Estimation of wall roughness functions acceptability in CFD simulation of mine ventilation networks

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Abstract

This paper presents the results of three-dimensional CFD models acceptability investigation for mine ventilation air distribution problem. The peculiarity of mine ventilation problems is wall roughness effect, which is predominating factor of mine workings pressure loss and air flow distribution. ANSYS turbulence models allow the single method for roughness accountance — wall function roughness modification based on empirical laws for sand grain roughened pipes. Initially this empirical method was developed in order to account micro scale roughness effects, while mine workings essential property is macro scale roughness. Due to unacceptability of direct wall macro scale roughness modeling at the geometry level we give an estimation of the CFD wall function roughness parameters influence on pressure loss. A turbulent flow through conduit with arbitrary-shaped cross section area is considered. A set of numerical calculations in ANSYS FLUENT CFD software is presented. As a result we present the comparison with classical approach for wall roughness modeling in mine ventilation networks based on air resistance coefficient.

1 Introduction

Increase of mine ventilation networks in conjunction with strict safety requirements leads to complication of theoretical analysis of air and temperature distribution. For this reason, in the last decades numerical simulation became the best way to predict air flow parameters of mine ventilation system [1, 2].

A classical and main approach of mine ventilation design is based on air distribution modeling using Kirchhoff's circuits laws [3]. This approach allows determination of one-dimensional air flow parameters field and is the most efficient for simulation of the whole network. Many composite elements of mine ventilation systems require more accurate solution with accounting of three-dimensional air flow, gas and temperature fields, for example, airflow distribution in fan drifts in mine shafts, in mine working coupling, conditioning air circuits etc. In this case three-dimensional CFD analysis is required.

At the same time, mine ventilations problems have a peculiarity: the wall roughness effects appear on a macro scale and are considered to be predominating factor in pressure loss and air flow distribution in mine ventilation networks [4, 5]. For this reason it's necessary to take into account macro scale wall roughness effects in CFD simulations. Wall roughness still stays a hindrance for correct 3D computer modeling of mine workings and damps the 3D methods intrusion in mine ventilation systems design.

A feasible solution is direct wall macro scale roughness modeling at geometry level. The difficulty of this approach is that we need to specify a certain roughness shape. However, real mine workings have variable roughness shapes and roughness heights. For that reason, in order to get shape-robust solution, we need to apply numerical analysis. Then a statistical processing must be carried out.

Another solution is law-of-the-wall function modified for roughness. ANSYS turbulence models allow the single wall function roughness modification based on empirical law for sand grain roughened pipes [6, 7]. This empirical method was initially developed to account micro scale roughness effects.

In this paper we investigate an acceptability of existent wall function roughness modification for macro scale roughness problems. The acceptability analysis is based on a "pressure loss — roughness height" function numerical definition. A codomain of the function is analyzed. In terms of achieved function we compare wall function roughness height with air roughness coefficient traditionally used in mine ventilation [4, 5] and with averaged value of roughness height defined in [4].

2 Mathematical model

We consider the fully-developed turbulent flow in straight cylindrical channel (mine working). The channel is defined by a set of geometric properties, such as length L, cross section area S, constant roughness height K_S. For determination of flow turbulent properties standard k-epsilon turbulence model is used [6, 8, 9].

Let U, p, ρ and μ_L be the Reynolds-averaged velocity, pressure, density and dynamic viscosity of the fluid, respectively. The Reynolds Averaged Navier-Stokes (RANS) equations can be written as [8]

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = -\nabla p + \mu_L \Delta \mathbf{U} + \rho\nabla\cdot\mathbf{R}, \tag{1}$$

where $\mathbf{R}$ is the Reynolds Stress Tensor for which the k-epsilon turbulence model proposes

$$\mathbf{R} = -\frac{2}{3}k\mathbf{I} + \nu_T\left(\nabla\mathbf{U} + \mathbf{U}\nabla\right) \tag{2}$$

