Interaction of Circular Cavities and a Crack near Bimaterials Interface under Incident SH-Waves

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Abstract

The SH-waves scattering by two cavities and a crack near bimaterials interface is reported applying the methods of Green's function and complex variables in this paper. Based on the idea of "conjunction", the bimaterials is divided along the horizontal interface into an elastic half space possessing a circular cavity and a crack and an elastic half space containing a circular cavity. Firstly, the scattering displacement field of half space containing an elliptical cavity is constructed, and the corresponding Green's functions of two half spaces are then deduced. Combined with "crack-division", a crack is created, and thus expressions of displacement and stress are derived while the cavity co-exists with the crack. Undetermined anti-plane forces are loaded on the horizontal surfaces for conjunction of two sections and then solved by a series of Fredholm integral equations based on continuity conditions of the interface. Finally, this paper presents discussion of influence of different parameters on dynamic stress concentration factors around the cavity.

Keywords: SH-waves; circular cavity; bimaterials interface; Green's function; dynamic stress concentration factor

1 Introduction

Interface problem exists in engineering practice and underground exploration engineering, such as underground strata, laminates, composites, polycrystal composites, various bonding structure and so on. In nature, defects like cavity, inclusion and crack are always clustered around underground strata, and in the study of material properties, flaws are also inevitable, which usually occur at the region where material property varies acutely in media or structure, namely the surface or the vicinity of interface.

In recent decades, ascribed to delamination character of the earth medium, wave propagation issue in solid interface increasingly fascinates researchers. The complex function theory has become popular in elastic wave scattering analysis since it was firstly used by Liu et al. for solution of static stress concentration around an irregularly shaped cavity in an infinite elastic plane[1], Chen et al. used it for upright incident of SH-waves at semi-cylindrical interface with a circular lining structure[2]; Wang and Liu [3] developed it to address steady SH-wave scattering and perform dynamic analysis of multiple circular cavities in half space, Xu et al. applied the method to study scattering of plane SH waves by two separated circular tunnel linings[4]; using which SH-wave scattering by cavities in the neighborhood of bimaterials interface was investigated by Liu et al. [5]. Numerous researches on interaction of a crack and elastic wave near interface have been also reported. Using crack division and weakly singular integral equation method, Liu et al. [6] obtained the solution of the scattering of SH-waves by an interface linear crack and a circular cavity near bimaterials interface; Yang et al. solved the problem of the scattering of SH-wave by an interacting mode III crack and an inclusion in a half space[7, 8]; Yang et al.[9] studied the scattering of SH-waves and ground motion by an elastic cylindrical inclusion and a crack in a half space by using the Green function, complex variables and multi-polar coordinates system; Qi and Yang investigated[10] the diffraction of time harmonic anti-plane shear waves by arbitrary positions of inclusions and a finite length interfacial crack embedded in an elastic half-space impacted by SH-wave; Bair and David [11] employed probabilistic method to analyze two-dimensional wave scattering by surface-breaking cracks and cavities.

The methods of Green's function and complex variables are applied in current paper to investigate the SH-wave scattering by two circular cavities and a crack near bimaterials interface focusing on discussion on dynamic response around the circular cavities in the case of different parameters.

2 Problem Statement

As Fig.1 shows, the media ỹcindicates an elastic half-space containing a circular cavity and a beeline crack with an incident wave $W^{(i)}$ from downside, while the media ỹт expresses another half-space possessing an beeline crack, and an wave $W^{(i)}$ symmetrical with $W^{(i)}$ is incident obliquely from upperside. The material parameters of the medias are defined as $(\rho_1, \mu_1; \rho_2, \mu_2)$, the length of crack is $l = 2A$. Four coordinates:XOY, $X_1O_1Y_1$, $X_2O_2Y_2$and $X'O'Y'$ are established, which have following relationship

$$x' = x = x_2, y' = y - h = y_2 + h_3, y_2 = y' - h_3$$
$$x_1 = x\cos\beta + y\sin\beta, y_1 = y\cos\beta - x\sin\beta,$$
$$h_2 = \frac{(h + h_1 - c\sin\beta)}{\cos\beta} \quad (1)$$

The boundary conditions are given,

$$\tau^{(t)}_{rz,a} = 0, r = R_1 \quad (2)$$
$$\tau^{(t)}_{rz,b} = 0, r = R_2 \quad (3)$$

3 Governing equation

In an isotropic medium, the scattering of SH-wave is the easiest problem of the study on scattering of elastic wave. In complex plane $(z, \bar{z})$,the displacement field W excited by SH-waves obeys

$$\frac{\partial^2 W}{\partial Z \partial \bar{Z}} + \frac{1}{4}k^2 W = 0 \quad (4)$$

where,$k = \frac{\omega}{c_s}$,in which ω is the circular frequency of wave function,$c_s = \sqrt{\frac{\mu}{\rho}}$ stands for the propagation velocity of the shear wave, and and ¦M are the mass density and shear modulus of the medium respectively.

