Average convection in rotating tilted plane layer

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Abstract

Thermal convection in an inclined plane layer rotating about an axis oriented perpendicular to the plane is studied experimentally. The threshold of convection excitation and the structure of supercritical flows depending on the angle of axis inclination, the temperature difference and the rotational speed are studied. The results are compared with the limiting cases of the rotational axis orientation. It is found that the excitation of average thermovibrational convection is possible at $\alpha < 0°$ (heating of layer from the top). The results of investigation of the quasi-equilibrium stability and heat transfer in the supercritical region are plotted against the dimensionless parameters: gravitational Rayleigh number, vibrational parameter and dimensionless rotation velocity.

1 Introduction

Thermal convection is one of the rapidly developing areas of fluid mechanics. The influence of such factors as rotation [1, 2] and vibrations [3] on convection attracts the attention of scientists. An original point of view on thermal convection of fluid in the cavity rotating around a horizontal axis is presented in [4]. In this case the gravity force causes the fluctuations of non-isothermal fluid in the cavity and generates the average vibrational effects. The average thermal convection is characterized by modified vibrational Rayleigh number $R_v = g^2\beta^2\theta^2h^2/(2\nu\chi\Omega^2)$ and dimensionless velocity of rotation $\omega \equiv \Omega h^2/\nu$ [5], where g - acceleration of gravity, $\Omega \equiv 2\pi f$ - rotation velocity, h - the layer thickness, θ - the temperature difference between the layer boundaries, β, ν, χ - the coefficients of volumetric expansion, kinematic viscosity and thermal diffusivity of fluid. On reaching the critical value of R_v, which increases with ω, the convective structures occur in the layer in the form of hexagonal cells which are stationary in the cavity frame.

This paper is a continuation of [4], but now the convection is studied in an inclined layer rotating around an axis normal to the layer plane. There is agreement of experiment with the results of previous studies in two limiting cases: of the horizontal ($\alpha = 90°$) and vertical ($\alpha = 0°$) layer. For the horizontal case the results are in good agreement with theory [1] and experiments [2]. The increase of dimensionless velocity of rotation (Taylor number) leads to an increase of the stability threshold. In case of vertical layer [4] the convection is excited in a threshold manner by the thermovibrational mechanism [5].

It is found that the average convection could be excited at an arbitrary tilt angle even when the hot layer boundary is at the top. The average convection is of thermovibrational nature and is excited by the gravity field rotating in the cavity frame. The results of the study are presented in the plane of dimensionless parameters.

2 Experimental technique

Research facility (described in detail in [4, 6]) rotates the table with the installed cuvette at a predetermined speed and certain orientation in space with simultaneous supply of the heat exchangers with the fluid from the thermostats. The layer (fig.1) with thickness h has a cylindrical lateral boundary. The layer plane is inclined at an angle α to the vertical. The layer boundaries are maintained at different temperatures T_1 and T_2. Tilt angle is considered positive if the hot layer boundary is below the cold one.

Layer thickness $h = 0.8cm$. Cavity diameter $D = 14.1cm$. Rotation velocity $f = 0.1 - 1.6rps$. Temperatures of heat exchangers $T_1 = 22°C, T_2 = 23 - 80°C$.

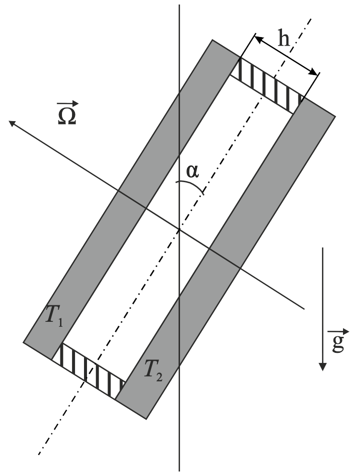


Figure 1: The problem statement.

The temperature of the boundaries of the layer is measured by the resistance thermometers. The cuvette is equipped with a heat flux sensor. The visual observations and photo-registration are carried out through a transparent heat exchanger.

In experiment (at a predetermined temperature difference) the rotational frequency f is reduced stepwise from a sufficiently high value, when there is no convection in the layer. With decreasing f the heat flux and the temperature of boundaries practically do not change at first. With further decreasing f at a certain threshold value the critical increase of heat transfer is observed. With further lowering of the speed the continuous growth of convective structures and heat flux take place.

3 Experiment result

The vertical layer ($\alpha = 0°$). The Nusselt number $Nu = \triangle T / \triangle T_0$, defined as the ratio of the heat flux through the layer to the heat flux in the absence of convection at a given temperature difference is used as the heat transfer characteristics. The critical change of Nu with changing (lowering) the rotation velocity indicates the emergence of convection in a layer (fig.2).

