Wave thermorelaxation effects in materials with large amount of gas-filled cracks

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Abstract

A problem of propagation of temperature waves in materials with large amount of gas-filled cracks is considered. The cracks are supposed small enough comparing to the wave length so that the waves might be considered propagating in an equivalent continuum with the effective characteristics determined by the presence of these cracks. The relative distances between the cracks are supposed to be large enough so that while considering any of them the influence of the other cracks to mechanical behavior be negligible (dilute concentration approach). It is also supposed that the mechanical impact on the cracks is not sufficient to cause their growth. Effective deformation behavior of the medium in question (contrary to metals, rocks and ceramics without gas-filled cracks) appeared linear viscoelastic (anelastic) with essential rate dependence of the volumetric deformation, determined by compression of the gas contained in the cracks, especially noticeable at not too high and not too law rates. Therefore the connectivity parameter in equations of thermoelasticity appears much higher comparing to the case of usually considered media without considerable ammount of the gas-field cracks. Hence in case in question the connectivity effect in the thermoelasticity equations may be significant enough to necessitate its taking into account, which is done in the current work. It is noted that the obtained results related to propagation of thermoelastic waves in the described media have potentialities of their application to diagnostics of the this type of cracks inherent to gas-field rocks and steels under conditions of appearance and growth of hydrogen saturation and hydrogen cracking.

Introduction

The paper is devoted to consideration of thermomechanical behavior of media containing large amount of gas-filled cracks, particularly to wave propagation in that media. The cracks are supposed isolated and small enough comparing to a typical wave length under consideration so that the waves might be considered propagating in an equivalent continuum with the effective characteristics determined by the presence of these cracks. The relative distances between the cracks are supposed to be large enough, so that while considering any of them the influence of the other cracks on its mechanical behavior may be neglected (dilute concentration approach). It is also supposed that the mechanical impact on the cracks is not sufficient to cause their growth. Therefore, such a medium for our purposes may be considered as effectively homogeneous in scales comparable with or exceeding a typical wave length and possessing some effective properties to be found.

Further, two cases will be considered: (i) aligned cracks, which leads to an effectively anisotropic (transversal isotropic) medium with the plane of isotropy coinciding with the planes of cracks, and (ii) cracks distributed randomly over their orientations, which leads to an effectively isotropic medium [1].

1 Basic system of equations

The cracks of radius a and initial opening $2\delta_0 << a$ are supposed randomly distributed in space with concentration

$$\Omega = Na^3 \tag{1.1}$$

Here N is the number of cracks per unit volume. During loading the crack opening may change, so that it becomes $\delta \neq \delta_0$. Considering the cracks as flat spheroids, the relative volume of gas in the element of the media may be estimated as

$$\Omega_g = \frac{4\pi}{3} Na\delta^2 = \frac{4\pi}{3}\Omega\left(\frac{\delta}{a}\right)^2 \tag{1.2}$$

It is supposed that the cracks are filled with an ideal gas under pressure p. The externally remotely applied load is considered consisting of the sum of a constant stress $\sigma_{ij}^0 = \sigma^0\delta_{ij}$ plus, for example, plane longitudinal wave with high enough cycle frequency ω and comparatively slow varying with time amplitude $A(t)$

$$u(x,t) = A(t)\exp i(\omega t - kx) \tag{1.3}$$

Here $i^2 = -1$. In the following, such quantities, as the amplitude in (1.3), which generally may depend on the coordinates too, will be considered varying with their arguments slow enough to make it possible neglecting their variations within regions, containing sufficiently great number of gas-filled cracks under consideration and allowing to consider these regions as elementary volumes of an effective continuum or effective medium, which the equations of thermoelasticity will be derived for below for the dependent variables being the quantities of the above type.