The corresponding stresses are given by

$$\tau_{rz} = \mu(\frac{\partial W}{\partial z}e^{i\theta} + \frac{\partial W}{\partial \bar{z}}e^{-i\theta}), \tau_{\theta z} = i\mu(\frac{\partial W}{\partial z}e^{i\theta} - \frac{\partial W}{\partial \bar{z}}e^{-i\theta}) \quad (5)$$

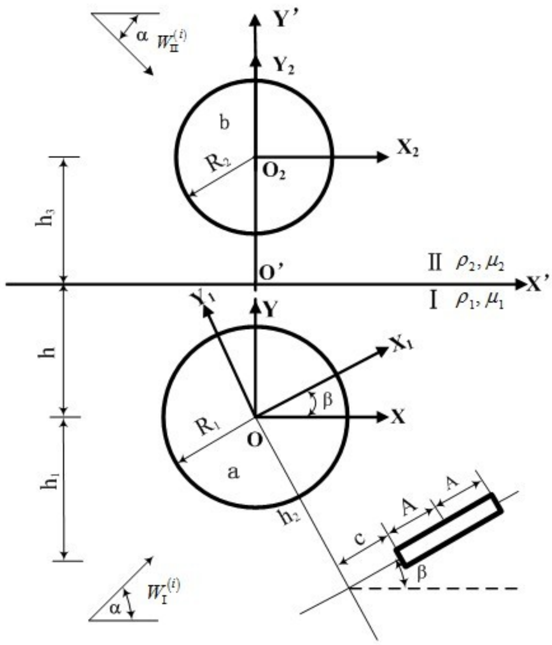


Figure 1: Model of circular cavities and a crack near bimaterials interface

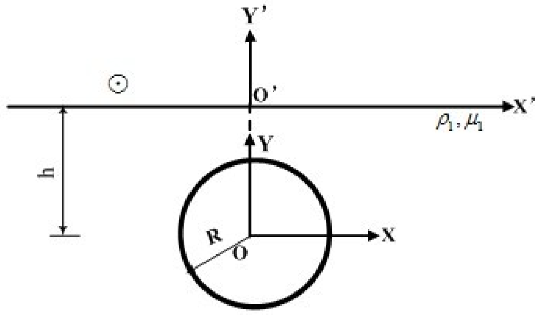


Figure 2: Half-space model of elliptical cavity impacted by an out-plane source load at surface

4 Green's function

4.1 Green's function G_1

The first Green's function in current paper is the displacement field of half-space of a circular cavity impacted by an out-plane source load at surface, marked as G_1.

The analysis model is given by Fig.2, which has boundary conditions as follows

$$\tau_{rz}^{(i)} + \tau_{rz}^{(s)} = 0, r = R \tag{6}$$

In an elastic half-space, the wave $G_1^{(i)}$ excited by the out-plane line load $\delta(z - z_0)$ can be regarded as an incident wave, and in mapping plane, it is defined as

$$G^{(i)} = \frac{i}{2\mu} H_0^{(1)}(k|z - z_0|) \tag{7}$$

where $H_0^{(1)}(\cdot)$ is the 0th order Hankel function of the first kind.

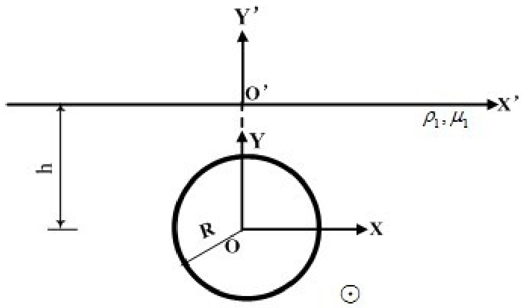


Figure 3: Model of elliptical cavity impacted by an out-plane source load in half-space

The scattering waves $G^{(s)}$ caused by the cavity can be given as

$$G^{(s)} = \sum_{n=-\infty}^{\infty} A_n [H_n^{(1)}(k|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k|z-2ih|)(\frac{z-2ih}{|z-2ih|})^{-n}] \tag{8}$$

where A_n are unknown coefficients decided by the boundary condition (6).

Now, the total wave field can be derived

$$\begin{aligned} G_1 = G^{(i)} + G^{(s)} = &\frac{i}{2\mu} H_0^{(1)}(k|z-z_0|) \\ &+ \sum_{n=-\infty}^{\infty} A_n [H_n^{(1)}(k|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k|z-2ih|)(\frac{z-2ih}{|z-2ih|})^{-n}] \end{aligned} \tag{9}$$

Substitution of the expression (9) into Eq.(5), the corresponding stresses can be obtained.

