Numerical Simulation of Hydroelastic Beam Dynamics in the Flow Using Vortex Element Method

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Abstract

Vortex element method is meshless lagrangian CFD method. In the presented research Vortex element method is used for flow-structure interaction simulation. Vortex fragmenton is proposed by authors as new vortex element for 3D incompressible flow simulation around deformable bluff-bodies. Using vortex fragmentons the numerical scheme is developed with the same vortex element used both for flow simulation and vorticity flux computation. The program package is developed using MPI technology for parallel computing on different types of multi-processor clusters. The coupled hydroelastic problem of the numerical simulation of elastic beam behavior in the flow is considered.

1 Introduction

Beams and pipes interacting with the flow are widespread structure elements of different engineering systems. In case of rigid beam (or pipe) with circular cross-section hydrodynamic loads are unsteady but close to monoharmonic with dimensionless frequency St ≈ 0.2, so the simplified mathematical models are often used in technical applications. But if elastic beam or pipe is considered and it is necessary to simulate its unsteady motion in the flow, the influence of hydroelastic vibrations should be taken into account. This problem — vortex induced vibrations (VIV) investigation for elastic beams and pipes in cross flow — is well-known [1]. Beam dynamics in the flow becomes complicated because of feedback arising between vortex shedding and beam motion.

Widespread grid methods can be used for 3D FSI-problems solving but they require too many computational resources, mainly time of computations, memory and disk space. In case of structure optimization process when it is necessary to investigate number of alternatives, this restriction becomes weighty. So in engineering analysis the other class of CFD methods — Lagrangian vortex methods — are more suitable.

Vortex element methods are well-known and well-developed for vortex structures dynamics simulation in unbounded regions because perturbation-decay boundary condition is satisfied automatically. However boundary condition on body surface satisfaction is non-trivial problem. There are approaches based on Discrete Vortices Method [2], Vorticity Flux approach [3], some other ideas [4]. Vorticity flux-based approach shows its advantages when solving coupled FSI-problems with deformable bluff bodies.

In this research the original numerical algorithm based on vortex element method is developed for coupled 3D problem of beam-flow interaction. This algorithm is verified on experimental data described in [5].

2 Governing equations

The 3D beam with length L and constant annular cross-section with radius R and wall thickness h is considered in unbounded incompressible flow with constant incoming flow velocity $\mathbf{V}_\infty$. This tube is supposed to be closed and clamped at cutting faces, so only external flow is considered (fig. 1).

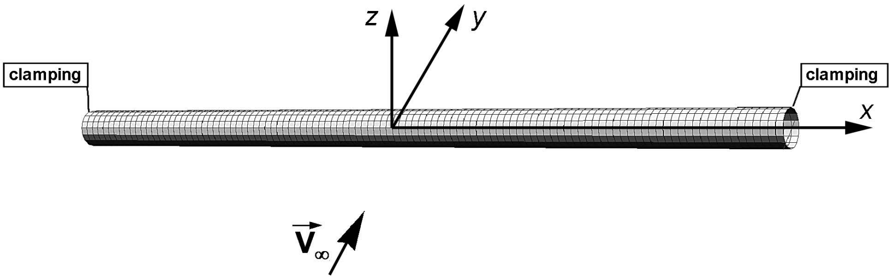

Figure 1: Beam in the cross flow

Beam has mass γ per unit length, modulus of elasticity E and Poisson ratio ν. Flow viscosity is sufficiently small so it is possible to take it into account only as a cause of vortex shedding on beam surface. The flow far from the beam surface is assumed to be inviscid.

Mathematical model of this FSI-problem consists of 2 groups of equations and corresponding boundary and initial conditions. Beam dynamics is described by the following equations of linear vibrations:

$$EJ\frac{\partial^4\mathbf{u}}{\partial x^4}+\gamma\frac{\partial^2\mathbf{u}}{\partial t^2}+\kappa J\frac{\partial^5\mathbf{u}}{\partial^4 x\,\partial t}=\mathbf{Q_H},$$

$$\mathbf{u}|_{t=0}=\mathbf{0},\quad \left.\frac{\partial\mathbf{u}}{\partial t}\right|_{t=0}=\mathbf{0},\quad \mathbf{u}|_{x=0,L}=\left.\frac{\partial\mathbf{u}}{\partial x}\right|_{x=0,L}=\mathbf{0}.$$

Here EJ is beam bending stiffness; κ is internal damping coefficient; $\mathbf{u}=(0,u,w)^T$, u and w are beam deflections in directions of axes Oy and Oz respectively; $\mathbf{Q_H}$ is hydrodynamic load. It is assumed that shear stress is neglected because of small flow viscosity so hydrodynamic load is normal to the surface (i.e., only pressure p is taken into account).

