Nonlinear effects in the vibrating cavity filled with a perfect gas

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Abstract

Numerical investigation of the influence of vibration on the behavior of a perfect gas inside the cavity is performed. The one-dimensional problem is solved. The comparison of the calculation results with the analytical solution of the problem in the linear approximation is presented. The analysis of the nonlinear effects is performed.

1 Introduction

The materials often contain cracks, pores or voids filled with air. The mechanisms sometimes also have cavities of technological origin. Machinery work is often accompanied by vibration. Vibrational effect can also be a part of a technological process. All this determines why it is necessary to study in detail the effects of vibration on the cavity filled with gas. The effects of vibration have earlier come into the focus of many researchers (see e.g. [1, 2, 3, 4, 5, 6, 7]). Fu and Shieh [1, 2] numerically investigated the thermal convection in a two-dimensional square enclosure induced simultaneously by gravity and vertical vibration. Aktas and Ozgumus [3] numerically investigated the effects of classical and irregular streaming motion on convective heat transfer in air-filled shallow enclosures carrying a standing sound wave. The fluid motion was driven by the periodic vibration of the enclosure left wall. Some authors investigated the disturbances produced in a closed, gas-filled tube by the oscillations of a piston at one end [4, 5, 6]. Numerical study of heat and mass transfer processes in the cavity under vibration exposure with frequency much lower than the natural frequency of the system was carried out in [7]. The maximum temperature achieved inside the cavity was found and the transitional mode which includes the complicated wave motion at small times was described.

The present paper deals with the influence of vibration with constant frequency on the heat and mass transfer inside the cavity filled with a perfect gas. Both walls are kept at constant temperature which equals the initial temperature. The range of vibration frequencies (with constant amplitude of vibration) covers low exposures and high exposures. The low exposures can be described by the linear theory. The high exposures leads to the nonlinear effects. The resonant phenomena is not considered in this paper.

2 Problem formulation. Mathematical model

The cavity of length L filled with a viscous perfect gas with thermal properties of air is considered (Fig.1). At the initial moment the gas in the cavity stayed at rest with a constant temperature T_0 and pressure P_0. The equilibrium state is disbalanced due to the vibrational effects with amplitude A and frequency ω. In the beginning the cavity was in the extreme right position. The boundaries are kept at constant temperature T_0. The thermal conductivity, the dynamic viscosity and the heat capacity are assumed to be constant.

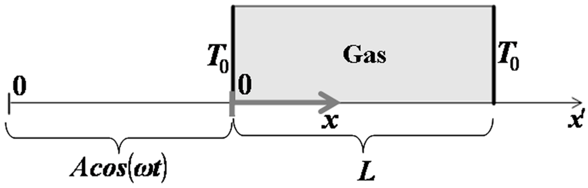

Figure 1: Schematic of the problem

The heat and mass transfer of the gas under these assumptions is described with the one-dimensional nonstationary system of equations in Cartesian coordinates. The Clapeyron ideal gas law is considered as the equation of state. The system is written in noninertial frame of reference associated with the vibrating cavity.

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \tag{1}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \frac{4}{3}\mu \frac{\partial^2 u}{\partial x^2} + \rho A \omega^2 \cos(\omega t), \tag{2}$$

$$c_v \rho \frac{\partial T}{\partial t} + c_v \rho u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial x^2} - p \frac{\partial u}{\partial x} + \frac{4}{3}\mu (\frac{\partial u}{\partial x})^2, \tag{3}$$

$$p = \rho R T \tag{4}$$

Here, t - time, x - coordinate in noninertial frame of reference, u - velocity in noninertial frame of reference, A - vibration amplitude, ω - frequency of vibration, ρ - density, p - pressure, T - temperature, R - gas constant, μ - coefficient of dynamic viscosity, k - thermal conductivity coefficient and c_v - specific heat capacity at constant volume.

