# Vortexes and waves in the “vibrational hydrodynamic top”

Victor G. Kozlov    Nikolay V. Kozlov    Stanislav V. Subbotin
kozlov@pspu.ru    nick.kzlv@gmail.com    subbozza@mail.ru

**Abstract**

In [1] it is found that the action of an external periodic force on a light spherical body centrifuged in a rotating about horizontal axis cavity filled with liquid leads to the excitation of the body differential rotation (vibrational hydrodynamic top). As a result, the flow in a 2D column form oriented along the rotation axis is generated. Inside the column the intense azimuthal fluid motion is observed [2].

In this paper the supercritical flow regimes are experimentally studied. With increase of the body differential rotation the axisymmetrical flow becomes unstable: 2D vortex structures appear inside the column. The rotation direction of each vortex coincides with the rotation direction of the cavity. At the same time, the vortex system as a whole rotates in the opposite direction relative to the cavity with a velocity considerably exceeding the body differential rotation velocity. The vortex structures in the column center are independent from its boundary that remains cylindrical. With increase of the supercriticality the number of vortices grows to three.

Another type of instability is associated with the appearance of an azimuthal wave on the column boundary, which can develop irrespective of the vortex system presence. The 2D wave on the boundary has another phase velocity, and the wave numbers are different. Interaction of the two different instability types manifests itself in their synchronization.

## 1 Introduction

The study of flows structure in rotating spherical liquid layers is an actual geophysical problem. The stability of flow in case of weak relative motion of the layer boundaries (differential rotation) was investigated in [3, 4]. In this case, the main flow has a shape of the Taylor column oriented along the axis of rotation. The instability manifests itself in the excitation of azimuthal wave on the column boundary. Differential rotation was set using several drives, one of which moves a cavity and the other – a body.

Differential rotation of a body can be generated due to the impact on the system of an external force causing the body oscillations relative to the cavity. This problem was solved theoretically and experimentally in a two-dimensional formulation with a cylindrical body in [5]. The phenomenon of differential rotation has been called “vibrational hydrodynamic top”. The differential rotation under the influence of external field also exists in case of a spherical body [1]. In this case, the flow in the form of Taylor column is also formed in liquid, but the difference from the experiments in “classical” formulation, in the absence of the influence of the external field [3, 4], is the vortex motion inside the column [2]. In addition, as shown in [6], the threshold of the wave instability in case of vibrational top is more than an order lower than when the differential rotation of the body is set [4]. The purpose of this paper is the experimental study of the dynamics of supercritical structures.

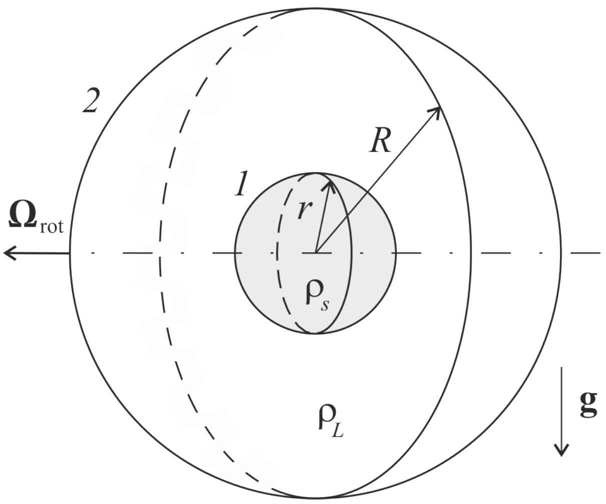


Figure 1: Problem statement.

## 2 Problem statement and experimental technique

A light spherical body (*1*) with radius $r = 1.77$ cm and average density $\rho_s = 0.23$ g/cm$^3$ is located in a fluid-filled spherical cavity (*2*) with radius $R = 3.60$ cm (Figure 1). The cavity rotates around the horizontal axis with velocity $f_{rot} = \Omega_{rot}/2\pi$, which is set by a step motor FL86STH80-4208A accurate to 0.01 rps and varies in the range $f_{rot} = 10 - 40$ rps. The working fluids are water and water-glycerol solutions with the kinematic viscosity varying in the limits $\nu = 1 - 80$ cSt; the fluid density varies in the range $\rho_s = 1 - 1.2$ g/cm$^3$ in this case.