Incompressible fluid dynamics is described by Navier — Stokes equations:

$$\nabla\cdot\mathbf{V}=0,$$

$$\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\,\mathbf{V}=\frac{\mu}{\rho}\nabla^2\mathbf{V}-\nabla\left(\frac{p}{\rho}\right),$$

$$\mathbf{V}(\mathbf{r},t_0)=\mathbf{V}_0,\quad \lim_{r\to\infty}\mathbf{V}=\mathbf{V}_\infty,\quad \lim_{r\to\infty}p=p_\infty,$$

$$\mathbf{V}(\mathbf{r}_K,t)=\dot{\mathbf{r}}_K.$$

Here $\mathbf{V}$ is flow velocity; p is pressure; $\rho=\text{const}$ is flow density; μ is viscosity coefficient; $\mathbf{r}_K$ is position of point on the deformed beam shape, $\mathbf{r}_K=\mathbf{r}_{0K}+\mathbf{u}(x,t)$; $\mathbf{r}_{0K}$ is position of the corresponding point at initial time on cylindrical surface.

So the coupling of both subsystems is provided by moving boundary $\mathbf{r}_K$ of the shell. Its displacement is determined by fluid pressure. At the same time pressure distribution depends on vorticity evolution near moving wall.

3 Numerical method

An original numerical algorithm is developed for above mentioned governing equations solving. Meshless lagrangian vortex element method is used for flow simulation. The vortex fragmenton model [6] is used as common vortex element for both vortex wake and vortex layer on the surface representation. In order to satisfy no-slip boundary condition on the body closed surface vortex frameworks are constructed on the panels which approximate the surface (fig. 2). Panels' vertexes displacements can be set by generalized vector $\{q\}$ which describes displacements of the elastic shell surface. There is collocation point $\mathbf{k}_0^j$ placed in the center of j^{th} panel and normal unit vector $\mathbf{n}_0^j$. Every vortex framework is placed on small distance β from the corresponding panel and consists of m_j vortex fragmentons which are characterized by marker positions $\mathbf{r}_s^j$ and fragmenton vectors $\mathbf{h}_s^j$, $s = 1, \ldots, m_j$.

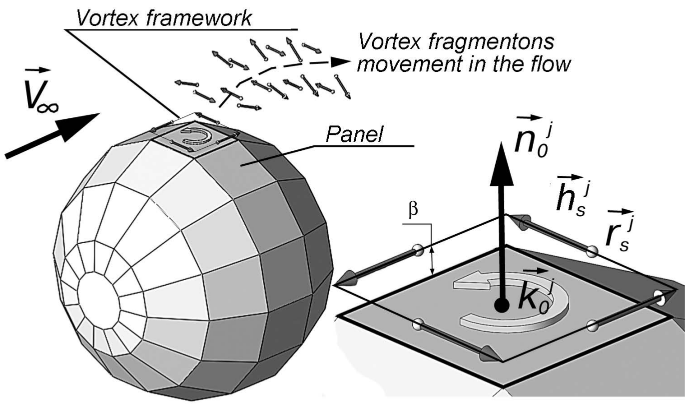

Figure 2: Vortex framework on the panel on body surface

Vortex frameworks circulation can be computed from the equivalence condition for normal velocities of the flow and the surface which is the same as in Discrete Vortex Method [2]. Then according to vorticity flux model developed by Lighthill & Chorin [3, 7] all the vorticity from surface vortex layer becomes free and moves in the flow. In order to simulate such vorticity flux vortex frameworks are split up into separate vortex fragmentons (fig. 2).

It should be noted that there exists another approach to vortex layer intensity on the surface computation. It is based on the equivalence of tangent velocities of the surface and flow and also leads to integral equation [8].

