Modeling of fatigue process by combining the crack initiation and growth

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Abstract

The present codes for fatigue design of structures and rules for assessment of residual fatigue life of existing structures are based on considering the fatigue process composed of the two phases: the crack initiation and crack propagation until a specified critical condition would be attained. Respectively, different approaches are applied in fatigue analysis within these phases. However, uncertainties inherent into the data base of the approaches, especially of the Stress-Life criteria-based approaches, do not provide the continuity of fatigue assessment of structures.

Versions of combined model of fatigue are suggested providing continuous analysis of fatigue damage of structures starting from initiation of service until a prescribed crack size would be attained. The problems of application and the prospects of practical implementation of models are discussed.

1 Introduction

The present rules for fatigue assessment of structures apply the design S-N curves to characterize fatigue resistance of material and typified welded joints [1, 2], etc. Material and welded joint specimens are tested to obtain the necessary information providing S-N curves under cyclic loading until almost complete failure. By this, the fatigue process described by S-N curve includes the crack nucleation caused by development of damage in material structure, macroscopic crack formation and growth until termination of test. When such S-N curve is applied to assess fatigue life of a critical location in structure the extent of the crack corresponding estimated life occurs uncertain.

For many years it has been recognized that fatigue crack growth may comprise a considerable portion of the fatigue lifetime of structure. Relatively slow crack propagation in many observations allows extending fatigue life of structure by including a certain portion of the developing damage into the safe life, until it may become menacing structural integrity (the safety factor should be applied, obviously). By this reason prediction of fatigue crack propagation in structural components is an essential part of analysis of residual strength and reliability of structures in service; it may provide the strategy of maintenance and repair of structures.

The crack propagation is analyzed almost solely by applying the linear elastic fracture mechanics (LEFM) procedure which needs in explicit definition of the initial crack extent, length at the surface, depth and crack front shape. Also, the LEFM-based analysis is frequently limited by development of plasticity well before failure of a structural component, e.g. of a pipeline, where it is not possible to estimate the residual life until the through crack would develop.

As said in above, the use of the S-N curve-based format does not predict the initial crack geometry and size. Respectively, there is a gap, a discontinuity in description and the means of analysis the fatigue process in structures and the crack propagation analysis may be incomplete in solution of engineering problems.

However, the mechanism of fatigue in above stages is unique: development of slip and microcracks within grains, coalescence of slip systems in adjacent grains and formation of macroscopic crack. The crack extensions may be related to co-operation of the two processes, the damage accumulation in material at the notch root and ahead the crack tip due to dislocation mechanics and failure of the material bonds in the tensile loading phase controlled by the effective stress intensity factor.

Therefore to attain at a better presentation of the fatigue damage and failure progress it is reasonable to suggest a model of fatigue based on combination of these mechanisms.

2 Description of the approach

The model incorporates the two well-known approaches to the crack initiation and the crack propagation phases of structural fatigue.

The crack initiation is modeled using the Strain-life fatigue criterion-based approach which again, does not define the crack size corresponding to completion of the initiation stage. However, by applying the procedure of the fatigue damage summation to arbitrary material elements located along the anticipated crack trace, it is possible to predict the early crack geometry and propagation rate.

The crack growth off the origination site may be modeled using the combination of two approaches, the mentioned Strain-life-based format and the Paris-Erdogan law [3]. The other equations and approaches other than the stress intensity factor related can be used, alternatively.

Combining the models in early crack growth permits to take into consideration both mechanisms of material failure: the plastic slip and microcrack formation due to the cyclic shear straining together with the effects of the stress field controlled by the stress intensity factor range, or by the maximum value of the stress intensity. The latter becomes effective when the microcrack overcomes several microstructural barriers and grows over approximately, 0.12-0.2 mm [4, 5]. Apart from that, considering the both mechanisms may allow analyzing the crack extensions until complete failure of structural element and avoiding limitations of the Linear Elastic Fracture model when the stress field becomes elastic-plastic one and the damage accumulation can be the governing mechanism of failure under conditions of developed plasticity.

The briefly outlined procedure may be described as follows. Let the crack initiation and extension into location of a material element at a distance δa from the notch root or growing crack growth should be assessed. The crack extension into the element is defined by the element failure under combined action of the above two mechanisms. The crack tip strain field component, the principal strain range at the distance r from the notch root of the crack tip ($0 \leq r \leq \delta a$) is $\Delta\varepsilon(r)$, Fig. 1. The Coffin-Tavernelli's [6] material fatigue failure criterion may be written as:

$$N(\Delta\varepsilon) = C^{1/\alpha}(\Delta\varepsilon(r) - \Delta\varepsilon_0)^{-1/\alpha} \tag{1}$$

where C and α are the material fatigue parameters, $\Delta\varepsilon_0$ is the elastic component of total strain range, $\Delta\varepsilon(r)$, corresponding to the minimum strain range when traces of the plastic slip may be observed in the material structure.