To minimize the optical distortions at the spherical boundary with the observation in the direction perpendicular to the face, the cavity is fabricated in the center of a cube-shaped plexiglas block. The deviation from sphericity of the cavity surface does not exceed 0.03% of the curvature radius.

The experiments were performed with sufficiently rapid rotation, when the light body occupies a stable position in the cavity center under the effect of centrifugal force. In the course of the experiment, the cavity rotation speed $f_{rot}$ decreases (increases) step-by-step. At each step, after the steady-state mode is established, the rotational velocity of the body $f_s$ is measured and the structure of the fluid flow is investigated. The velocity of the body relative to the cavity is calculated as the difference $\Delta f = f_s - f_{rot}$. Negative values of $\Delta f$ mean that in the laboratory frame the sphere rotates slower than the cavity. At the same time, in the cavity frame the body rotates in the opposite direction.

The observations are conducted in a stroboscopic light. Measurement of rotation velocities of the sphere and the cavity is performed by synchronization with the frequency of stroboscope flashes. The observations are carried out both from the side and end wall (along the axis of rotation). When viewed along the axis the illumination is done from the side with a 1 mm wide light sheet, located at equal distance between the poles of the sphere and the cavity. The light-scattering particles Resin Amberlite of density about 1 g/cm$^3$ and the diameter of 50 $\mu$m are used for visualization.

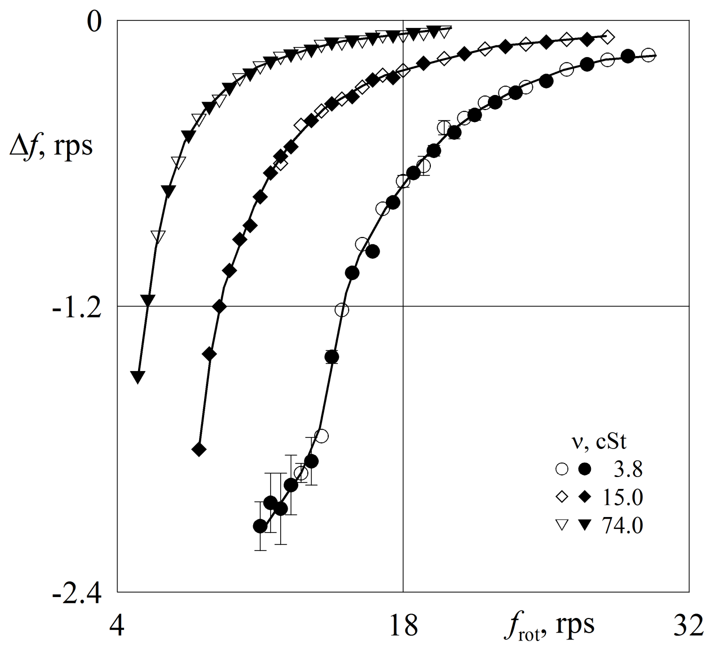


Figure 2: Dependence of the sphere differential rotation on the cavity rotation speed for different viscosity of the liquid; light symbols correspond to the increase of $f_{rot}$, dark – to the decrease.

# 3 Differential rotation

The disturbing effect of an external force field (gravity field) leads to the oscillations of the body relative to the cavity with frequency $f_{osc} = f_{rot}$. The oscillations result in generation of the body differential rotation. With $f_{rot}$ increase the differential rotation velocity of the sphere $|\Delta f|$ monotonically decreases (Figure 2). At large values of $f_{rot}$ the differential rotation becomes weak, the sphere and the cavity rotate practically as a solid. With $f_{rot}$ decrease the velocity $|\Delta f|$ grows, the experimental points are consistent with the dependence obtained at the increase of $f_{rot}$. In the transitions of the sphere in centrifuged state and back a hysteresis is observed. An increase of the fluid viscosity leads to a decrease of $|\Delta f|$ at specified $f_{rot}$. The thresholds of floating-up and centrifugation of the sphere shift to the lower values of $f_{rot}$.

Explanation of the differential rotation mechanism is given in [5]. The presence of a rotating force field (the gravitational field in this case) in the rotating frame leads to the circular oscillations of the body. As a result, an inertial azimuthal wave in fluid and, consequently, the pulsating motion in the viscous boundary layer appear. This leads to the emergence of an averaged mass force acting along the body wall, spinning it.