Here $C_\mu = 0.09$ is empirical constant, k is turbulent kinetic energy, ε is dissipation of turbulent kinetic energy, $\mathbf{I}$ is identity tensor, μ_T is turbulent kinetic viscosity, defined as

$$\nu_T = \frac{C_\mu k^2}{\varepsilon} \tag{3}$$

The equations for k and ε are the following [8, 10]:

$$\frac{\partial k}{\partial t} + \mathbf{U}\cdot\nabla k = \nabla\cdot\left(D_k\nabla k\right) - \varepsilon + \frac{\nu_T}{2}\left(\nabla\mathbf{U} + \mathbf{U}\nabla\right) \tag{4}$$

$$\frac{\partial \varepsilon}{\partial t} + \mathbf{U}\cdot\nabla\varepsilon = \nabla\cdot\left(D_\varepsilon\nabla\varepsilon\right) - c_2\frac{\varepsilon^2}{k} + \frac{c_1\kappa}{2}\left(\nabla\mathbf{U} + \mathbf{U}\nabla\right), \tag{5}$$

where $\kappa = 0.4187$ is von Karman constant and the diffusion coefficients D_k and D_ε are given by

$$D_k = \frac{\nu_T}{\sigma_k} + \nu_L \tag{6}$$

$$D_\varepsilon = \frac{\nu_T}{\sigma_\varepsilon} + \nu_L \tag{7}$$

with empirical constants $c_1 = 0.126$, $c_2 = 1.92$, $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$.
System of equations (1) — (7) become determined by adding a continuity equation for incompressible flow

$$\nabla \cdot \rho \mathbf{U} = 0 \tag{8}$$

and boundary conditions on inlet, outlet and wall areas.

Inlet boundary condition is formulated for velocity vector $\mathbf{U}$, turbulent kinetic energy k and dissipation ε of turbulent kinetic energy [6, 10].

$$\mathbf{U} = (0, 0, U_{in}) \tag{9}$$

$$k = k_{in} \tag{10}$$

$$\varepsilon = \varepsilon_{in} \tag{11}$$

The uniform velocity profile with velocity magnitude U_{in} and direction normal to the boundary is specified.

Outlet boundary condition is formulated for static pressure p, turbulent kinetic energy k and dissipation ε of turbulent kinetic energy:

$$p = p_{out} \tag{12}$$

$$k = k_{out} \tag{13}$$

$$\varepsilon = \varepsilon_{out} \tag{14}$$

Wall boundary conditions for turbulent flow problem are given in terms of wall function. Experiments in roughened pipes and channels [6, 11] indicate that the mean velocity distribution near rough walls has the classical logarithmical slope but a different intercept (additive constant ΔB in the log-law). For that reason, the law-of-the-wall for mean velocity modified for roughness can be written in the form

$$U^* = \frac{1}{\kappa} \ln \left(E y^* \right) \tag{15}$$

$$U^* = \frac{U_z u^*}{\tau_w / \rho} \tag{16}$$

$$y^* = \frac{\rho u^* y}{\mu_L}, \tag{17}$$

where $E = 9.793$ is empirical constant, y is distance to the wall, U_z is mean velocity of the fluid at point y, μ_L is laminar dynamic viscosity, $u^* = C_\mu^{1/4} k^{1/2}$ is dynamic velocity [12].

Constant ΔB depends, in general, on the roughness shape and roughness height. There is no universal roughness function valid for all types of roughness. For a sand-grain roughness and similar types of uniform roughness elements constant ΔB has been found to be well-correlated with the nondimensional roughness height

$$K_S^+ = \frac{\rho K_S u^*}{\mu_L}, \tag{18}$$

where K_S is the physical roughness height. Analyses of experimental data show that the roughness function is not a single function of K_S^+ , but takes different forms depending on the K_S^+ value. For fully-rough regime [6] constant ΔB can be determined as

$$\Delta B = \frac{1}{\kappa} \ln \left(1 + C_S K_S^+ \right) \tag{19}$$

Here C_S is roughness coefficient which identifies the roughness shape and equal 0.5 for spherical sand grains.

Wall boundary conditions for turbulent kinetic energy k and dissipation ε of turbulent kinetic energy are defined as

$$\frac{\partial k}{\partial n} = 0 \tag{20}$$

$$\varepsilon = \frac{C_\mu^{3/4} k^{1/2}}{\kappa y} \tag{21}$$

3 Numerical calculation

The finite element discretization of described mathematical model and further numerical solution were achieved using ANSYS Fluent CFD software. The SIMPLE algorithm for pressure-velocity coupling was used [13]. A second order schemes for spatial discretization were used.

