Kelvin–Helmholtz type instability on a centrifuged liquid–liquid interface

Nikolai V. Kozlov Anastasiia N. Kozlova Darya A. Shuvalova
kozlovn@pspu.ru shuvalovada@gmail.com

Abstract

The interface of two immiscible liquids of different density in a rotating cylindrical container is studied experimentally. The dynamics is considered under external force action, periodic in the cavity frame. In experiments, liquids of various viscosity were used, the relative volume of filling was varied. Stroboscopic light was applied for observation and for measurement of rotation frequency. Light particles, that settled on the interface in the light liquid, were used for the interface velocity measurement.

At sufficiently fast rotation of the container, the liquids are centrifuged. The angular velocity of the inner liquid is less than the cavity rotation rate. The column of the light liquid undergoes a radial displacement (stationary in the laboratory frame) due to gravity action, the vector of which rotates in the cavity reference system. However, the column performs circular inertial oscillations in relation to the cavity (i.e. in the rotating system of reference). As a consequence, the tangential oscillations of the liquid near the interface lead to the generation of a mean vibrational mass force in the viscous Stokes layer. This force is oriented tangentially and excites the average differential rotation of the fluid. Meanwhile, the cross section of the light liquid column keeps the form of a circle.

With the decrease of the cavity rotation velocity, a small increase in the intensity of differential rotation of liquids interface is observed. In a threshold way, a wave is excited on the interface, similar to Kelvin вЂ“ Helmholtz instability. The axial symmetry of coaxial liquid layers is broken, on the boundary of the liquid column the crests are formed extended parallel to the cavity rotation axis. The direction of the liquid motion relative to the cavity coincides with the direction of the azimuthal wave propagation. In the research the waves with azimuthal wave numbers from 2 till 4 were observed. The further decrease of the cavity rotation velocity leads to the non-linear increase of the wave amplitude and is accompanied with a non-stationary mode, auto-oscillations take place.

1 Introduction

Rotating hydrodynamic systems with an interface are widespread in nature and technology, and knowledge about the behavior of such systems in the vibrational fields allow to use the vibrations for the management of these systems or prevent undesired oscillating force fields.

Multiphase systems in rotation are exposed to the impact of inertial forces including the Coriolis force. This contributes to the appearance of inertial waves. An intense azimuthal flow was found in the study of stability of a centrifuged liquid layer under the simultaneous action of vibrations and rotation in [1]. The occurrence of inertial waves was found also in the study of a light body behavior in a rotating horizontal cylinder with liquid [2].

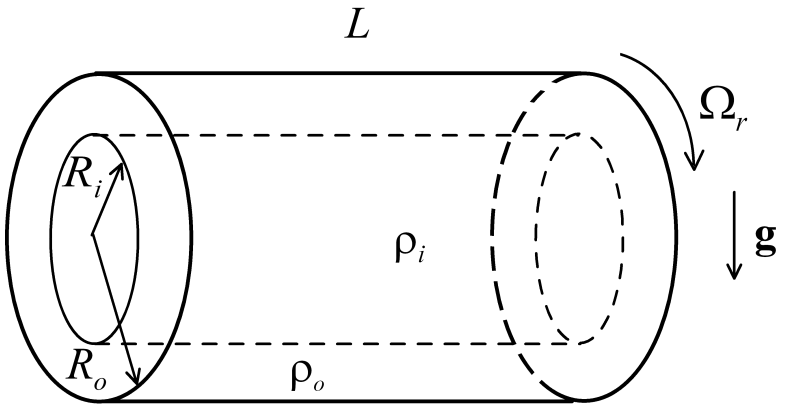

Figure 1: A centrifuged two-liquid layer. R_iвЂ“ radius of the undisturbed interface; вЂ“ angular frequency of the cavity rotation