The differential rotation of the body leads to the formation of a quasi-two-dimensional flow in the form of the Taylor–Proudman column elongated along the cavity rotation axis. The transverse size of the column coincides with the body diameter. At weak relative rotation of the body, the column boundary is shaped as a circular cylinder (the white ring on the Figure 3a). Particles with neutral floatability are concentrated in the Stewartson layer on the boundary of the tangential break of the azimuthal velocity thereby visualizing the column. Inside the Taylor column the visualizing particles accumulate on the rotation axis, forming the inner column. The latter has a size $r/3$ and is characterized by intensive lagging differential rotation of the fluid within it, which is superior to $|\Delta f|$.

# 4 Vortex structures and waves

With increase of the differential rotation rate $|\Delta f|$ the main flow becomes unstable. Initially, in the central part of the column two two-dimensional vortices arise in a threshold way (Figure 3b). The vortex formations have a shape of flagella elongated along the axis of rotation. Near the threshold of occurrence the vortices in the central part of the column do not affect the shape of its boundary, it remains a cylindrical $m = 0$. The rotation direction of each of the vortices is cyclonic, while the rotation of the vortex system is anticyclonic with velocity $|\Delta f_v|$, much higher than $|\Delta f|$ (Figure 4).

With $|\Delta f|$ increasing (with lowering of $f_{rot}$) the vortex system grows in transverse size and its rotation velocity $|\Delta f_v|$ decreases. With grow of the differential rotation of the sphere the vortex number increases to $k = 3$ (Figure 3c).

The next threshold transition is associated with the instability of tangential discontinuity on the external boundary of the column. The boundary takes the form of ellipse in a threshold way (Figure 3d), and the number of vortices decreases to $k = 2$. The development of wave instability is accompanied by an abrupt change in the phase velocity $\Delta f_v$ (threshold $a$ on the Figure 4).

With further increase of supercriticality $|\Delta f|$ the number of column faces increases to $m = 8$ (the hatched area on the Figure 4). The motion of the vortex system inside the column leads to the fact that the column boundary takes asymmetric shape (Figure 3e). Thus, both types of instability manifest themselves independently from each other: the short-wavelength structures are developed on the boundary of the column, while inside it there is a system of two or three vortices.

With $|\Delta f|$ increasing the short-wavelength mode is suppressed by long-wavelength one. On the boundary one wave mode ($m = 2$) prevails, and the vortex system rotates in phase with the wave on the boundary ($\Delta f_v = \Delta f$). Note that, despite the prevalence of the long-wavelength mode the short-wavelength disturbances may be present on the boundary. In the strong supercriticality the particles of visualizer are distributed throughout the liquid volume that means the destruction of the column and predominance of three-dimensional motion.

Thus, with the supercriticality change the wave number m varies nonmonotonically: the long-wavelength structures are replaced by the short-wavelength ones, which, in turn, are replaced by the long-wavelength ones. In the experiments with a cylindrical cavity [1, 6]when changing $\Delta f$, the wave number is always changed by one. The same scenarios of transitions between the wave numbers were observed in the experiments with the fixed axis of body rotation [3, 4].

With decrease of the supercriticality a hysteresis is observed in the transitions between the structures. So, the trihedral column and system of three vortices inside it are formed in the threshold of centrifugation (Figure 3f). With increasing of rotation speed of the cavity the wave phase velocity $\Delta f_w$ decreases in absolute value. Disappearance of the wave on the boundary occurs in the threshold $a$ (Figure 4), that leads to the change of rotation velocity of the vortex system $\Delta f_v$ in a threshold way.

Further dynamics of the system is as follows. With $f_{rot}$ increase the vortex system is compressed in transverse size, rotation velocity $|\Delta f_v|$ increases. At some critical size of the system, two vortices are combined in one, this occurs until $k = 1$. As a result, the system takes an axisymmetric shape and consists of two nested liquid columns.