In order to get mesh-independent solution, numerical calculations for various meshes were accomplished. The boundary layer mesh parameters were determined according to formula [6]

$$\Delta y_1 = \frac{\max Y^+ \mu_L}{\rho U_\tau} \tag{22}$$

which allows approximate analytical determination of first near wall finite element height Δy_1 premised by required parameter $\max Y^+$. A set of finite volume meshes with calculated parameter $\max Y^+$ of 35, 96, 237, 598, 1465 were used (values of $\max Y^+$ are given for $K_S = 0.01m$). Computational parameters for described problem are the following: length $L = 50m$, cross section square $S = 10m^2$, density $\rho = 1.225kg/m^3$, laminar dynamic viscosity $\mu_L = 1.78 \cdot 10^{-5}kg/m/s$, inlet velocity $U_{in} = 4m/s$, outlet pressure $p_{out} = 0Pa$, inlet/outlet turbulent intensity $I = 10\%$, inlet/outlet turbulence scale $l = 0.1m$.

4 Analysis of pressure loss

The pressure loss as a function of roughness parameters and inlet air velocity magnitude was investigated. The correctness of the numerical calculation results largely depends on boundary conditions. For this reason, pressure drop was calculated between two cross-sections taken over a distance from inlet and outlet boundary areas. It allows us to exclude influence of uncertain uniform boundary conditions on calculating pressure drop. Distances L_1 and L_2 (Fig. 1) were selected according to convergence estimation for pressure drop in the channel.

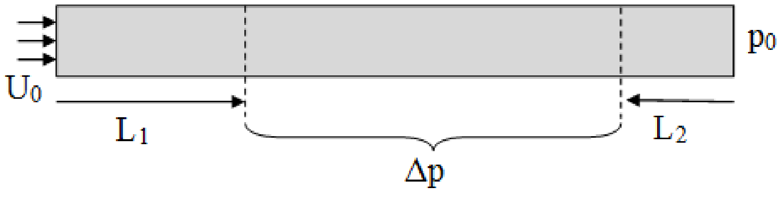

Figure 1: Pressure drop determination method

The "pressure loss — roughness height" relation from numerical experiment for various K_S values is shown on Fig. 2.

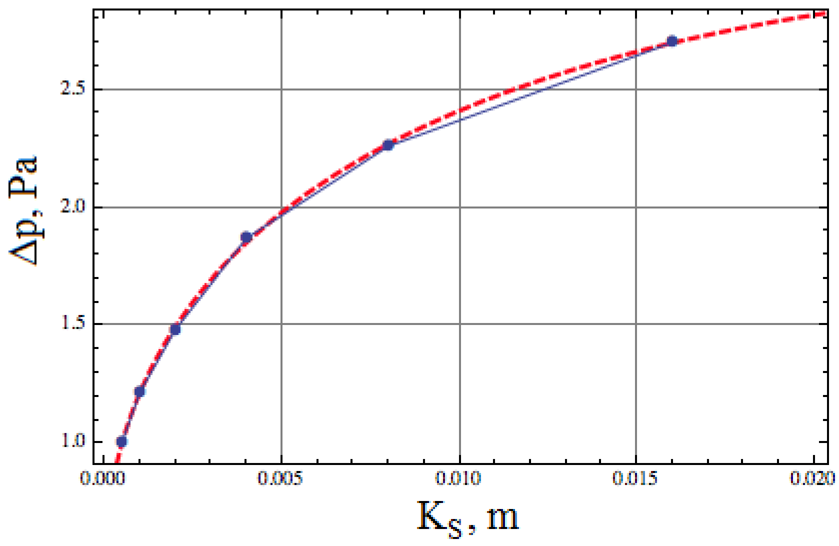

Figure 2: Pressure loss as a function of roughness height; continuous curve - approximated analytic formula, isolated points - results of numerical simulation

In the process of numerical simulation roughness height K_S was varied in a wide range from 0 to $1m$. Dimensional analysis has shown that the "pressure loss — roughness height' function can be estimated using following approximating function

$$Eu = 0.0539 + 0.0695 \cdot \ln\left(1 + 0.222 \cdot C_S K_S\right) \tag{23}$$

Here $Eu = p/(\rho U_{in}^2)$ is dimensionless Euler number, K_S^+ is dimensionless roughness height calculated for inlet kinetic turbulent energy value k_{in}.

