the involvement of catastrophe theory methods to modeling of rock deformation process includes out of limit part of the curve. Therefore, this representation is associated with a gradient dynamical system [6]. Since there is a strong non-linear dependence $\omega = \omega(\eta)$ (Fig.2), the potential function $\Phi(\eta)$ and antigradient $R(\eta)$ of this function can be represented as an elementary cusp catastrophe:

$$\Phi(\eta) = \frac{a\eta^2}{2} + \frac{b\eta^4}{4}, \quad R(\eta) = -a\eta - b\eta^3, \tag{13}$$

where a and b defined constants.

If we meet requirements of (13) damage parameter based on (10), (11) is defined

$$\omega = A \cdot \exp\left[\frac{1}{\lambda}\left(-a^2\eta^2 - \frac{1}{2} b\eta^4\right)\right]. \tag{14}$$

Substitution of (14) into (12) will give us equity

$$A \cdot \sqrt{\frac{a}{b}} \cdot \exp\left(\frac{a^2}{4\lambda b}\right) \cdot K_{\frac{1}{4}}\left(\frac{a^2}{4\lambda b}\right) = \sqrt{2}. \tag{15}$$

Here $K_{\frac{1}{4}}\left(\frac{a^2}{4\lambda b}\right)$ is modified Bessel function.

Further assume that inflection point corresponds to the change of dissipative structures on the curve $\omega = \omega(\eta)$ (Fig.2). In other words:

$$\frac{d^2\omega}{d\eta^2}|_{\eta=\eta_0} = 0. \tag{16}$$

Considering (14) and (16) we will get:

$$\frac{2b}{\lambda}\left(t\eta_0 + \eta_0^3\right)^2 - \left(t + 3\eta_0^2\right) = 0, \tag{17}$$

$$t = a/b. \tag{18}$$

Taken into consideration (18), equity (15) can be overwritten as:

$$A \cdot \sqrt{t} \cdot \exp\left(\frac{t^2 b}{4\lambda}\right) \cdot K_{\frac{1}{4}}\left(\frac{t^2 b}{4\lambda}\right) = \sqrt{2}. \tag{19}$$

If we use the obvious fact, according to which $\omega|_{\eta=1} = 1$ we could write

$$\ln\left(\frac{1}{A}\right) = -2\frac{b}{\lambda}(2t+1). \tag{20}$$

To define A, t and b/λ parameters numerical procedure was developed and their values for some rocks are presented in the Table.

Rock	Parameters						
	t	$b/2\lambda$	A	n	N	c	r
Not explosive sandstone	0.098	225.3	1.046	-2.64	0.93	-0.89	4.186
Explosive sandstone	0.131	99.792	5.525	-2.81	1.02	-0.55	1.29
Koelgan Marble	0.139	96.349	1.132	-3.089	1.2	-0.78	2.532
Biotitic granite	0.113	159.36	1.082	-2.17	0.86	-1.5	0.768

Suppose that for some value of $\omega = \omega_e$ have the condition $\beta = \beta(\omega_e)$ and β_e and ω_e correspond to the rock's elastic limit. Assume that the change in the value of the parameter ω for $d\omega$ value β parameter responds with change for amount proportional to β. Therefore, we set:

$$d\beta = -\beta K(\omega - \omega_e)d\omega. \tag{21}$$

where $K(\omega - \omega_e)$ is a kernel, that decreases with $(\omega - \omega_e)$ growth.

Solution of differential equation (21) is

$$\beta(\omega) = \beta(\omega_e) \cdot Q(\omega, \omega_e). \tag{22}$$

and

$$Q(\omega, \omega_e) = \exp\left(-\int_{\omega_e}^{\omega} K(\omega - \omega_e)d\omega\right). \tag{23}$$

Kernel of (23) is presented in form:

$$K(\omega - \omega_e) = \frac{1-n}{(\omega - \omega_e)^n}, \tag{24}$$

here n and N are dimensionless material constants.

Considering (24) solution of (22) is presented in form:

$$\beta(\omega) = \beta(\omega_e)\exp\left[-(\omega - \omega_e)^{1-n}\right]. \tag{25}$$

For β_e from (8) we have:

$$\beta(\omega_e) = -\eta_e(F_0 + \eta_e). \tag{26}$$

Values of N and n are defined numerically and shown in Table 1.

Let's assume further that N corresponds to $\eta = \eta_e$ deformation. Therefore for second approximation we could see that $N = N(\eta)$ and consider

$$N = r + c \cdot \eta. \tag{27}$$

Here r and c are material's constants and shown in the Table 1.

Thus relationship between imperfections parameters β and damage parameters ω will be in the form:

$$\beta = \beta_e \left[(r + c \cdot \eta) \cdot \exp \left[-(\omega - \omega_e)^{1-n} \right] \right]. \tag{28}$$

5 Conclusions

Thus, the linkage between maximum shear stress and shear strain is formulated for variety of rocks. Experimental data was taken from [10]. Model is presented as state equation (8), which includes imperfection parameter β. Suggested a connection between β and damage parameter ω, where kinetic equation in static form of Fokker-Plank equation is proposed for defining ω. Integration of this equation considering additional mitigation conditions managed to express β through ω and as a result to set constants of material that were necessary to define for describing stress-strain rock diagram.

Figs.3–6 present comparison of experimental data [10] and model representations as $\beta \sim \omega$ dependences. In this case, the solid lines correspond to the theoretical dependences.

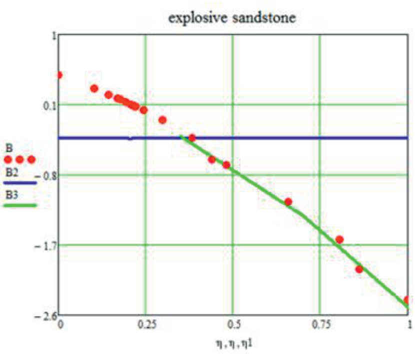

Fig.3

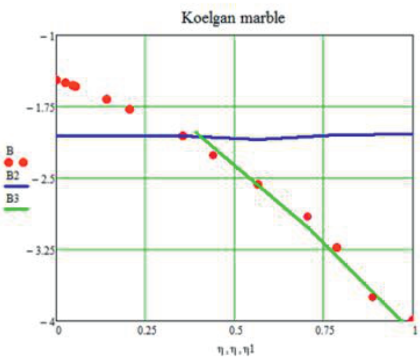

Fig.4

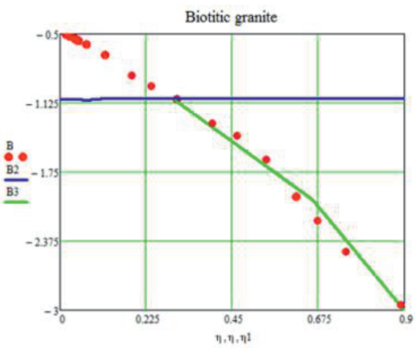

Fig.5

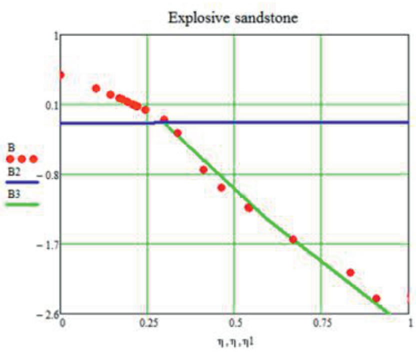

Fig.6

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