In order to compute pressure distribution $p(\mathbf{r}, t)$ in the flow the analog of the Bernoulli and Cauchy — Lagrange integrals is used [9]. For numerical solution of vortex fragmenton dynamics equations explicit Euler method is used [6] with fixed time step Δt.

In order to solve beam dynamics equation at every time step normal mode transient analysis method is used. Denoting normal modes generalized coordinates as $\{\phi\}$ it can be written down in the following form:

$$\{\ddot{\phi}\} + \delta[\omega]_{\text{diag}}\{\dot{\phi}\} + [\omega^2]_{\text{diag}}\{\phi\} = \{f_H\}. \tag{1}$$

Here $\{f_H\}$ is reduced hydrodynamic forces vector; δ is constant damping coefficient (modal damping decrements are assumed to be proportional to eigenfrequencies); $[\omega]_{\text{diag}}$ is diagonal matrix of lower eigenfrequencies, $[\omega^2]_{\text{diag}} = [\omega]_{\text{diag}} \cdot [\omega]_{\text{diag}}$.

Eigenfrequencies $[\omega]_{\text{diag}}$ and eigenforms matrix $[A]$ are computed using Finite Element Method software MSC.Nastran with SOL103 solver. Surface mesh on the beam surface is built with Patran preprocessor, and it is used both for dynamics analysis and flow simulation. Nodes of QUAD4 shell elements are the same as the panels vertexes. In this case generalized coordinates vector $\{\phi\}$ is connected with vertexes displacements vector $\{q\}$:

$$\{q\} = [A]\{\phi\}.$$

It should be noted that only beam eigenforms are taken into account while MSC.Nastran allows to compute both beam forms and shell eigenforms for the considered thin-walled tube model. Algorithm scheme of FEM analysis and mesh constructing is shown on fig. 3.

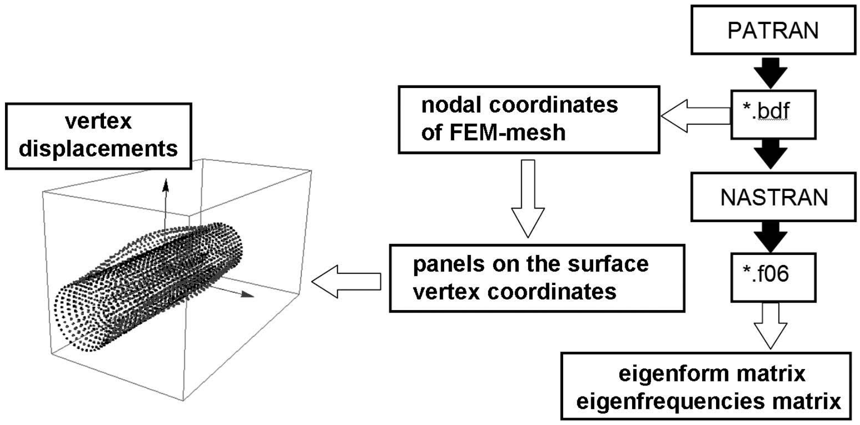

Figure 3: Scheme of FEM analysis and mesh constructing using MSC.Nastran and Patran

It is important that $\{f_H\}$ vector assumed to be constant during time step. It allows to use normal mode transient analysis method for computing vertexes' displacements $\{q(t_{n+1})\}$ at the next time step for given 'initial' displacements $\{q(t_n)\}$. At initial time moment $\{q(t_0)\} = \{0\}$.

So the developed numerical algorithm consists of two subsystems: elastic subsystem and hydrodynamic one. These subsystems are coupled via vortex layer intensity computation procedure.

At every time step t_n firstly no-slip boundary condition are satisfied in panels' collocation points $\mathbf{k}_0^j$, then new vortex fragmentons are generated from the corresponding vortex frameworks and finally pressure distribution and $\{f_H\}$ vector for system (1) are computed. Vortex fragmentons displacements are computed according to the algorithm described in [6].