The function (23) is also shown on Fig. 2 and valid in the interval $K_S \in (0.005m, 1m)$. The function has a logarithmical slope due to expression (19) in mathematical model. A case of close to zero K_S gets Eu as a function of Reynolds number Re. When K_S grows, pressure loss rapidly become independent from Re. It is associated with dominating factor of ΔB in wall function (19). As far as mine workings always have essentially non-zero roughness height K_S, we consider only the case when Eu doesn't depend on Re. For this case, the approximating function (23) demonstrates well-known Darcy parabolic law for pressure drop in rough pipe networks [3].

The function (23) was achieved for channel with circular cross section and, consequently, has constrained applicability, because real mine working cross sections differs from circle. Therefore, generalization of this function for arbitrary cross section is required.

There is a classical formula for arbitrary mine working pressure loss determination in mine ventilation [4, 5].

$$\Delta p = R \cdot Q^2 = 9.81 \cdot \alpha \cdot \frac{L \cdot P}{S} V^2, \tag{24}$$

where α is air resistance coefficient, which depends only on roughness properties, P is perimeter of cross section and dimensionless complex $P \cdot L/S$ represents mine working cross section shape and square.

Then we modify the function (23) using formula (24) for the arbitrary mine working cross section.

$$\frac{\Delta p \cdot S}{\rho \cdot U_{in}^2 \cdot P \cdot L} = 0.00241 + 0.00311 \ln \left(1 + 0.222 \cdot \frac{\rho C_\mu^{1/4} k_0^{1/2} \cdot C_S K_S}{\mu_L} \right) \tag{25}$$

Received function was numerically verified: different values of inlet velocity magnitude U_{in}, length L, cross section square S and perimeter P of mine working cross section were considered. The multiparameter verification analysis is demonstrated on Fig. 3 in axes $\Delta p \cdot S$ and $\rho \cdot U^2 \cdot P \cdot L$. Relative error is less than 6% for considered range of parameters $U_{in} \in (0.25, 10)\, m/s$, $L \in (20, 500)\, m$, $S \in (5, 50)\, m^2$ and $P \in (5, 50)\, m$.

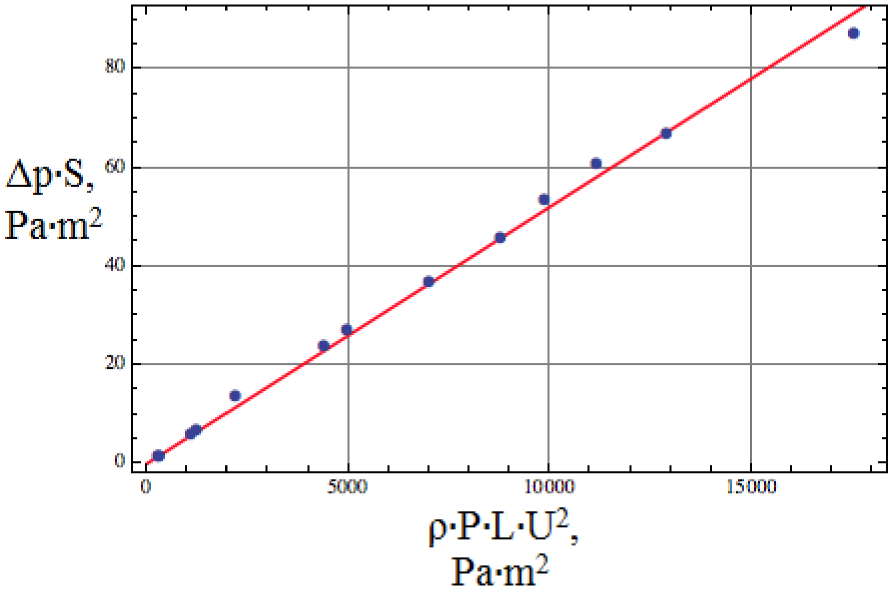

Figure 3: Multiparametric verification of pressure loss function; continuous curve - approximated analytic formula, isolated points - results from numerical simulation