The initial and boundary conditions are as follows:

$$t = 0 : u = 0,\, T = T_0,\, p = p_0,\, \rho = \frac{p_0}{RT_0} = \rho_0, \tag{5}$$

$$x = 0 : u = 0,\, T = T_0, \tag{6}$$

$$x = L : u = 0,\, T = T_0 \tag{7}$$

The following dimensionless variables and parameters are introduced:

$$X = \frac{x}{L},\, \tau = \frac{tc_0}{L},\, U = \frac{u}{c_0},\, P = \frac{p}{\gamma p_0},\, \tilde{\rho} = \frac{\rho}{\rho_0},\, \Theta = \frac{T - T_0}{T_0},$$

$Re_a = \frac{\rho_0 L c_0}{\mu}$ - acoustical Reynolds number,
$\gamma = \frac{c_p}{c_v}$ - ratio of specific heats,
$\Omega = \frac{\omega L}{c_0}$ - nondimensional vibration frequency,
$\tilde{A} = \frac{A}{L}$ - nondimensional vibration amplitude,
where $c_0 = (\frac{\gamma p_0}{\rho_0})^{\frac{1}{2}}$ is adiabatic speed of sound.

The dimensionless governing equations can be expressed as follows:

$$\frac{\partial \tilde{\rho}}{\partial \tau} + \frac{\partial \tilde{\rho} U}{\partial X} = 0, \tag{8}$$

$$\tilde{\rho}\frac{\partial U}{\partial \tau} + \tilde{\rho} U \frac{\partial U}{\partial X} = -\frac{\partial P}{\partial X} + \frac{4}{3}\frac{1}{Re_a}\frac{\partial^2 U}{\partial X^2} + \tilde{\rho} A \Omega^2 \cos(\Omega\tau), \tag{9}$$

$$\tilde{\rho}\frac{\partial \Theta}{\partial \tau} + \tilde{\rho} U \frac{\partial \Theta}{\partial X} = \Gamma \frac{\partial^2 \Theta}{\partial X^2} - \gamma(\gamma-1) P \frac{\partial U}{\partial X} + \frac{4}{3}\frac{1}{Re_a}\gamma(\gamma-1)(\frac{\partial U}{\partial X})^2, \tag{10}$$

$$P = \frac{\tilde{\rho}(\Theta+1)}{\gamma}, \tag{11}$$

here $\Gamma = \frac{\gamma k}{\rho_0 L c_p c_0}$
The dimensionless initial and boundary conditions are as follows:

$$\tau = 0 : U = 0,\ \Theta = 0,\ P = \frac{1}{\gamma},\ \tilde{\rho} = 1, \tag{12}$$

$$X = 0 : U = 0,\ \Theta = 0, \tag{13}$$

$$X = 1 : U = 0,\ \Theta = 0 \tag{14}$$

Thus, the vibration is specified by the body force per unit mass (driving force) $f = \tilde{A}\Omega^2\cos(\Omega\tau)$, where $\tilde{A}\Omega^2$ is amplitude of the driving force. The linear theory can be used with small velocity of gas in a wave ($\rho - \rho_0 << \rho_0, \tilde{A}\Omega^2 << 1$). Let $\tilde{A}$ be constant, so Ω will be a parameter of nonlinearity.

3 Results and discussion

The numerical method and algorithm which we used for numerical solution are described in [7]. The finite volume method with the power-law differencing scheme [8] was taken as a basis for the numerical method. The numerical scheme is implicit, conservative and it can be used for flows with shock waves. The dimensionless parameters were taken following: $Re_a = 10^5$, $\gamma = 1.4$, $\tilde{A} = 2$, $\Omega = 0.14, 0.29, 0.43, 0.58, 0.72, 0.86$. The number of grid points was increased with the frequency of vibration (from 1000 to 5000). The time step was taken in terms of Courant condition. The conservation of mass and energy was checked. The numerical scheme was verified by some tests with analytical solutions (exact solution of the linearized equations, shock wave formation, the Sod shock tube test [9], reflection of a shock wave from a wall).