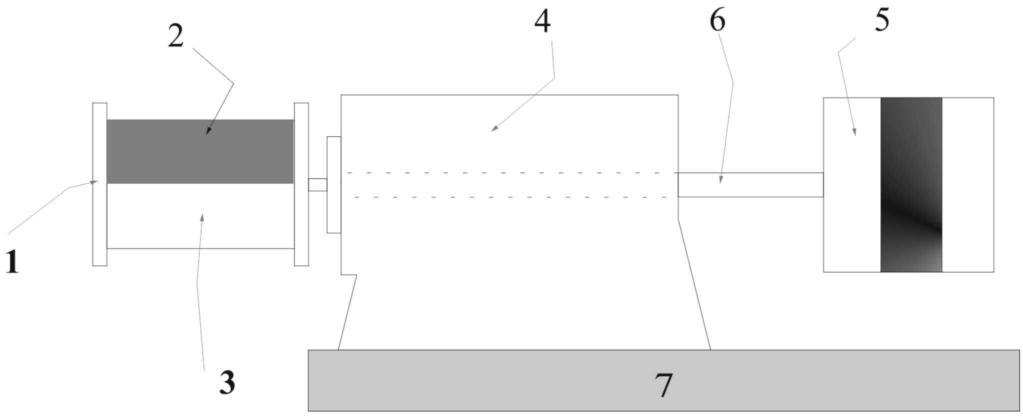

Figure 2: The experimental setup. Side view. 1 – cylindrical cell, 2 – industrial oil, 3 – aqueous glycerol solution, 4 – support with bearing, 5 –stepper motor, 6 – flexible coaxial transmission

The investigation of a two-liquid system under the influence of vibrations [3] shows that vibrations lead to the appearance of intense mean flows in liquid.

In [4] the wave excitement on the interface of multiphase liquid system under horizontal rotation in gravity field is revealed and thresholds of the system stability for low-viscous liquids are investigated. The dynamics and stability of a two-liquid system in a rotating horizontal cylinder is investigated in [5] at various viscosity relations.

In this paper the behavior is studied of a rotating system of two liquids with relatively large and similar viscosities. The flows structure in a light liquid column and the profile of the interface are studied.

2 Experimental setup

The experiments are carried out with a Plexiglas cylinder of the inner radius $R_o = 2.6$ cm and the length $L = 7.2$ cm (Fig. 1). The cylinder is hermetically closed with transparent lids. The working liquids are aqueous glycerol solution ($\rho_o = 1.2$ gr/cm3, $\nu_o = 18.2$ cSt) and industrial oil ($\rho_i = 0.88$ gr/cm3, $\nu_i = 23.6$ CЃSt). NaCl is added into the aqueous glycerol solution to adjust the density.

The cylindrical cuvette *1* is filled with the immiscible liquids: industrial oil *2* and aqueous glycerol solution *3*, and is mounted on a metal axis in the support *4*. The rotation speed of the system is set by the stepper motor *5* which is connected with the cuvette by means of the flexible coaxial transmission *6*. The construction is mounted on the stationary platform *7* (Fig. 2).

The cavity is rotated with the angular velocity $\Omega_r \approx 63$ rad/s for establishment of a centrifuged state (Fig. 1). After the centrifugation, the cavity rotation frequency is decreased step-by-step. For the establishment of a stationary mode of the fluid motion a delay of few minutes is done on each step before measurements. The cavity rotation frequency varies in the interval $\Omega_r = 0 - 63$ rad/s.

The volume ratio of the liquids is $q = V_i/V_o$, where V_i is the volume of the light liquid and V_o вЂ“ the total cavity volume. The volume ratio varies from 0.2 to 0.85. The ratio of densities of the aqueous glycerol solution and industrial oil is calculated as $\rho = \rho_i/\rho_o = 0.73$. The stroboscopic light is used both for the observation in the cavity frame and for measurement of rotation frequency by synchronization. The rubber particles of size about 0.1 mm and polypropylene particles of size about 0.01 mm are used for visualization of the flows.