4.2 Green's function G_2

In the full elastic space II, the wave field $G^{(i)}$ excitated by the out-plane line load $\delta(z-z_0)$ and the reflected wave $G^{(r)}$ caused by surface can be described as following forms

$$G^{(i)} = \frac{i}{4\mu} H_0^{(1)}(k|z' - z'_0|) \tag{10}$$

$$G^{(r)} = \frac{i}{4\mu} H_0^{(1)}(k|z' - \bar{z}'_0|) \tag{11}$$

in which,z'_0 indicates the source point and $\bar{z}'_0$ is the complex conjugate.

The scattering waves $G^{(s)}$ caused by the circular cavity can be given as Eq.(8).

In half-space I, the total wave field is obtained

$$\begin{aligned} G_1 = G^{(i)} + G^{(r)} + G^{(s)} = &\frac{i}{4\mu} H_0^{(1)}(k|z' - z'_0|) + \frac{i}{4\mu} H_0^{(1)}(k|z' - \bar{z}'_0|) \\ &+ \sum_{n=-\infty}^{\infty} A_n [H_n^{(1)}(k|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k|z-2ih|)(\frac{z-2ih}{|z-2ih|})^{-n}] \end{aligned} \tag{12}$$

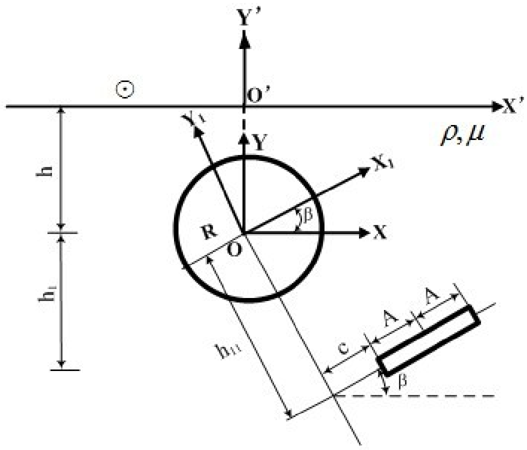


Figure 4: Half-space model of circular cavity and crack impacted by an out-plane source load at surface

4.3 Green's function G_3

The Green's function in this section is the displacement field of half-space of a circular cavity and crack impacted by an out-plane source load at surface, marked as G_3, shown as Fig.4.

According to Section 4.1, the wave function of half space without crack is provided as

$$G_1 = G^{(i)} + G^{(s)} = \frac{i}{2\mu} H_0^{(1)}(k|z-z_0|) + \sum_{n=-\infty}^{\infty} A_n [H_n^{(1)}(k|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k|z-2ih|)(\frac{z-2ih}{|z-2ih|})^{-n}] \quad (13)$$

Subsequently, the total stresses of arbitrary point in the half space can be solved when the surface bears out-plane harmonic line load. Based on "crack-division" method, if additional stresses with the same magnitude and opposite direction as the total stresses are applied at the same point, the ultimate stresses of this point are zero, and consequently a crack will be created when a pair of forces with the same magnitude and opposite direction are loaded along the region where the crack will appear. Using the Green's function G_2 derived in Section 4.2, the wave field produced in creating crack by means of antiplane loads' infliction, which is indicated as $G^{(c)}$, is expressed as

$$G^{(c)} = \int_l \tau_{\theta_2 z_2} G_2 dl \quad (14)$$

Then the total wave field is obtained

$$G_3 = G^i + G^s - G^{(c)} = \frac{i}{2\mu} H_0^{(1)}(k|z-z_0|) + \sum_{n=-\infty}^{\infty} A_n [H_n^{(1)}(k|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k|z-2ih|)(\frac{z-2ih}{|z-2ih|})^{-n}] - \int_l \tau_{\theta_2 z_2} G_2 dl \quad (15)$$

5 Scattering by circular cavities and a crack near bimaterials interface

5.1 Incident SH-waves

Interaction of circular cavities and a crack near bimaterials interface under incident SH-waves can be regarded as "conjunction" problem to investigate. Firstly, along the interface we subdivide the bimaterials into elastic half space I and II, in which, I possesses a circular cavity and a crack while II represents a full elastic space containing a circular cavity.

In medium I,$W^{(i)}$ and $W^{(r)}$ are described as following forms:

$$W_I^{(i)} = W_0 \exp\{\frac{ik_1}{2}[(z - ih)e^{-i\alpha} + (\bar{z} + ih)e^{i\alpha}]\} \tag{16}$$

$$W_I^{(r)} = W_0 \exp\{\frac{ik_1}{2}[(z - ih)e^{i\alpha} + (\bar{z} + ih)e^{-i\alpha}]\} \tag{17}$$

where W_0 is maximum amplitude of incident wave and α denotes incident angle. In the half-space II, for symmetry in angle, the expressions of the incident wave $W_{II}^{(i)}$ and the reflected wave $W_{II}^{(r)}$ have the same form as $W_I^{(i)}$ and $W_I^{(r)}$ with different parameters, but opposite in direction, which can be respectively expressed as

$$W_{II}^{(i)} = W_0 \exp\{\frac{ik_2}{2}[(z_2 - ih_3)e^{i\alpha} + (\bar{z}_2 + ih_3)e^{-i\alpha}]\} \tag{18}$$

$$W_{II}^{(r)} = W_0 \exp\{\frac{ik_2}{2}[(z_2 - ih_3)e^{-i\alpha} + (\bar{z}_2 + ih_3)e^{i\alpha}]\} \tag{19}$$