The process includes free oscillations (acoustic waves) and forced oscillations, which appear due to the periodic driving force. It is possible to find the analytical solution of the linearized equations without taking into account free oscillations which are damped with time. Whereas the driving force $f = \tilde{A}\Omega^2\cos(\Omega\tau)$ is uniformly distributed on the domain with zero boundary conditions for velocity, the analytical solution for the linear theory can be expressed as follows:

$$P(X,\tau) = -\frac{\tilde{A}\Omega\cos(\Omega\tau)\sin(\Omega(0.5-X))}{\cos(0.5\Omega)} + \frac{1}{\gamma}, \tag{15}$$

$$U(X,\tau) = -\frac{\tilde{A}\Omega\sin(\Omega\tau)\cos(\Omega(0.5-X))}{\cos(0.5\Omega)} + \tilde{A}\Omega\sin(\Omega\tau), \tag{16}$$

$$\rho(X,\tau) = -\frac{\tilde{A}\Omega\cos(\Omega\tau)\sin(\Omega(0.5-X))}{\cos(0.5\Omega)} + 1, \tag{17}$$

$$\Theta(X,\tau) = (1-\gamma)\frac{\tilde{A}\Omega\cos(\Omega\tau)\sin(\Omega(0.5-X))}{\cos(0.5\Omega)}, \tag{18}$$

We don't consider the resonance frequencies when the denominator becomes zero. The frequencies less than resonance frequencies will be taken ($\Omega = 0.14, 0.29, 0.43, 0.58, 0.72, 0.86$).

First, we shall consider the vibration process with the lowest frequency for the range $\Omega = 0.14$ ($A\Omega^2 = 0.04$). Because initially the gas was at a state of rest the vibrational action leads to the appearance of acoustic waves.

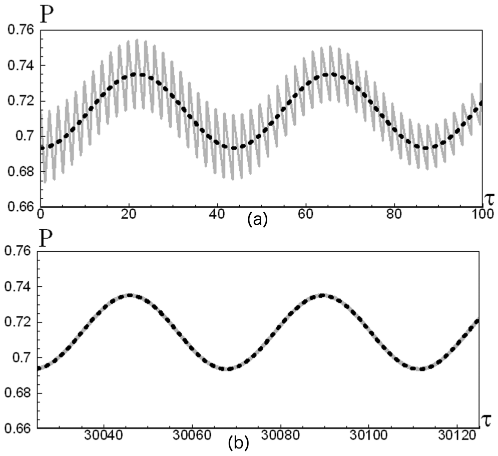

Figure 2: Variation of pressure with time for $\Omega = 0.14$ at X=0.0005 (numerical - solid line, analytic - dashed line)

Fig. 2 shows the time dependence of pressure for $\Omega = 0.14$ at X=0.0005. The solid line shows the numerical solution, the dashed line - analytic solution for the linearized equations taking no account of acoustic oscillations which are shown at Fig. 2 (a). The acoustic waves with taking frequency $\Omega = 0.14$ are damped roughly at $\tau = 1100$ but the steady-state oscillation mode reaches later, roughly at $\tau = 30000$. Note that the analytic solution is the mean line for the numerical solution. The amplitude of driving force is small for $\Omega = 0.14$, so the numerical solution must be same with the analytic solution at the steady-state oscillation mode. It was used as a test and the result is quite good (Fig. 2 (b)).

The analytic solution for temperature, density and pressure is the standing waves with a node in the center of the domain. The analytic solution for velocity is the standing waves with nodes at the boundaries and antinode in the center. At Fig. 3 is shown that the numerical solution is sufficiently close to the analytic solution at the steady-state oscillation mode except the boundary region where the boundary temperature has influence. The cavity is situated at the extreme positions at chosen moments of time.

Let us consider the vibration process with the highest frequency for the range $\Omega = 0.86$ when the process is significantly nonlinear ($A\Omega^2 = 1.48$). The character of the process is quite different from the previous linear case. The free oscillations include shock waves which also are damped with time. The free oscillations are damped at $\tau = 190$. The steady-state oscillation mode reaches roughly at $\tau = 15000$. Now, the numerical solution is not the same with the analytic solution at the steady-state oscillation mode (Fig. 4).