With increasing the temperature difference between the layer boundaries the threshold is shifted to larger values of f. The threshold curves are of similar view and can be divided into three areas. Horizontal section (area I) corresponds to high values of the rotational velocity. In this the cellular convection in a layer is absent, convective flows are associated with the action of centrifugal force. The system of concentric rolls of relatively low intensity is observed near the lateral boundary of the cavity (fig.3a).

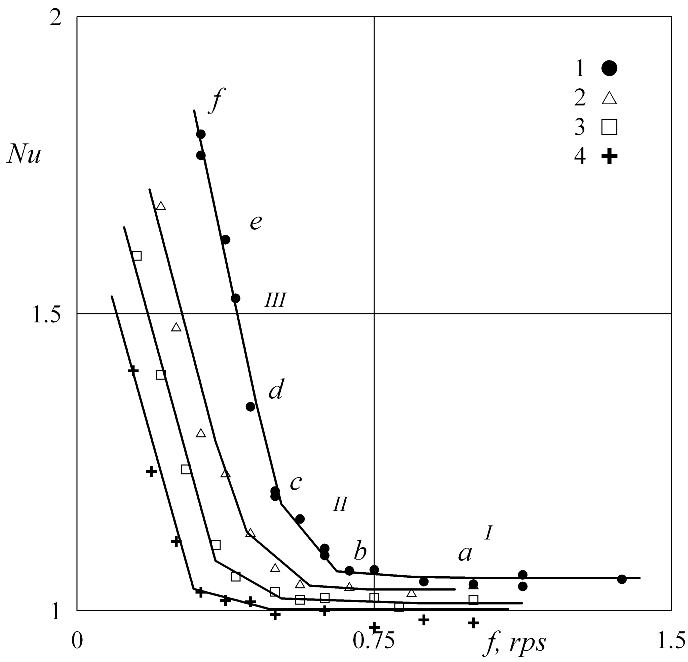


Figure 2: The dependence of heat flux on the rotational speed: the temperature difference between the layer boundaries θ=18.4(1),13.0(2),10.4(3),7.4(4)$°C$. Letters on curve 1 present the corresponding photos in Fig.3.

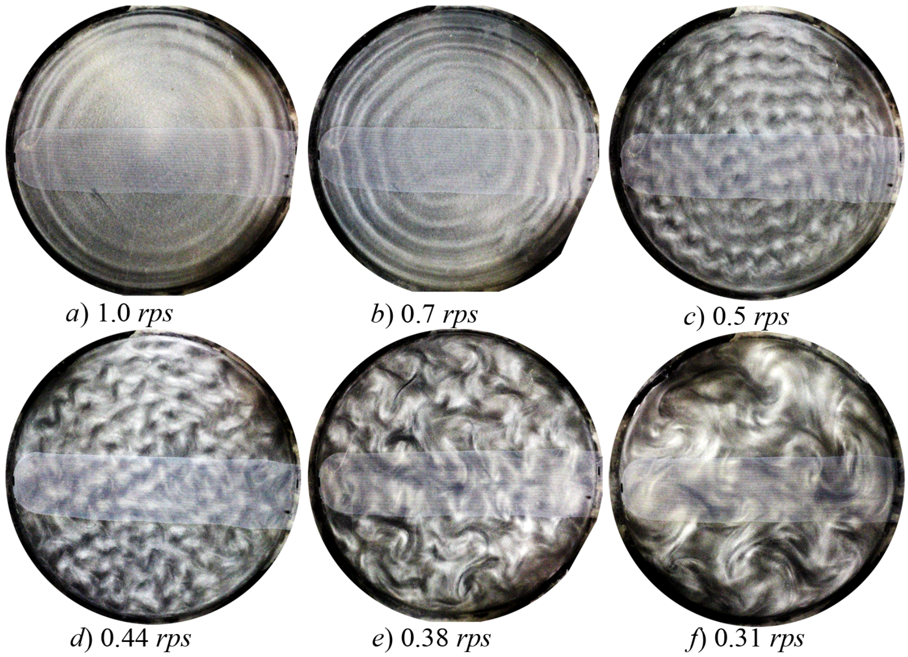


Figure 3: Convective structures ($\alpha = 0°, \theta = 18.4°C, Ra = 0$).

With decreasing rotation velocity the heat flux through the layer remains substantially unchanged despite the formation of regular concentric rolls (fig.3b). The emergence of cellular convection occurs with a further decrease of f and is accompanied by the critical increase in the heat transfer (area II). This is associated with the threshold occurrence of the hexagonal cells (fig.3c). The cell size increases with decreasing f (fig.3d and e). At very slow rotation the cells are converted into large-scale vortex structures (fig.3 e, f, area *III*).

The inclined layer. Modes, the structure and the threshold of convection onset in a rotating inclined layer ($\alpha = 15°$) with a hot lower boundary in subcritical and supercritical regions are similar to the case $\alpha = 0°$. The difference is that at $\alpha = 15°$ the cells are smaller.