5.2 The scattering waves around the circular cavities

Now, based on the symmetry of the scattering wave and multi-polar coordinates, the scattering waves $W_I^{(s)}$ and $W_{II}^{(s)}$ around the cavity a and b can be constructed, which should obey the governing equation (4) and the radiation condition for infinite distance except the stress free condition on the horizontal interface.

In the media I, $W_I^{(s)}$ takes the form of

$$W_I^{(s)} = \sum_{n=-\infty}^{\infty} A_n [H_n^{(1)}(k_1|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k_1|z - 2ih|)(\frac{z - 2ih}{|z - 2ih|})^{-n}] \tag{20}$$

where A_n are the unknown coefficients determined by the boundary condition of the circular cavity a.

In the media I, $W_{II}^{(r)}$ is given by

$$W_{II}^{(s)} = \sum_{n=-\infty}^{\infty} B_n [H_n^{(1)(k_2|z_2|)}(\frac{z_2}{|z_2|})^n + H_n^{(1)(k_2|z_2 - 2ih_3|)}(\frac{z_2 - 2ih_3}{|z_2 - 2ih_3|})^{-n}] \tag{21}$$

where B_n are the unknown coefficients determined by the boundary condition of the circular cavity b.

5.3 The wave fields in district I and II

In the half space I

$$\begin{aligned}W_I &= W_I^{(i)} + W_I^{(r)} + W_I^{(s)}\\ &= W_0 \exp\{\frac{ik_1}{2}[(z-ih)e^{-i\alpha} + (\bar{z}+ih)e^{i\alpha}]\} + W_0 \exp\{\frac{ik_1}{2}[(z-ih)e^{i\alpha} + (\bar{z}+ih)e^{-i\alpha}]\}\\ &\quad + \sum_{n=-\infty}^{\infty} A_n[H_n^{(1)}(k_1|z|)(\frac{z}{|z|})^n + H_n^{(1)}(k_1|z-2ih|)(\frac{z-2ih}{|z-2ih|})^{-n}] \quad (22)\end{aligned}$$

Subsequently, the total stresses of arbitrary point in the half space can be solved when the surface bears out-plane harmonic line load. Based on "crack-division" method, if additional stresses with the same magnitude and opposite direction as the total stresses are applied at the same point, the ultimate stresses of this point are zero, and consequently a crack will be created when a pair of forces with the same magnitude and opposite direction are loaded along the region where the crack will appear. Using the Green's function G_2 derived in Section 4.2, we can obtain the total wave field of the half-space I

$$W_I^{(t)} = W_I^{(i)} + W_I^{(r)} + W_I^{(s)} - \int_{(c,h_2)}^{(2A+c,-h_2)} \tau_{\theta z,I} G_2 dz_1 \quad (23)$$

where $\tau_{\theta z,I}$ denotes the force with the same magnitude and opposite direction as that caused by G_2 loaded along the region where the crack will appear.

For the half space II, the total wave field cab be written as

$$\begin{aligned}W_{II}^{(t)} &= W_{II}^{(i)} + W_{II}^{(r)} + W_{II}^{(s)}\\ &= W_0\exp\{\frac{ik_2}{2}[(z_2-ih_3)e^{i\alpha}+(\bar{z_2}+ih_3)e^{-i\alpha}]\}+W_0\exp\{\frac{ik_2}{2}[(z_2-ih_3)e^{i\alpha}+(\bar{z_2}+ih_3)e^{-i\alpha}]\}\\ &\quad + \sum_{n=-\infty}^{\infty} B_n[H_n^{(1)}(k_2|z_2|)(\frac{z_2}{|z_2|})^n + H_n^{(1)}(k_2|z_2-2ih_3|)(\frac{z_2-2ih_3}{|z_2-2ih_3|})^{-n}] \quad (24)\end{aligned}$$

5.4 Definite integral equations

In previous paper, the total wave fields of two half-space have been derived, the wave field for two cavities and a crack near bimaterials interface can be deduced based on "conjunction" combining with Green's function, and hence a series of definite integral equations for the problem is obtained.

For convenient solution, the problem model is divided along the interface $y = h$ into two parts: the part I is a half space containing an cavity and a crack, while the part II represents a full elastic half space containing the other circular cavity.