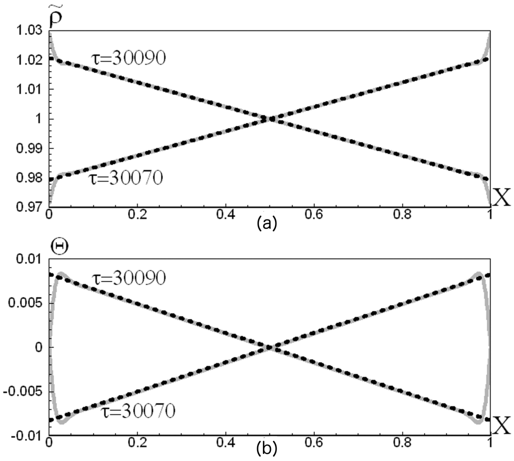

Figure 3: Coordinate dependence of density and temperature for $\Omega = 0.14$ at the steady-state oscillation mode (numerical - solid line, analytic - dashed line)

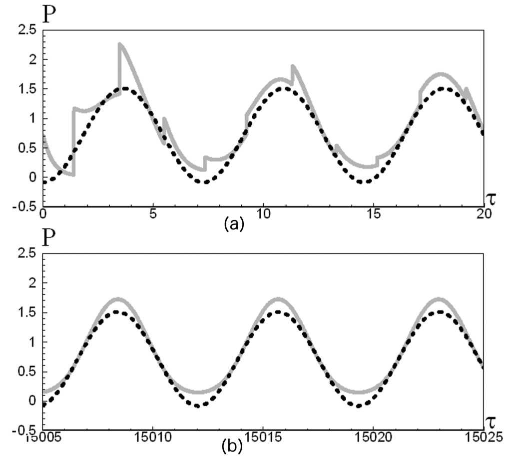

Figure 4: Variation of pressure with time for $\Omega = 0.86$ at X=0.0005 (numerical - solid line, analytic - dashed line)

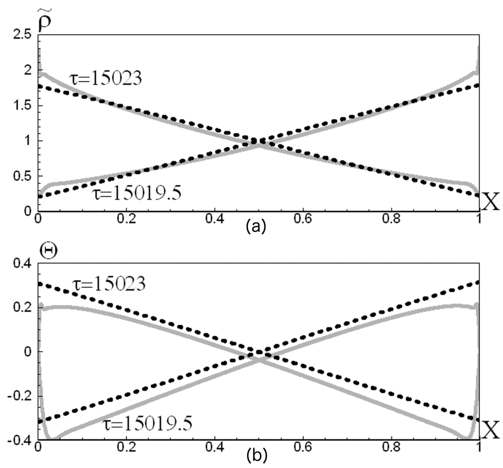

Figure 5: Coordinate dependence of density and temperature for $\Omega = 0.86$ at the steady-state oscillation mode (numerical - solid line, analytic - dashed line)

Because of density change at $\Omega = 0.86$ is not small, there are the nonlinear effects. Linear theory cannot describe the process, and the numerical solution is different from the analytic solution for linearized equations (Fig. 5). Consider now the feature of the numerical solution at the steady-state oscillation mode. Due to the influence of centrifugal force there is an outflow of the gas from the center of the domain. The period average density is greater than the initial density near the boundaries and less than initial density near the center of the domain (Fig. 5(a), Fig. 6).

The period average density is less than the initial density in the interval $0.2 \leq X \leq 0.8$. Density always less than the initial density in the interval $0.46 \leq X \leq 0.54$. Although there is the constant force acting by the cavity all time, temperature and pressure are also less than the initial temperature and pressure in the central part of the domain. The period average pressure is less than the initial pressure in $0.23 \leq X \leq 0.77$. Pressure always less than the initial pressure in $0.46 \leq X \leq 0.54$. The period average temperature is less than the initial temperature almost in all domain in $0.0055 \leq X \leq 0.9945$. Temperature always less than the initial temperature in $0.45 \leq X \leq 0.55$. The period average velocity is equal zero.