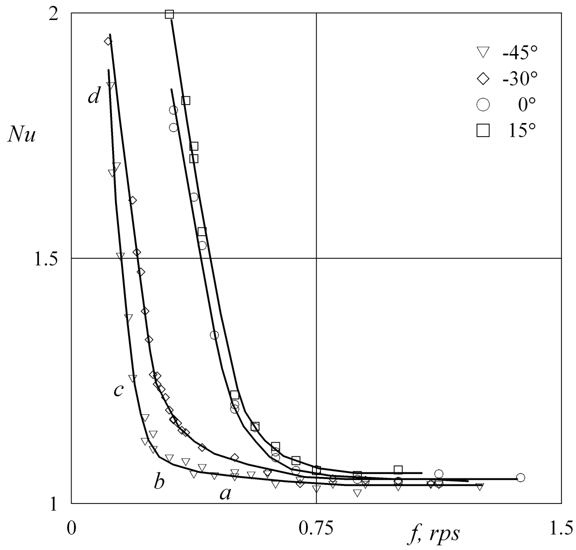
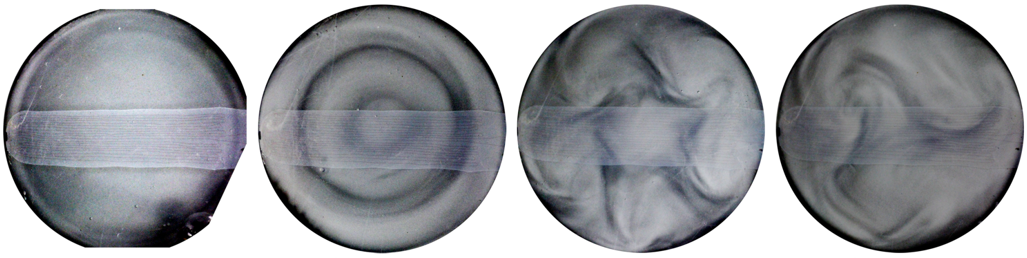


Figure 4: Heat transfer through the layer depending on the frequency of rotation at different tilt angles.

Fig.4 shows the heat transfer curves at $\theta = 18.4$ and different angles of the layer inclination. An interesting fact is that the convective structures emerge and evolve even at $\alpha < 0°$, when the hot border is on top. Convection in this case arises at a lower velocity of rotation (as compared with $\alpha = 0°$) in a form of cell of relatively large size (fig.5). With increasing negative α the threshold velocity of rotation of the cavity is shifted to lower values, the size of the vortex structures increases. With decreasing f the quasi-equilibrium (fig.5a) is changed by toroidal vortices (fig.5b) that, at further decrease in speed, are replaced by the large-scale vortex structures (fig.5c), the size of which increases with further lowering f (fig.5d).



a) 0.5 *rps* *b*) 0.22 *rps* *c*) 0.17 *rps* *d*) 0.15 *rps*

Figure 5: Convective structure when heated from above; $\theta = 18.4$, $\alpha = -45°$.

Heat transfer curves at different inclinations are similar in shape (fig.4). The fractures of all the curves on the border of areas are associated with the development of cellular structures.

4 Discussion

Vibrational effects which appear due to oscillation of gravity in a rotating inclined layer are defined by the following parameters: dimensionless velocity of rotation $\omega \equiv \Omega h^2/\nu$, vibrational parameter $R_v = g^2 \cos^2 \alpha \beta^2 \theta^2 h^2 / 2\nu\chi\Omega^2$ [5] and gravitational Rayleigh number $Ra = g \sin \alpha \beta \theta h^3/\nu\chi$. Dimensionless velocity of rotation from one side characterizes the action of the Coriolis force, from another - the ratio of the working layer thickness to the thickness of the Stokes layer.

Parameters R_v and Ra characterize the action of two different mechanisms associated with the action of the tangential and normal to the layer components of gravity field. The tangential component $g \cos \alpha$ rotates in the cavity frame, exciting the oscillations of non-isothermal fluid and as a result, causing the vibrational mechanism of thermal convection [5]. The normal component $g \sin \alpha$ is stationary in the cavity frame and determines the action of Rayleigh mechanism of convection. Positive values of Ra correspond to the case when the hot boundary is at the bottom, and normal to the layer boundaries component of the gravity has a destabilizing effect. In the case of negative $Ra(\alpha < 0°)$ the stabilizing action of this mechanism is expected.

In the particular case of the rotation of the vertical layer around a horizontal axis ($Ra = 0°$) the results of the experiment are in satisfactory agreement with the results [4] (fig.6). Convection excitation boundary (threshold value of R_v) changes non-monotonically with dimensionless frequency. Some disagreement can be explained by different boundary conditions (in [4] both layer boundaries were of high thermal conductivity).