3 Results

At rather fast rotation the light liquid occupies a steady position along the cavity axis forming an internal column, which is stationary in the laboratory frame and slightly shifted along the radius from the cavity axis (Fig. 1). The interface has a cylindrical shape; the particles are distributed on all surface of the column. In the cavity reference system the column of the light liquid makes the circular oscillations of small amplitude with the frequency of cavity rotation. The oscillations are caused by gravity. As a result the interface slowly moves in the direction of the oscillations, i.e. opposite to the direction of the cavity rotation. Detailed description of the mechanism of differential rotation is given in [2].

The differential rotation velocity of the light liquid column depends on the cavity rotation rate. At relatively high rate of the cavity rotation the light liquid column has angular velocity Ω_i close to Ω_r . There is only weak lagging rotation of the light liquid. With the decrease of the cavity rotation rate the increase of the difference in velocities of the cavity and internal liquid column $\Delta\Omega = \Omega_i - \Omega_r$ is observed. The dependence of the differential velocity of the interface $\Delta\Omega$ on the cavity rotation rate Ω_r is presented in [5].

With the decrease of the cavity rotation rate the markers move on the boundary of the internal column from end faces parallel to the rotation axis and form a ring in average area of the interface. This suggests the existence of the flows in the internal liquid column. With further decrease of Ω_r the particles continue their migration into the light liquid, where they form the cylindrical surface (Fig. 3, b, *1*).

The axial motion is generated in the viscous Ekman layers which are formed at the end faces of the cylinder in the presence of relative rotation [6]. Under the influence of the Coriolis force the liquid is ejected radially out of viscous layer, thickness of which is $\sqrt{2\nu/\Omega_r}$, to the interface and moves on it to the central part of the column. The mass flow in the boundary layer is compensated by normal fluid inflow from nonviscous area [6]. Therefore, the flow arises in the light liquid near the interface. This flow is directed along the interface from the cavity ends whereas the liquid inflow to the ends occurs at some fixed distance from the interface (Fig. 3, a). As near both cavity ends the Ekman layers are formed under the same conditions, in the bulk of the liquid the toroidal vortices are formed which are symmetrical relative to the central cross section of the cavity. With the decrease of the cavity rotation frequency the velocity of the lagging rotation of the light liquid column and intensity of the Ekman flows increase. As a result the liquid flows

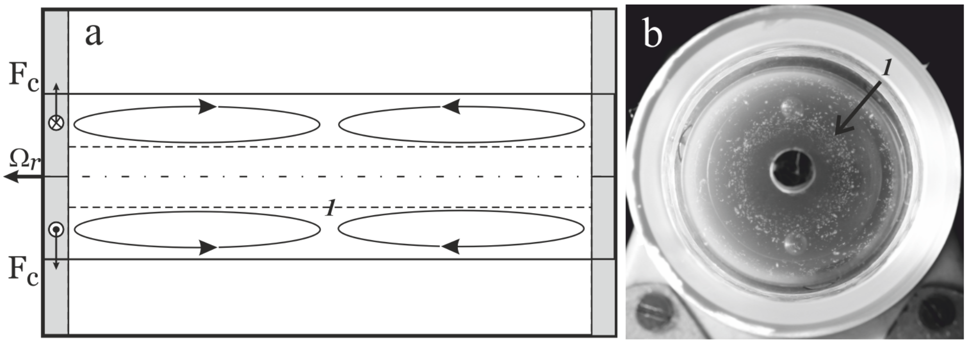

Figure 3: Schematic view of the Ekman flows in the axial section relative to the cavity (a); the shaded areas are the viscous layers; F_c – the Coriolis force; Ω_r – the angular velocity of rotation; the photograph of the cylindrical surface formed by the particles *(1)*, end view, $\Omega_r = 33.0$ rad/s, $q = 0.65$ (b)

departing from the cavity ends carry away the particles from the interface into the light liquid column, where they form the cylindrical surface (Fig. 3, b, *1*).