On "subdivision" surface $y = h$ of half space I, we define the total displacement as $W_I^{(t)}$. Note that the stresses of the incident wave, the reflection wave, the scattering waves of the lining and the wave field caused by construction of the crack are free in the interface, we have

$$\tau_{\theta z,I}^{(t)} = 0, \theta = 0, \pi \quad (25)$$

Likewise, on "subdivision" surface of half space II, total stress and displacement of "subdivision" surface in half space II are:

$$\tau_{\theta z,II}^{(t)} = 0, \theta = 0, \pi \quad (26)$$

$$W_{II}^{(t)} = 0 \quad (27)$$

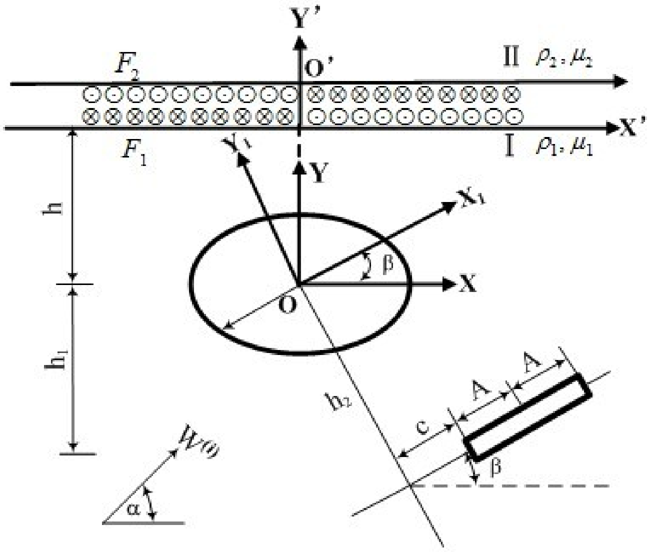


Figure 5: The conjunction model of "subdivision" surface

To guarantee continuous conditions of displacement and stress at "subdivision" interface, the horizontal surfaces of the medium I and medium II are loaded with a series of undetermined anti-plane forces $F_1(r_0, \theta_0)$ and $F_2(r_0, \theta_0)$ (shown as Fig.5). Consequently, the total displacements and total stresses of the horizontal surfaces transform into

$$\begin{aligned} W_I = {} & W_I^{(i)} + W_I^{(r)} + W_I^{(s)} - W^{(c)} \\ & + \int_0^\infty F_1(r_0, \pi) G_3(r_1, \theta_1; r_0, \pi) dr_0 + \int_0^\infty F_1(r_0, 0) G_3(r_1, \theta_1; r_0, 0) dr_0 \end{aligned} \quad (28)$$

$$\begin{aligned} W_{II} = {} & W_{II}^{(i)} + W_{II}^{(r)} + W_{II}^{(s)} \\ & + \int_0^\infty F_2(r_0, \pi) G_{II}(r_1, \theta_1; r_0, \pi) dr_0 + \int_0^\infty F_2(r_0, 0) G_{II}(r_1, \theta_1; r_0, 0) dr_0 \end{aligned} \quad (29)$$

$$\tau_{\theta z}^{(I)} = F_1(r_0, \theta_0), \tau_{\theta z}^{(II)} = F_2(r_0, \theta_0) \quad (30)$$

where G_3 is defined by Eq. (15).G_{II} indicates the Green's function that the displacement field of half-space II of a circular cavity and crack impacted by an out-plane source load at surface, which takes the form of

$$G_{II} = \frac{i}{2\mu_2} H_0^{(1)}(k_2|z_2 - z_0|) + \sum_{n=-\infty}^{\infty} B_n [H_n^{(1)}(k_2|z_2|)(\frac{z_2}{|z_2|})^n + H_n^{(1)}(k_2|z_2 - 2ih_3|)(\frac{z_2 - 2ih_3}{|z_2 - 2ih_3|})^{-n}] \quad (31)$$

According to the stress balance condition at interface $\tau_{\theta Z}^{(I)} = \tau_{\theta Z}^{(II)}$, we have

$$F_1(r_0, \theta_0) = F_2(r_0, \theta_0) \quad (32)$$

Combining with expressions (27) and (28), the continuous conditions of displacements at the interface turn to be

$$\int_0^\infty F_1(r_0,\pi)[G_3(r_1,\pi;r_0,\pi)+G_{II}(r_1,\pi;r_0,\pi)]dr_0+ \\ \int_0^\infty F_1(r_0,0)[G_3(r_1,\pi;r_0,0)+G_{II}(r_1,\pi;r_0,0)]dr_0 \\ =-[W_I^{(i)}+W_I^{(r)}+W_I^{(s)}-W^{(c)}-W_{II}^{(i)}-W_{II}^{(r)}-W_{II}^{(s)}]_{\theta=\pi} \quad (33)$$

$$\int_0^\infty F_1(r_0,\pi)[G_3(r_1,\pi;r_0,\pi)+G_{II}(r_1,\pi;r_0,\pi)]dr_0+ \\ \int_0^\infty F_1(r_0,0)[G_3(r_1,\pi;r_0,0)+G_{II}(r_1,\pi;r_0,0)]dr_0= \\ -[W_I^{(i)}+W_I^{(r)}+W_I^{(s)}-W^{(c)}-W_{II}^{(i)}-W_{II}^{(r)}-W_{II}^{(s)}]_{\theta=0} \quad (34)$$