Since system (1) consists of independent ordinary linear differential equations with constant coefficients, it can be solved analytically:

$$\phi_k(t_{n+1}) = \frac{f_{Hk}}{\omega_k^2} + \frac{\exp(-\delta\omega_k\Delta t)}{\Omega_k\omega_k} \times$$
$$\times \left\{ \left[\Omega_k\omega_k\phi_k(t_n) - f_{Hk}\right]\cos(\Omega_k\Delta t) + \left[\omega_k\dot{\phi}_k(t_n) + \delta\left(\omega_k^2\phi_k(t_n) - f_{Hk}\right)\right]\sin(\Omega_k\Delta t) \right\}$$

$$\dot{\phi}_k(t_{n+1}) = \frac{\omega_k\exp(-\delta\omega_k\Delta t)}{\Omega_k} \times$$

$$\times \left\{ \left[\frac{\Omega_k}{\omega_k} \phi_k(t_n) \right] \cos(\Omega_k \Delta t) + \left[\frac{f_k}{\omega_k} - \left(\delta \dot{\phi}_k(t_n) + \omega_k \phi_k(t_n) \right) \right] \sin(\Omega_k \Delta t) \right\},$$

where $\Omega_k = \omega_k \sqrt{1-\delta^2}$, index k denotes number of eigenform.

The obtained results allow to compute vertex displacements vector $\{q\}$ and to construct the deformed shape of the beam surface at the next time step t_{n+1}.

4 Numerical experiment

In numerical experiment the beam with dimensions $R = 0.025$ m, $L = 1.1$ m, $h = 0.001$ m made from isotropic material ($E = 2 \cdot 10^9$ Pa, $\nu = 0.3$, $\gamma = 1.1$ kg/m) was considered. Damping coefficient was $\delta = 0.05$. Beam's axis coincides with Ox axis and it is symmetric with respect to Oyz plane.

CAE-model of the shell consisting of 1792 panels was constructed in Patran (16 panels in circumferential direction, 110 panels in lengthwise direction, 32 panels on tips).

For flow simulation using vortex element method vortex fragmenton smoothing radius $\varepsilon = 0.004$ m was chosen, distance from the panel to vortex framework was $\beta = 0.004$ m.

Incoming flow velocity was $V_\infty = 1.0$ m/s, time step was chosen equal to $\Delta t = 0.001$ s. In order to prevent large initial perturbation of the beam whose bending stiffness is relatively low, incoming flow was assumed to be uniformly accelerated from $V_\infty = 0$ to $V_\infty = 1.0$ m/s during first 50 time steps (which corresponds to the period from initial time moment $t = 0$ to $t = 0.05$ s).

In modal analysis 6 lowest eigenforms were used. The beam is axisymmetric, so eigenfrequencies are multiple and eigenforms are the same in Oxy and Oxz planes; the eigenfrequencies values are $\omega_{1,2} = 26.6$ Hz, $\omega_{3,4} = 70.8$ Hz, $\omega_{5,6} = 132.7$ Hz.

The transient mode was simulated from initial time moment $t = 0$ to $t = 1.0$ s; 1000 steps in numerical algorithm were performed. Vortex wake around the beam at time moment $t = 1.0$ s is shown on fig. 4. Specific filament-type vortexes can be recognized behind the beam, and in Oyz cross-section these vortexes are similar to von Karman-type vortex street (fig. 5)

Fig. 6 shows the deformed shape of the beam at time $t = 0.1$ s. For clarity, displacements of the beam cross-sections are increased by 30 times. It is evident that under the influence of the incoming flow the beam statically bends in the plane Oxy mainly by the first eigenform. In Oxz plane motion is more complicated: the second eigenform becomes apparent.

Fig. 7 shows the time dependence for the deflection of the middle cross-section ($x = 0$) of the beam. The dashed line shows the deflection in flow direction (Oy), the solid line — in the direction transverse to the flow (Oz). After a transient mode that lasted until time $t = 0.3$ the forced oscillation mode comes. There are frequencies about 66 Hz and 3.3 Hz in the displacement spectrum, which are close to the second eigenfrequency and to the von Karman frequency of vortex shedding respectively (for an infinite rigid cylinder von Karman frequency corresponds to Strouhal number value St = 0.2).