5 Relationship between wall function roughness height and air resistance coefficient

The classical air flow distribution calculation methods are based on formula (24) with air resistance coefficient α. We can get roughness height K_S as a function of air resistance coefficient α by combining formulas (24) and (25) and excluding Δp.

$$K_S(\alpha) = 4.49 \frac{\mu_L}{\rho C_\mu^{1/4} k_0^{1/2} \cdot C_S} \cdot \left[0.342 \cdot \exp\left(4341 \cdot \frac{\alpha}{\rho}\right) - 1 \right] \tag{26}$$

We have the geometry-independent relation between K_S and α. Geometric independence has a good correlation with physical meaning of this parameters. Function $K_S(\alpha)$ (26) is shown on Fig. 4. For $\alpha > 0.0015$ we have a physically abnormal values of parameter K_S. It is probably caused by micro scale character of logarithmical law (19). It leads to physical divergence of roughness height value K_S for large air resistance coefficients α.

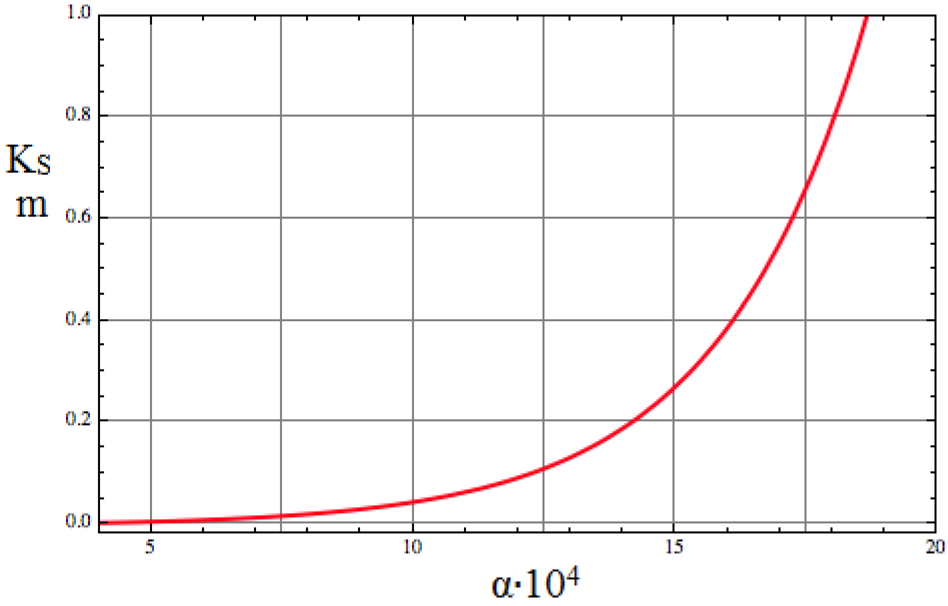

Figure 4: Roughness height K_S as a function of air resistance coefficient α

Nevertheless, we can obtain some unequivocal K_S value for arbitrary $\alpha > 0.003$. In such case K_S parameter can represent some artificial roughness height with no certain physical meaning. Consequently, function (26) establishes the possibility in principle mine working air flow simulation using wall functions modified for roughness in standard k-epsilon turbulence model. At that, pressure drop from numerical simulations will be in good correlation with natural experiments.

It should be emphasized that K_S parameter in (26) is undefined for $\alpha > 0.0003$. The low borderline value represents the case of smooth walls with $K_S = 0$ and, properly speaking, this value is a function of Re. But function (26) was received using assumption of Re-independence and strictly parabolic Darcy law.

6 Conclusion

The pressure loss problem in straight mine working with rough wall surface was considered. The approximated analytical formula for roughness height determination using air resistance coefficient was proposed. This formula establishes the possibility in principle mine working air flow simulation using wall functions modified for roughness in standard k-epsilon turbulence model. The results of numerical simulation has shown that for arbitrary $\alpha > 0.0003$ exists some unequivocal K_S value, which gives correct pressure loss in mine working. This limitation is not critical because air resistance coefficient in real mine working is always higher than 0.0006.

Wall function roughness height K_S determination should be performed not in terms of experimentally measured values, but from approximated analytical function (26). It is caused by physical divergence of roughness height value for large air resistance coefficients.

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