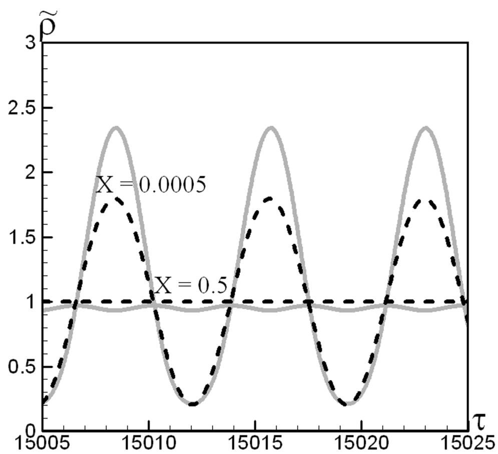

Figure 6: Variation of density with time for $\Omega = 0.86$ at the steady-state oscillation mode (numerical - solid line, analytic - dashed line)

Let us describe another nonlinear effect. In the linear case there was a wave node in the center of the domain. In this case, the oscillations in the center are observed. As shown in Fig. 6 the oscillations of density has a double frequency in the center of the domain in comparison with the oscillations of density near boundary. The oscillations of density near boundary has the same frequency with the driving force. Density, pressure and temperature oscillations have a double frequency in the center of the domain. Variation of the period average density in the center of the domain with frequency of vibration is shown in Fig. 7. The mass outflow from the center increases with the frequency of vibration. Variation of the period average temperature in the center of the domain with frequency of vibration is shown in Fig. 8.

Thus, the nonlinear effects increase with the frequency of vibration. The linear theory can be applied for frequencies $\Omega = 0.14$ and $\Omega = 0.29$ in the chosen frequency range. Generally, the linear theory can be applied when $\tilde{A}\Omega^2 < 0.3$, then the difference between numerical and analytic solutions is significant only near resonance frequencies.

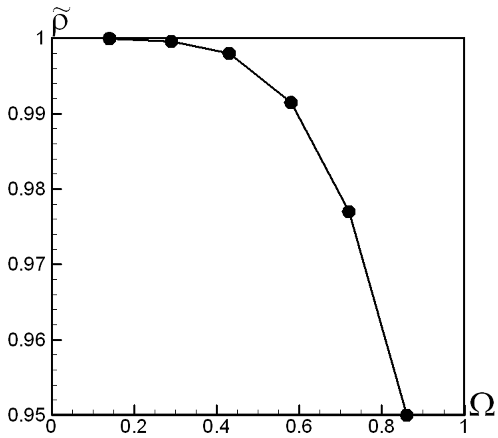

Figure 7: Variation of the period average density in the center of the domain with frequency of vibration

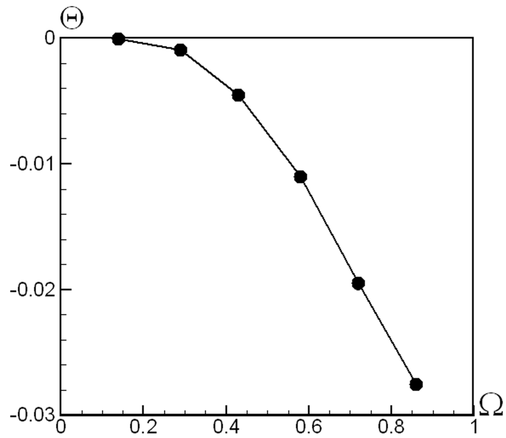

Figure 8: Variation of the period average temperature in the center of the domain with frequency of vibration

4 Conclusions

The process of heat and mass transfer is quite nonlinear at the case of strong vibrational action. There is the gas rarefaction and reducing of the period average pressure and temperature in the central part of the domain in comparison with their initial significants. The oscillations of temperature, density and pressure have the double frequency in the center of the domain as opposed to the linear case when there is a node of the standing wave.

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