The above definite integral equations belong to the first species Fredholm equations with weak singularity in the semi-infinite domain. According to attenuation characteristics of the scattering wave, the direct discrete method of weak singular integral equations is adopted here to transform the integral equations into linear algebra equations for solving the additional forces F_1 and F_2 at a series of discrete points.

6 Dynamic Stress Concentration Factor (DSCF)

When the stress free condition is given at the surface around the circular cavities, usually the dynamic stress concentration factor (DSCF) $\tau_{\theta z}^*$can be written as

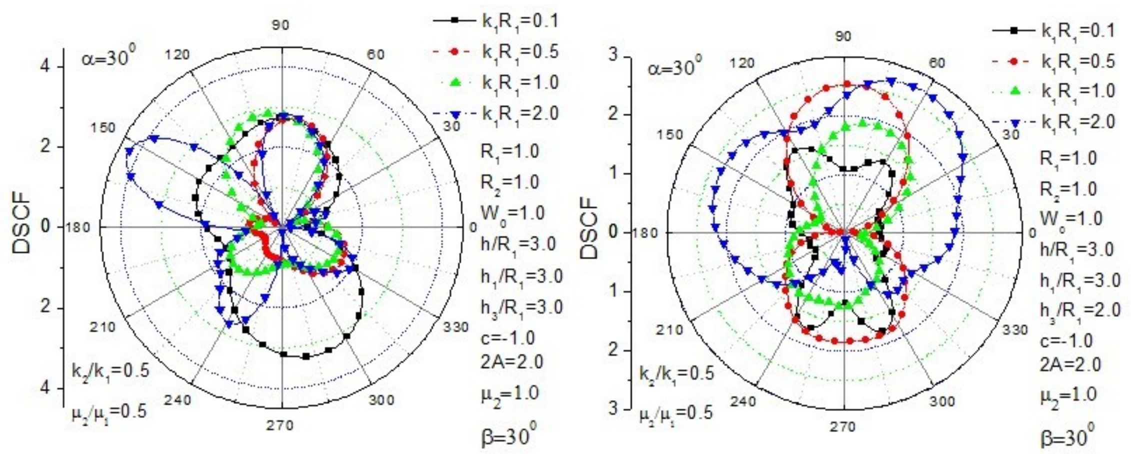
$$\tau_{\theta z}^* = |\tau_{\theta z}^{(\cdot)}/\tau_0| \quad (35)$$

where $\tau_{\theta z}^{(\cdot)}$ is the stress around the outer boundary of the elliptical cavity; $\tau_0=\mu_1 k_1 W_0$ stands for the largest amplitude of the incident stresses.

7 Numerical results and discussion

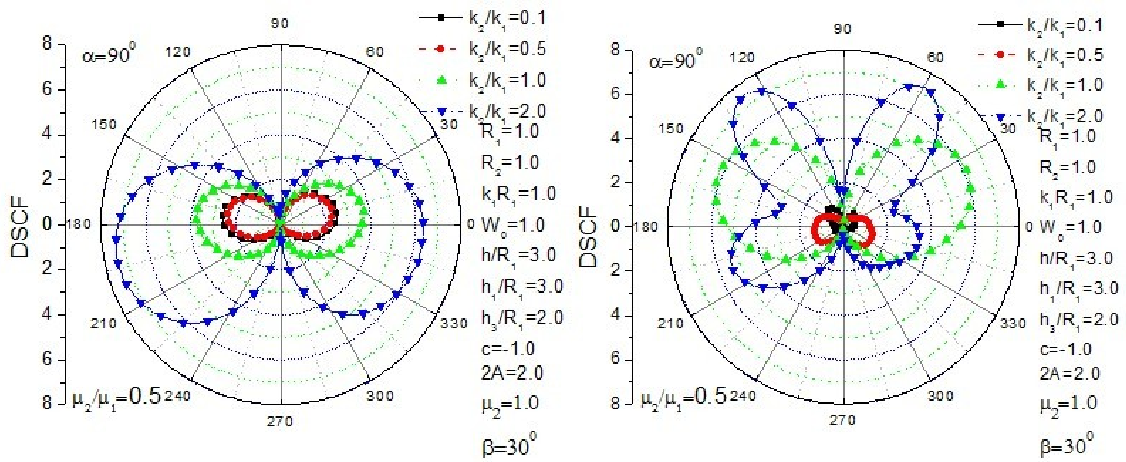
Based on above theoretical derivation, some numerical examples are given to discuss dependence of dynamic stress concentration factors around the cavities on different parameters. The expressions of DSCF is defined by the expression (35).