At rather fast rotation of the cavity the profile of the interface looks as a harmonic function (Fig. 4, a). Here h is the distance from the cavity wall to the liquid interface. With the decrease of the rotation frequency, the velocity of lagging motion of the column increases. Upon reaching the critical value the redistribution of the visualizing particles throughout the entire volume of the light liquid is observed. The process is non-stationary and can take several seconds. This is associated with excitation of a two-dimensional azimuthal wave on the interface. The crests of the wave are extended parallel to the rotation axis of the system (Fig. 4). In the cavity frame the wave propagates in the direction opposite to the cavity rotation. The loss of stability occurs in a threshold way and is accompanied with the sharply increase of relative velocity of the markers on the interface (the velocity of azimuthal lagging rotation of the column). The wave appears as a result of tangential discontinuity of velocity on the interface of immiscible liquids of various densities (Kelvin–Helmholtz instability) [7].

Let us turn now to (Fig. 4). In the photographs the cuvette rotates counterclockwise. The direction of rotation is taken as positive for the coordinate φ. The origin of coordinates $\varphi = 0$ corresponds to the bottom point of the cavity, where the directions of the gravity vector and radius-vector coincides (Fig. 4). In the cavity frame the wave propagates in the direction opposite to the cavity rotation. With decrease the difference in velocities of the cavity and the wave increases. The waves with azimuthal number m from 2 to 4 were observed in the experiment (Fig. 4, b–d). The wave number depends on filling and changes discretely. The crests of the wave differ in amplitude as a result of superposition of the stationary (in the laboratory frame) displacement of the light liquid column under action of gravity and the wave excited due to the instability of tangential velocity discontinuity. The azimuthal wave generates the average liquid motion in the direction of its propagation [8], thereby intensifying the lagging rotation of the light liquid column.

With the decrease of Ω_r the differential rotation intensity increases and the wave amplitude grows. In case of large amplitude the wave passes from the steady state into a self-oscillations mode.

Stability of the system is described by the parameter $\Gamma = g/(\Omega_r^2 R_i)$ – dimensionless acceleration of gravity. The larger critical value of this parameter corresponds to higher stability of the system. The thresholds of wave appearance are presented on the plane (q, Γ). On (Fig. 5) the threshold curves for the various couples of liquids are shown:

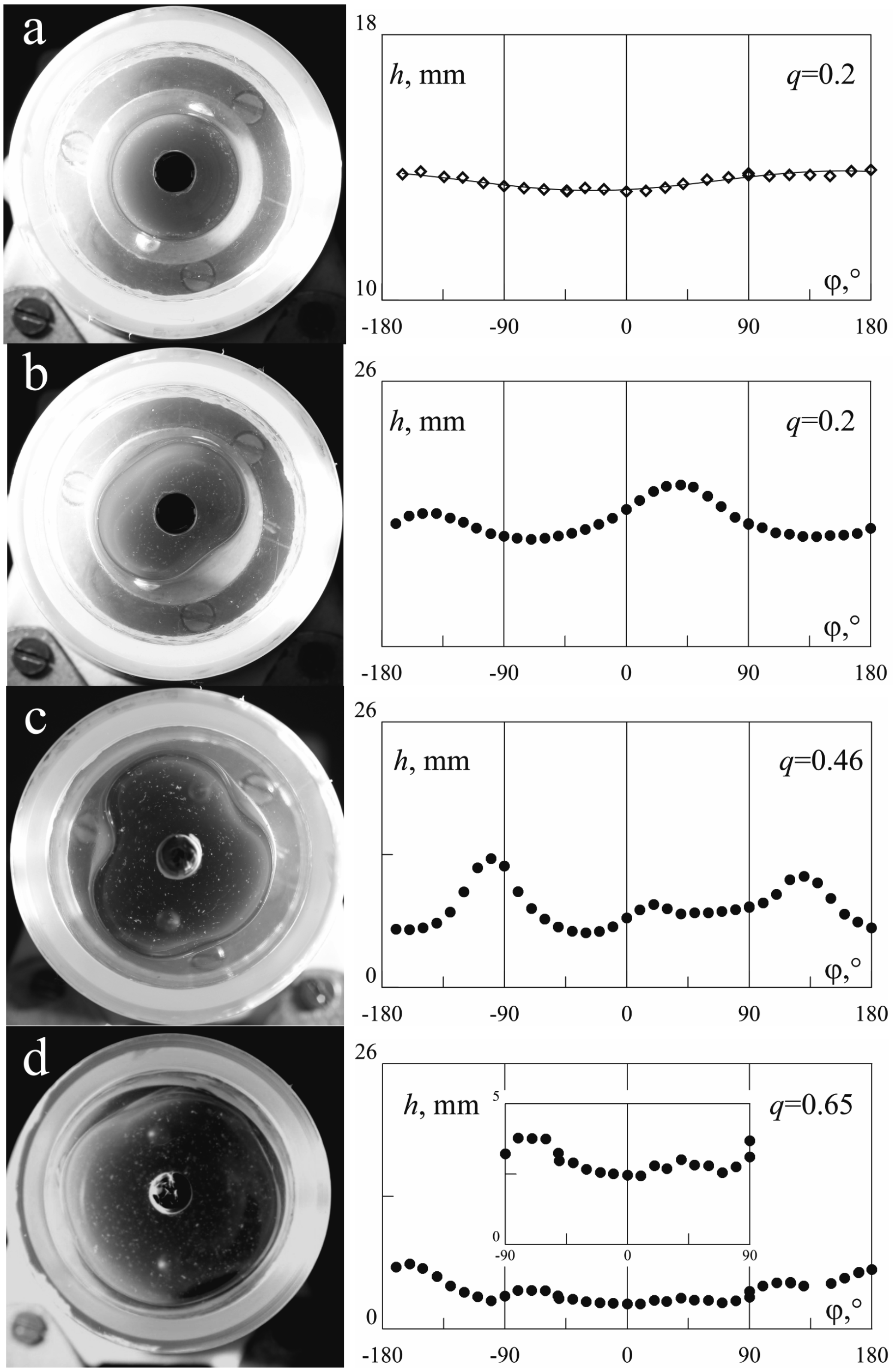

Figure 4: SPhotographs of the liquids interface and the corresponding profiles of the azimuthal wave on the plane Γ, q; $\Omega_r(sec^{-1}) = 62.8(a), 38.1(b), 35.0(c), 27.2(d); \Omega_w(sec^{-1}) = 27.8(b), 28.3(c), 21.8(d)$