Fig. 8 shows the trajectories of the centers of the beam cross-sections at $x = -0.275$, $x = 0$ and $x = 0.275$, which are close to the coordinates of the antinodes of the first and second modes of vibration. Shown trajectories correspond to time interval $0.8 < t < 1.0$ s. It is seen that the trajectories are complicated, and the maximum displacements from the static deviation are in range from 0.3 to 0.5 mm.

Beam vibration in the flow slightly reduces the average drag force F_d, acting on the beam in the direction Oy (along the incoming flow). Fig. 9 shows the time dependence

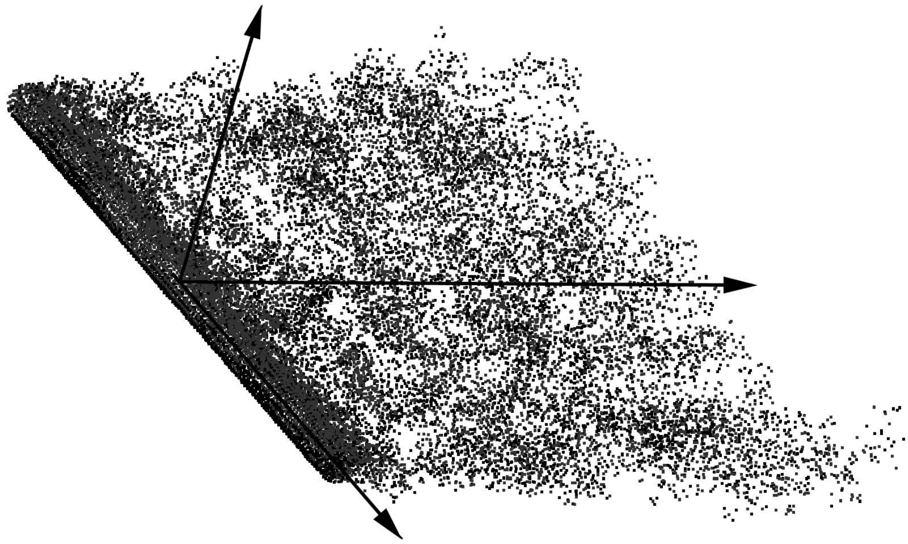

Figure 4: Vortex wake around the beam at time moment $t = 1.0$ s. Points denote vortex fragmentons markers

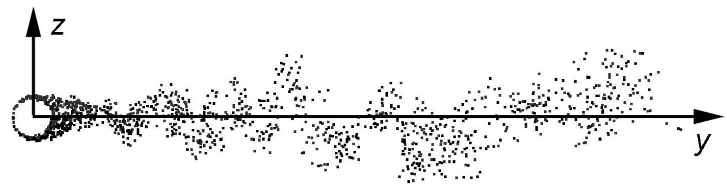

Figure 5: Vortex street in Oyz cross-section behind the beam

for F_d for elastic and rigid beams. Horizontal lines on the graphs show the average values obtained by averaging on time interval $0.3 < t < 1.0$ s.

5 Conclusion

The numerical algorithm for complex coupled FSI-problem solving is developed. It is based on the combined usage of normal modes analysis method for shell dynamics simulation and vortex element method for flow simulation. This algorithm and original software allow to simulate directly vortex induced vibrations in the 3D flow for shells and structures with arbitrary shape. It can be used, for example, to analyze the dynamics of the pipelines in the heat exchangers.

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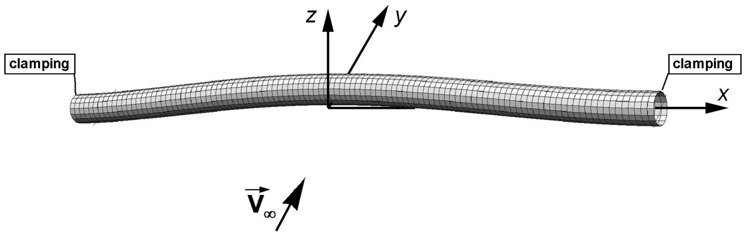