Fig.6 illustrates that as the wave number k_1R_1 increases, the distribution of dynamic stress concentration factor $\tau_{\theta z}^*$ around the cavities presents more and more obvious oscillation behavior change. Fig. 7 shows that the ratio of wave number $\frac{k_2}{k_1}$ has certain effect on $\tau_{\theta z}^*$. With the increase of $\frac{k_2}{k_1}$,$\tau_{\theta z}^*$ around the cavity a and b become greater, but compared with the cases of $\frac{k_2}{k_1}=0.1, 0.5$ and 1.0, $\tau_{\theta z}^*$ changes little, when SH wave is incident vertically, the maximums occurs, which respectively are:7.41(the cavity a, $\theta=192°$,$\frac{k_2}{k_1}=0.2$) and 7.30(the cavity b, $\theta=59°$,$\frac{k_2}{k_1}=0.2$). The ratio of shear modulus $\frac{\mu_2}{\mu_1}$ also has certain influence on $\tau_{\theta z}^*$ around the cavities as shown as Fig.8. Compared with two distribution graphs of $\tau_{\theta z}^*$ around the cavity a and b, it can be seen that, beyond $\frac{\mu_2}{\mu_1}=0.1$ to 2.0, $\tau_{\theta z}^*$ change little with $\frac{\mu_2}{\mu_1}$, while $\frac{\mu_2}{\mu_1}=4.0$, $\tau_{\theta z}^*$ becomes great, and $\tau_{\theta z}^*|_{max}$ appears at $\theta=172°$ (the cavity a) and $\theta=197°$ (the cavity b), 3.82 and 1.99 respectively.