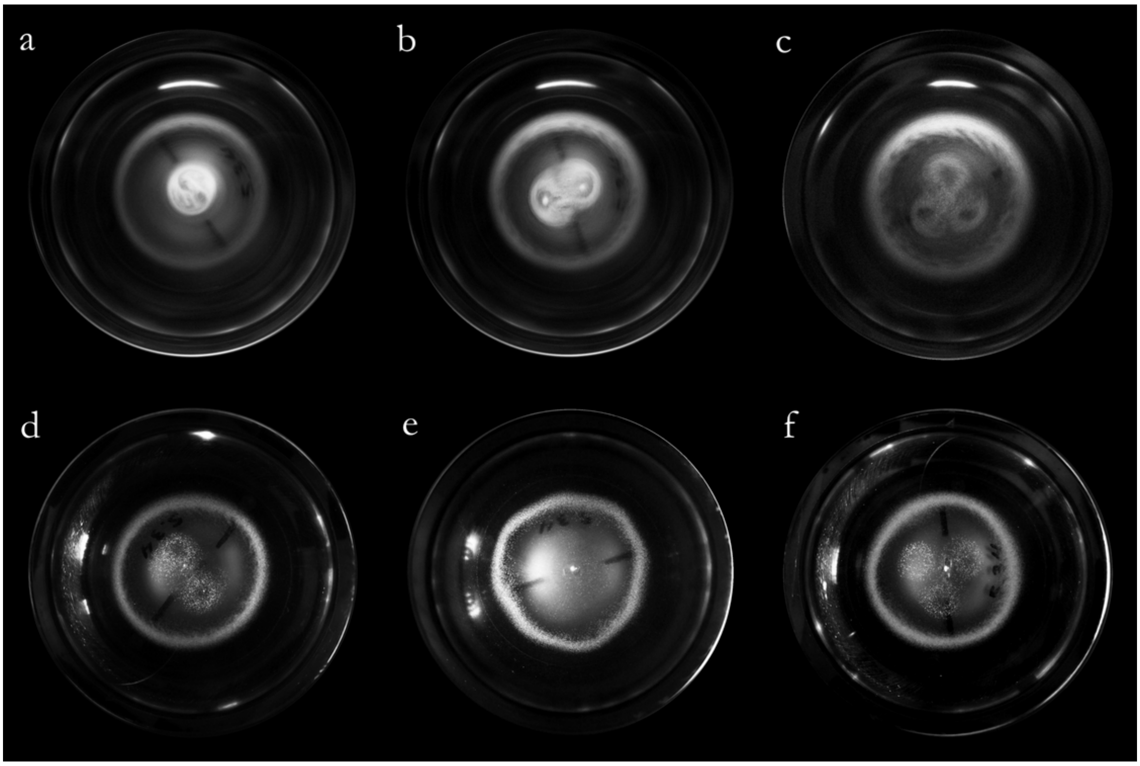


Figure 3: Development of the vortex system (a – c) and Taylor column instability (d – f) with increase of the supercriticality: a – $f_{rot} = 28.5$rps, b – $f_{rot} = 24.5$rps, c – $f_{rot} = 20.0$rps ($\nu = 5.9$cSt), d – $f_{rot} = 21.5$rps ($\nu = 5.0$cSt), e – $f_{rot} = 19.0$rps ($\nu = 4.8$cSt), f – $f_{rot} = 21.0$rps ($\nu = 5.0$cSt).

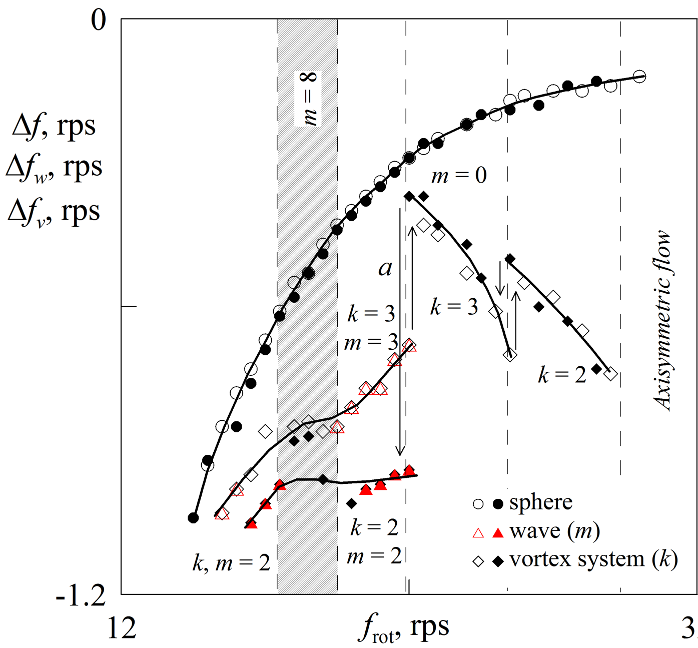


Figure 4: Dependence of the sphere differential rotation $\Delta f$, wave phase velocity $\Delta f_w$ and vortex system velocity $\Delta f_v$ on the cavity rotation speed for $\nu = 4.8$ (a) cSt; light symbols correspond to the increase of $f_{rot}$, dark – to the decrease.