Figure 6: The deformed shape of the beam at time $t = 0.1$ s

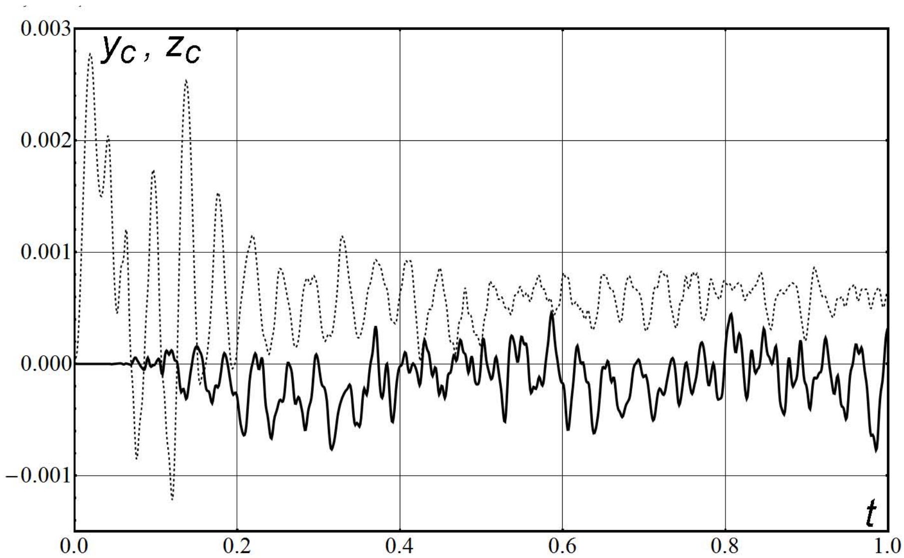

Figure 7: Time dependence for the deflection of the middle cross-section ($x = 0$) of the beam

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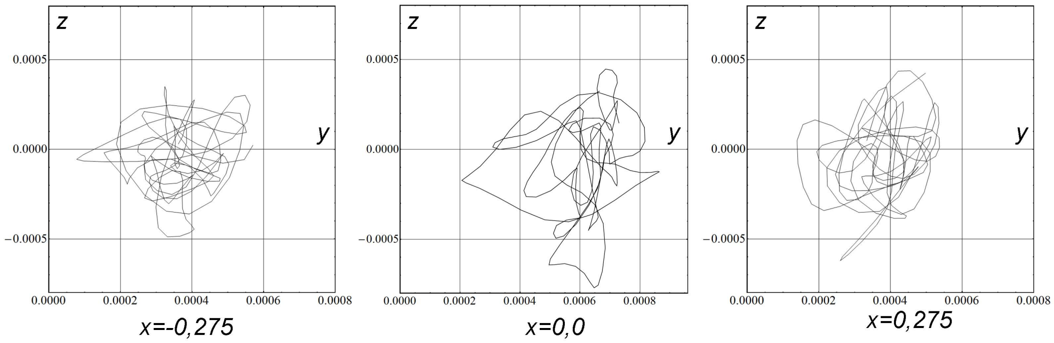

Figure 8: Trajectories of the beam cross-sections at $x = -0.275$, $x = 0$ and $x = 0.275$

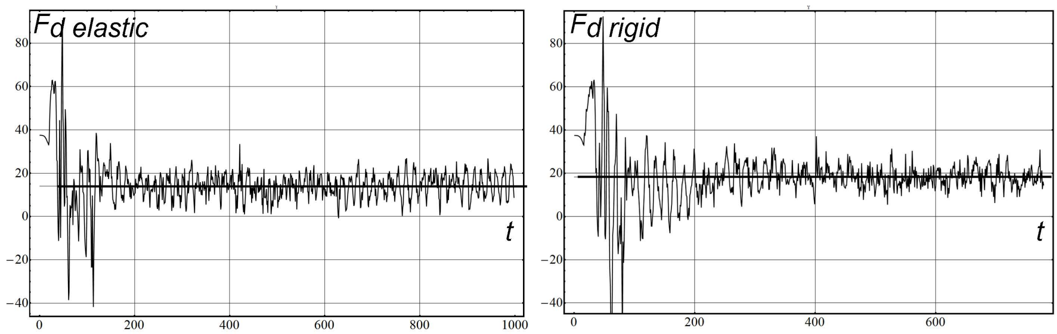

Figure 9: Time dependence for drag force F_d and its average values for elastic and rigid beams

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