Under the action of the vibrations produced by the wave, the compression-depression of the gas contained in the crack occurs resulting in relative heating-cooling cycles of the gas in the cracks, which, in turn, results in heat exchange between the gas in the cracks and the solid containing them and, as a result, to heating the latter. This heating has to be accounted for in the heat conduction equation [2] of the effective medium under consideration, which may be written in the form

$$C_v\frac{\partial T}{\partial t} + Q = \kappa\Delta T \tag{1.4}$$

Here T is the temperature; κ the heat conductivity; t the time; Q the rate of total heat generation respectively; the latter may be written as follows:

$$Q = \frac{C_p - C_v}{\alpha}\frac{\partial}{\partial t}\operatorname{div}\mathbf{u} + Q' \tag{1.5}$$

Here C_p, C_v are specific heats at constant pressure and volume respectively; $\mathbf{u}$ is the displacement vector; α the coefficient of thermal expansion; Q' the above mentioned heat generation due to heat exchange between the gas contained within cracks and containing them solid per unit of time. Note that the mass of gas in every crack is presumed to be constant. The effect due to the first term in the right hand side of (1.5) is ordinarily negligibly small that is presumed here. However, as it will be shown below, the second term may be of importance. Taking this into account, Equation (1.4) may be rewritten as

$$C_v\frac{\partial T}{\partial t} + Q' = \kappa\Delta T \tag{1.6}$$

It is important that this equation involves not only purely thermal variables, but also a variable $\left(Q'\right)$ depending on mechanical impact. In order to close the system of equations to be considered the equation of motion [2] should be added:

$$2\left(1-\nu^{ef}\right)\operatorname{grad}\operatorname{div}\mathbf{u}-\left(1-2\nu^{ef}\right)\operatorname{rot}\operatorname{rot}\mathbf{u} = \frac{2\alpha\left(1+\nu^{ef}\right)}{3}\operatorname{grad}T-\frac{2\rho\left(1+\nu^{ef}\right)\left(1-2\nu^{ef}\right)}{E^{ef}}\ddot{\mathbf{u}} \quad (1.7)$$

Here ρ is the density; ν^{ef} and E^{ef} are the effective Young's modulus and Poisson's ratio, respectively. Dots over variables correspond to partial derivatives with respect to time. All these values (as well as the values involved in Equation (1.6)) have to be considered as the effective values relevant to the equivalent (effective) homogeneous medium. Thus equations (1.6) and (1.7) form the closed system, which is fully coupled, because equation (1.6) involves also $Q' = Q'\left(\dot{u}\right)$ depending on mechanical quantities.

2 Determining heat generation due to heat exchange between gas and solid

The heat generation during the period of vibrating (which is obviously equal to the energy loss) may be written (in our notations) [3] as

$$Q'' = 2\pi W\tan\phi \quad (2.1)$$

Since we are interested in the heat generation during the period of time we need to divide the above value by the period $2\pi/\omega$ therefore

$$Q' = W\omega\tan\phi \quad (2.2)$$

Here W is the maximal value of the energy stored in the unit of volume; ϕ is usually called internal material friction. The former is found as follows [3]

$$W = \int_{\omega t=0}^{\pi/2}\sigma d\varepsilon \quad (2.3)$$

where σ, ε are tensors of instantaneous stress and strain, respectively. For the case under consideration the energy W consists of the energy stored by solid W_s, and the energy stored by gas W_g. The same is valid for the increments (all increments should be multiplied by $e^{i\omega t}$)

$$\Delta W = \Delta W_g + \Delta W_s \quad (2.4)$$

The contribution of pore (crack) gas (assumed to be ideal) into the specific internal energy is

$$W_g = \frac{1}{\gamma-1}pVN \quad (2.5)$$

Here $\gamma = C_p/C_v$. The change in the specific internal energy due to loading is (neglecting the terms containing $\Delta p\Delta V$):

$$\Delta W_g = \frac{N}{\gamma-1}\left(pV-p_0V_0\right) = \frac{N}{\gamma-1}\left(p_0\Delta V+V_0\Delta p\right) \quad (2.6)$$

Expressions for $\Delta p,\ \Delta V$ are given in [1]

$$\Delta p = -\Delta\sigma \left[1 + \frac{\delta_0}{a}\frac{E*}{p_0}\frac{4\pi}{3}\left(1 - \frac{\gamma-1}{\gamma}f\right)\right]^{-1} \tag{2.7}$$

$$f = f_R + i\, f_I \tag{2.8}$$

$$f_R = 1 - \frac{3}{2q^3}\int\limits_0^q x\frac{\tanh x\left(1+\tan^2 x\right) + \tan x\left(1-\tanh^2 x\right)}{1+\tan^2 x \tanh^2 x}dx \tag{2.9}$$