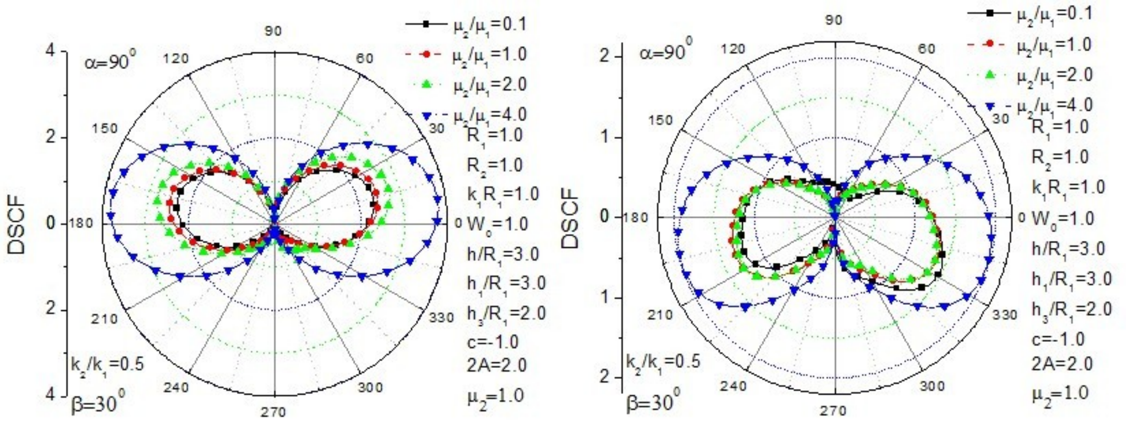
a) Cavity *a* b) Cavity *b*

Figure 6: Variation of DSCF withk_1R_1



a) Cavity *a* b) Cavity *b*

Figure 7: Variation of DSCF with$\frac{k_2}{k_1}$



a) Cavity *a* b) Cavity *b*

Figure 8: Variation of DSCF with$\frac{\mu_2}{\mu_1}$

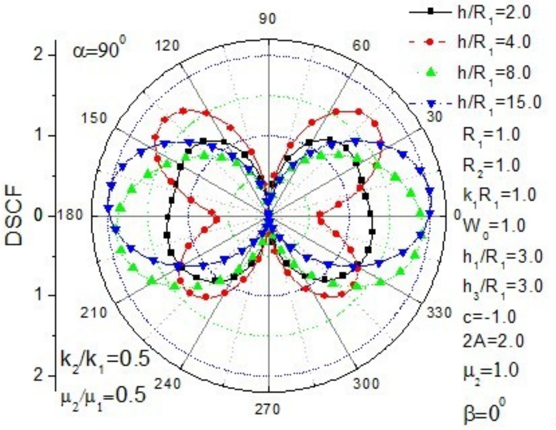


Figure 9: Variation of DSCF with $\frac{h}{R_1}$

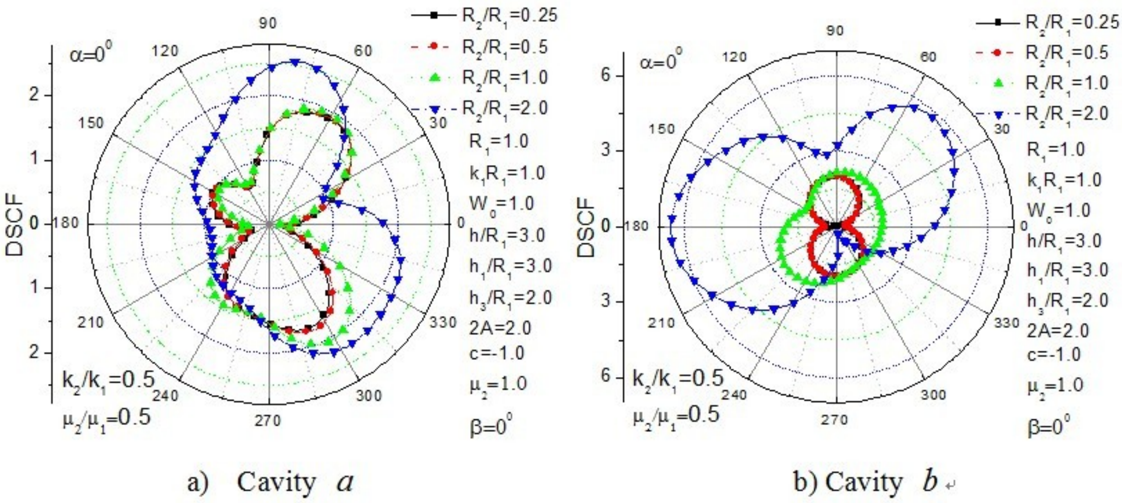
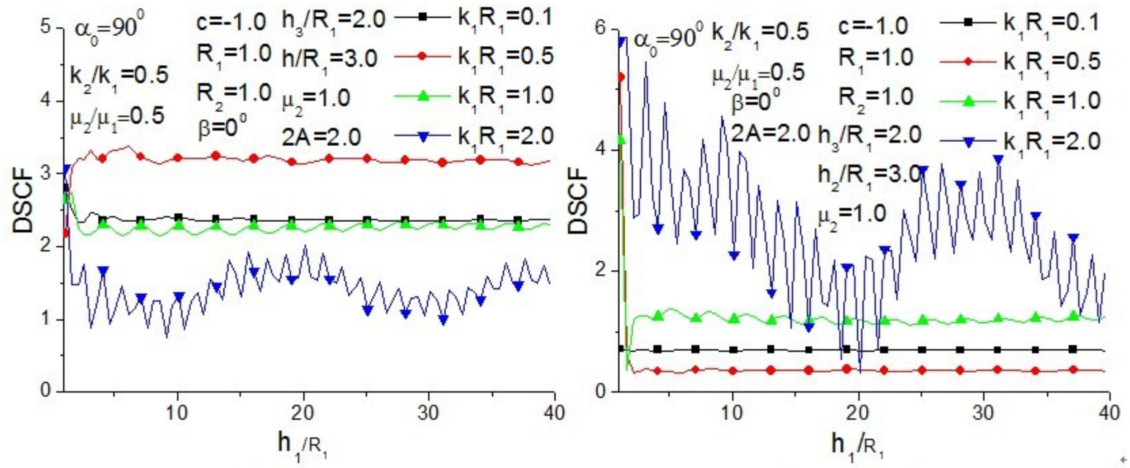


Figure 10: Variation of DSCF with $\frac{R_2}{R_1}$



a) Cavity a b) Cavity b

Figure 11: Variation of DSCF with $\frac{h_1}{R_1}$ at $\theta = 0°$

Fig.9 shows that, when $\alpha = 90°$, as the distance of the cavity a and the interface $\frac{h}{R_1}$, $\tau^*_{\theta z}$ increases slightly, for $\frac{h}{R_1} = 2.0$, 4.0,8.0 and 15.0,the maximums are 1.31,1.76,1.91 and 2.01.

As shown in Fig. 10, for the cavity a, the ratio of radius of two cavities $\frac{R_2}{R_1}$ impacts little on $\tau^*_{\theta z}$, but much greater for the cavity b, especially the case of $\frac{R_2}{R_1} = 2.0$.

Plotted by Fig.11 is the distribution law of DSCF at $\theta = 0°$ of the cavities with $\frac{h_1}{R_1}$ and in the cases of $k_1R_1 = 0.1$,0.5,1.0 and 2.0 under vertically incident SH-waves. Fig.11 reveals that, for $k_1R_1 = 0.1$,0.5 and 1.0, $\tau^*_{\theta z}$ varies slightly with the increasing of $\frac{h_1}{R_1}$, but when $k_1R_1 = 2.0$, $\tau^*_{\theta z}$ changes as wave.

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