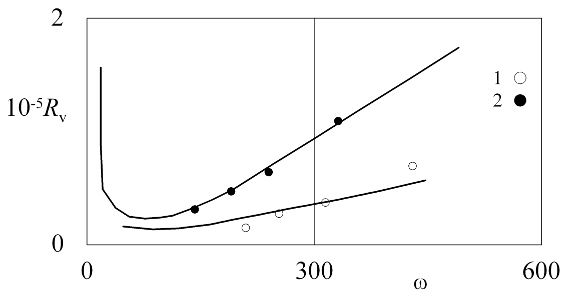


Figure 6: The threshold values of R_v depending on ω in vertical layer orientation ($Ra = 0$): points 1 - excitation threshold of the convective rolls, 2 - of convective cells, the curves are the results of the experiment [4].

With changing α the threshold values of R_v change (fig.7P°). With increasing negative angle the stability threshold increases. The critical increase of heat transfer with R_v growth, as noted above, is associated with the development of the vibrational thermal convection in the form of convective cells.

In the experiment with changing the velocity of rotation (at some constant angle of layer inclination and the temperature difference θ) the centrifugal force of inertia, the effect of which is characterized by centrifugal Rayleigh number $Ra_c = \Omega^2 R \beta \theta h^3/\nu\chi$, changes simultaneously with R_v. In the experiments Ra_c changes within the range $Ra_c = 0 \div 10^5$ (fig.7b) and does not significantly effect the heat transfer.

The boundary of vibrational convection excitation is represented on the plane Ra, R_v (fig.8a). Values of ω, corresponding to different threshold points are shown in fig.8b. The value $Ra = 0$ (shown by the dashed line) corresponds to a vertical position of the layer when only thermovibrational mechanism occurs. The stability threshold, as expected,

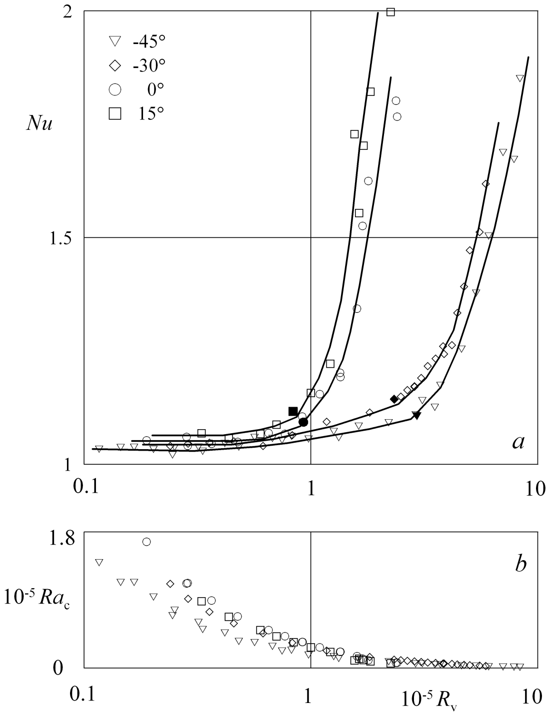


Figure 7: The dependence of Nu on the vibrational parameter at different inclinations of layer (a), the corresponding values of the centrifugal Rayleigh number Ra_c(b).

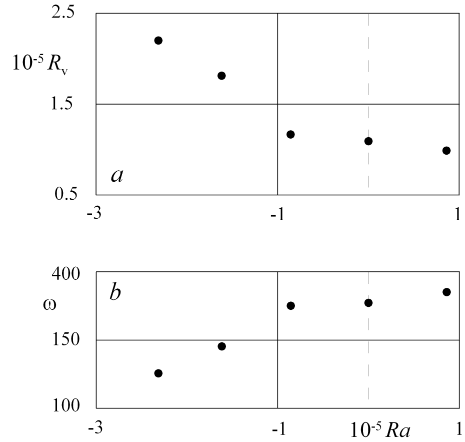


Figure 8: Stability threshold of thermovibrational convection depending on Ra (a) and the values of ω corresponding to the threshold points (b).

increases with the negative value of the Rayleigh number and decreases at $Ra > 0$.

5 Conclusion

The problem of convection in a rotating inclined layer is determined by the action of two mechanisms: gravitational and vibrational ones. Thermovibrational mechanism of convection is caused by a gravity field rotating in a cavity frame. It is found that the average convection is excited at an arbitrary angle of inclination even in the case when the hot layer boundary is at the top. The important role belongs to the dimensionless velocity of rotation which characterizes the effect of Coriolis force. With increasing the speed of rotation the excitation threshold of thermovibrational and gravitational convection grows.

Acknowledgements

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