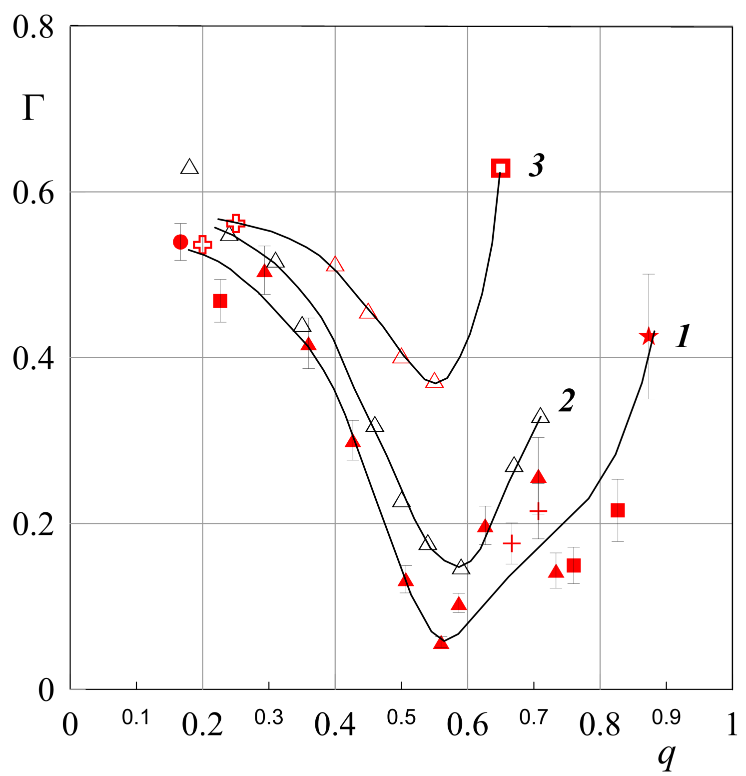

Figure 5: The thresholds of the wave appearance for dodecane – water: $\nu_i/\nu_o(St) = 0.015/0.010$ (*1*); industrial oil – aqueous solution of NaCl: $\nu_i/\nu_o(St) = 0.236/0.016$ (*2*);industrial oil – aqueous glycerol solution: $\nu_i/\nu_o(St) = 0.236/0.178$ (*3*)

water–dodecane (points *1*) [4], aqueous salt solution–industrial oil (points *2*) [5], aqueous glycerol solution–industrial oil (points *3*). The threshold value of Γ changes with the volume ratio non-monotonously. In a vicinity of $q = 0.6$ there is a pronounced minimum of the system stability for all pairs of liquids. With the growth of average viscosity of liquids pair the stability of the system increases.

4 Conclusion

The behavior of the interface of two immiscible liquids in the rapidly rotating horizontal cylinder is experimentally investigated. With the decrease of cavity rotation rate the velocity of column differential rotation increases, meanwhile the markers are redistributed, they accumulate in the light liquid column and form a cylindrical surface at a certain distance from the interface. The profiles of stationary propagating waves are presented at various volume ratio and azimuthal numbers. The experiments, which are done with different volume ratio, showed that this parameter significantly influences the stability threshold. The minimum stability is observed when $q = 0.6$. With viscosity growth the stability of system increases.

Acknowledgements

The work is done on the task of Minobrnauki of the Russian Federation 2014/372 (project 2176), with the support from the Program of strategic development of PSHPU (project

029-F).

References

[1] A.A. Ivanova, V.G. Kozlov, and D.A. Polezhaev. Vibrational Dynamics of a Centrifuged Fluid Layer // Fluid Dynamics. 2005. Vol. 40. No. 2, P. 297–304.

[2] Kozlov V.G. and Kozlov N.V. Vibrational dynamics of a light body in a liquid-filled rotating cylinder // Fluid Dynamics. 2008. Vol. 43. No. 1, P. 9–19.

[3] Ivanova A.A. and Sal'nikova A.N. Dynamics of a two-fluid system in a rotating horizontal cylinder under longitudinal vibration // Fluid Dynamics. 2007. Vol. 42. No. 3. P. 369–375.

[4] Kozlov, N., Salnikova, A., Stambouli, M. Vibrational dynamics of two immiscible liquids under rotation // 61st International Astronautical Congress 2010, IAC 2010. Elsevier. 2010. Vol. 4. P. 2886–2892.

[5] Kozlov N.V., Kozlova A.N., Pichkalev S.V. Dynamics of two-liquid system in a rotating horizontal cylinder // Convective flows... Edition в„–6/Perm: PSHPU, 2013. P. 168–184. [In Russian].

[6] H.P. Greenspan. The Theory of Rotating Fluids (Cambridge Univ. Press, Cambridge, 1968).

[7] L. D. Landau and E. M. Lifshitz, Fluid Mechanics, 2nd ed. (Nauka, Moscow, 1986; Pergamon Press, Oxford, 1987).

[8] Lighthill J. Waves in fluids. Cambridge, UK: Cambridge University Press, 1979. 504 p.

Kozlov N.V., Laboratory of Vibrational Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, Russia
Kozlova A.N., Laboratory of Vibrational Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, Russia
Shuvalova D.A., Laboratory of Vibrational Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, Russia