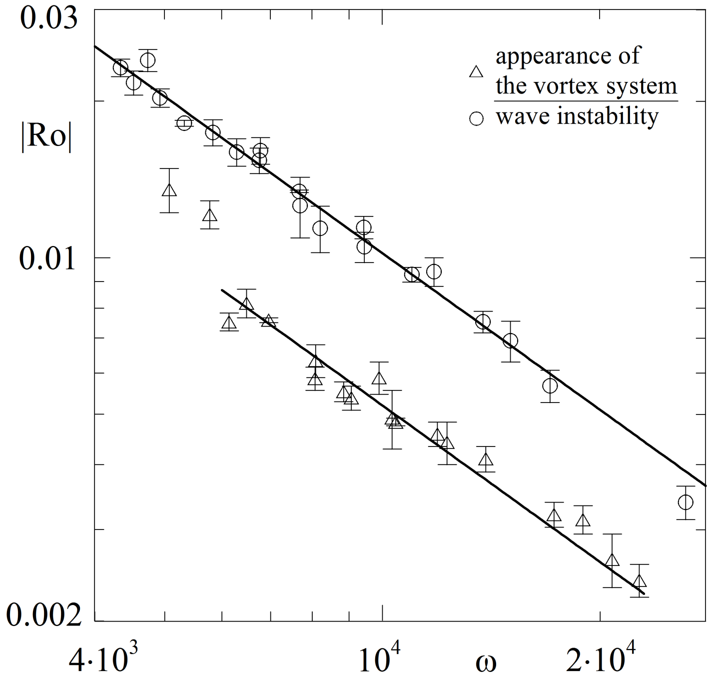


Figure 5: Thresholds instability of the flow.

## 5 Analysis of the results

The wave instability of the Taylor–Proudman column boundary and the two-dimensional system of vortices in the central part of the column have different mechanism. In order to describe the fluid motion and its stability, we use the dimensionless velocity of differential rotation (Rossby number) $\mathrm{Ro} \equiv |\Delta f|/f_{rot}$ and the dimensionless frequency of cavity rotation $\omega = 2\pi f_{rot} r^2/\nu$. With increase of $\omega$ the stability threshold of the structures of both types decreases according to the law $|\mathrm{Ro}| \sim \omega^{-1}$ (Figure 5). This means that the thresholds are characterized by one dimensionless complex – the Reynolds number $\mathrm{Re} \equiv \mathrm{Ro}{\cdot}\omega = |\Delta f| r^2/\nu$, calculated through the differential rotation velocity. Moreover, the threshold value of the Reynolds number for appearence of the system of vortices is $\mathrm{Re} = 52 \pm 5$, which is two times less than the stability threshold of the column boundary ($\mathrm{Re} = 102 \pm 9$).

The appearance of the wave on the column boundary due to the Stewartson layers instability, which was studied earlier theoretically [7] and experimentally [3, 4, 8]. The main difference of this paper from [3, 4, 8] is the method of excitation of the body differential rotation. The method determines the flow structure in the column.

The differential rotation results from the generation of the averaged shear stresses on the body surface. These shear stresses also generate the flows within the column. Experiments show that the structure of the classical Taylor column, which is characterized by solid-body rotation, is different from the column generated by a free body. In the latter case, the vortex motion inside the column is found which is more intensive than the differential rotation of the body [2].

## 6 Conclusion

The flow structures excited by the differential rotation of the free core in the spherical liquid layer are experimentally studied. The differential rotation, caused by the coreвЂ™s oscillations under the action of external force, leads to the formation of the flow in the form of Taylor's column.

In the subcritical region the flow is axisymmetric, the column is shaped as a cylinder of circular cross-section. A new type of instability, which is manifested in the appearance of the system of two-dimensional vortices inside the column, is found. The rotation direction of the system is anticyclonic and intensity of rotation exceeds the differential rotation rate of the body. With the growth of supercriticality the wave instability of the column boundary occurs which is not associated with the development of the vortices inside the column. The discovered types of instabilities are independent of each other, which is confirmed by the difference in phase velocities and wave numbers.

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*Kozlov V.G., Laboratory of Vibrational Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, Russia*
*Kozlov N.V., Laboratory of Vibrational Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, Russia*

*Subbotin S.V., Laboratory of Vibrational Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, Russia*