$$f_I = -\frac{3}{2q^3}\int\limits_0^q x\frac{\tan x\left(1-\tanh^2 x\right) - \tanh x\left(1+\tan^2 x\right)}{1+\tan^2 x \tanh^2 x}dx \tag{2.10}$$

$$q = \delta_0\sqrt{\omega/2\kappa} \tag{2.11}$$

$$\Delta V = \frac{a^3}{E*}\left(\Delta p + \Delta\sigma\right) \tag{2.12}$$

$$E* = \frac{3E}{16\left(1-\nu^2\right)} \tag{2.13}$$

The integrals in (2.9), (2.10) exist for any values of q. The change in the energy stored by the solid according to (2.12) is

$$\Delta W = \frac{a^3}{E*}\left(p+\sigma\right)\left(\Delta p + \Delta\sigma\right) \tag{2.14}$$

The expression for the internal material friction was also obtained in [1]. For the cracks randomly distributed over their orientations it is

$$\tan\phi = \frac{16\left(1-\nu^2\right)}{15}\frac{\psi_I}{\psi_R^2+\psi_I^2}\Omega \tag{2.15}$$

Here

$$\psi_R = 1 + \frac{\pi}{4\left(1-\nu^2\right)}\left(1 - \frac{\gamma-1}{\gamma}f_R\right)\frac{E}{p_0}\frac{\delta_0}{a} \tag{2.16}$$

$$\psi_I = \frac{\pi}{4\left(1-\nu^2\right)}\frac{\gamma-1}{\gamma}f_I\frac{E}{p_0}\frac{\delta_0}{a} \tag{2.17}$$

For the system of aligned cracks, coefficient 15 in the denominator of (2.15) should be replaced with 3. Summarizing it up, the value of Q' for formula (1.6) is given by (2.2)-(2.4), (2.6)-(2.17).

3 Determining the other parameters involved

The expressions for effective Young's modulus and Poisson's ratio required for (1.7) are given in [1]. For isotropic over crack orientations, crack distribution they are

$$E^{ef}/E = 1 - \frac{16\left(1-\nu^2\right)}{15}\left[\frac{10-3\nu}{2\left(2-\nu\right)} - \frac{\psi_R}{\psi_R^2+\psi_I^2}\right]\Omega + i\frac{16\left(1-\nu^2\right)}{15}\frac{\psi_I}{\psi_R^2+\psi_I^2}\Omega \quad (3.1)$$

$$\nu^{ef} = \nu\left\{1 - \frac{16\left(1-\nu^2\right)}{15}\left[\frac{3-\nu}{2-\nu} - \frac{1+3\nu}{3\nu}\frac{\psi_R}{\psi_R^2+\psi_I^2}\right]\Omega\right\} - i\frac{16\left(1-\nu^2\right)\left(1+3\nu\right)}{45}\frac{\psi_I}{\psi_R^2+\psi_I^2}\Omega \quad (3.2)$$

The case of parallel crack alignment is also considered in [1]. Such crack alignment leads to transversal anisotropy of the effective medium and hence equation (1.7) has to be correspondingly modified.

Functions f_R and f_I from (2.9), (2.10) may be approximated by simple functions, yielding the correct asymptotic behavior both for small and large q with the relative error not exceeding 6%, as follows

$$f_R = \frac{4q^3}{12+3q^2+4q^3} \quad (3.3)$$

$$f_I = \frac{6q^2}{15-3.5q+8q^3} \quad (3.4)$$

Conclusions

Effective deformation behavior of the medium in question (contrary to metals, rocks and ceramics without considerable amount of gas-filled cracks), appearing linear viscoelastic (anelastic) with essential rate dependence of the volumetric deformation, determined by compression-depression of the gas contained in the cracks, is especially noticeable at not too high and not too law rates of loading. Therefore, the connectivity parameter in equations of thermoelasticity (responsible for essential dependence of heat-evolution on mechanical impact) appears much higher comparing to the case of usually considered media without the gas-filled cracks. Hence in the case in question the connectivity effect in the thermoelasticity equations may be significant enough to necessitate its taking into account, which is done in the current work. The obtained results related to propagation of thermoelastic waves in the described media are in particular of interest from the point of view of potentialities of their application to diagnostics of the presence and parameters of multitude of gas-field cracks in rocks, as well as in steels under conditions promoting appearance and development in them of hydrogen cracking.

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