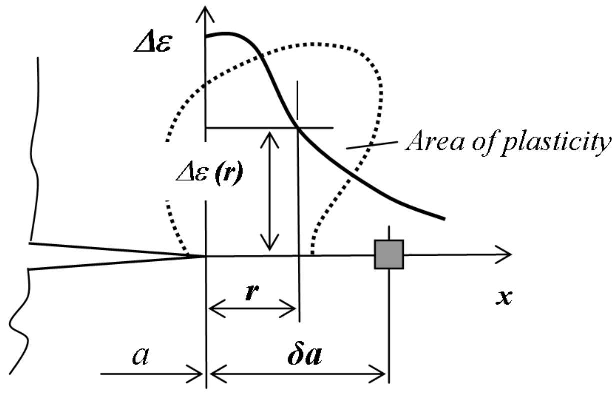

Figure 1: Scheme of material element location ahead the crack tip

The strain range, $\Delta\varepsilon(r)$, may be obtained by finite element analysis of the cyclic stress-strain field at the notch root or the crack tip. The cyclic stress-strain diagram may be expressed, e.g., in the form modified Ramberg-Osgood equation:

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p = \Delta\sigma/E + K(\Delta\sigma - \Delta\sigma_c)^m \tag{2}$$

where $\Delta\varepsilon$ is the total, elastic + plastic, strain range, K and m are the strain-hardening coefficient and exponent in a load reversal, respectively, $\Delta\sigma$ and $\Delta\sigma_c$ are the stress range and cyclic proportionality stress range, respectively.

Crack propagation rate within the plastic zone may be defined depending on the average cyclic plastic strain range value related to a finite distance from the free surface or the crack tip to the location of a provisional material element inside the plastic zone. The average strain range over distance $x = \delta a$, strain redistribution due to the crack origination and progress to be considered, may be defined as

$$\overline{\Delta\varepsilon_p} = (1/\delta a)\int \Delta\varepsilon_p(\xi)\, d\xi \tag{3}$$

where ξ is the integration variable, $0 \le \xi \le \delta a$.

According to (2), the number of load repetitions prior to the failure of the material element depends on plastic strain range. Consequently, the cyclic strain range (3) causes failure of material element after the number of cycles defined by equation (1):

$$N = (C/\overline{\Delta\varepsilon - \Delta\varepsilon_e})^{1/\alpha} = (C/\overline{\Delta\varepsilon_p})^{1/\alpha} \tag{4}$$

and respectively, the average crack growth rate over a distance $x = \delta a$ may be approximated as:

$$\overline{\frac{da}{dN}} \approx \delta a/N = \delta a(\overline{\Delta\varepsilon_p}/C)^{1/\alpha} \tag{5}$$

To describe the early crack growth rate in more detail, the material element should be located at a shorter distance from the free surface where the crack origination is expected. When the crack becomes macroscopic, the element should be located at a short distance from the crack tip inside the tip plastic zone. It should be noted that in the latter case a direct application of equations (3) and (4) would be incorrect: the past damage due to plastic zone passage over the element location is to be considered at distance $r_p - a$, where r_p is the plastic zone size measured from the crack tip. The linear damage summation rule may be applied to correct the number of load cycles in (4). When the singular stress field is formed beyond the plastic zone boundary, the crack growth rate may be increasingly influenced by the failure of the microstructural bonds controlled by the effective stress intensity factor (SIF) range according to the Paris-Erdogan law:

$$da/dN = C_c(\Delta K_{ef})^m \tag{6}$$

where $\Delta K_{ef} = \Delta K(a) - \Delta K_{th}$ is the effective stress intensity factor range, precisely, $\Delta K_{ef} = K_{max} - K_{op}, K_{op}$ is the stress intensity corresponding to the crack opening in the tensile loading phase.

By combining expressions (5) and (6) the total growth rate attributed to both mechanisms is defined as:

$$da/dN = (\int_a^{a+\delta a} \Delta\varepsilon_p(\xi)\, d\xi)^{1/\alpha}/C^{1/\alpha} + B(\Delta K_{ef})^m \tag{7}$$

where B is the constant different from C_c in (6).

Equation (7) is addressed to the growth rate at any crack size. Material constants C, B and m should be found from the crack propagation tests; perhaps, the latter may not differ from the respective constant in (6). The effect of the second term in the right-hand part of (7) depends on the presence of a singularity of the $1/\sqrt{r}$ type in the stress field ahead the crack tip plastic zone. In the initiation phase the second term should be actuated when the microscopic crack turns into the macroscopic phase and singularity develops in the stress field at the notch root [5]. Alternatively, this transition can be estimated when the crack extends over half-size of the plasticity zone at the notch root [7]. In proceeding the crack growth rate, respectively, becomes dependent on the both, plasticity induced damage and the stress field, namely, values of the stress intensity factor. When the crack extensions essentially reduce the crossection area of an affected element and the stress singularity ceases, the first term in the right-hand part of (7), again, solely characterizes progress of fatigue failure.

Material constants in (7), alternatively, may be found by comparing the crack growth rate (7) and that defined by (6) since the stress intensity format is applied to define *actual material resistance* to the crack extensions. Respectively, by equating (6) and (7) the following relationship is found:

$$((\int_{a_0}^{a} \Delta\varepsilon_p(\xi)\, d\xi)/C)^{1/\alpha} + B(\Delta K_{ef})^m = C_c(\Delta K_{ef})^m \tag{8}$$

which may be used to estimate the constant on assumption that the crack growth rate in the stable growth phase is equally modeled by (6) and (7).

3 Numerical example

The outlined procedure was applied to estimate fatigue crack propagation rate in a steel plate with a sharp notch at the longitudinal edge; the plate was assumed cyclically loaded in

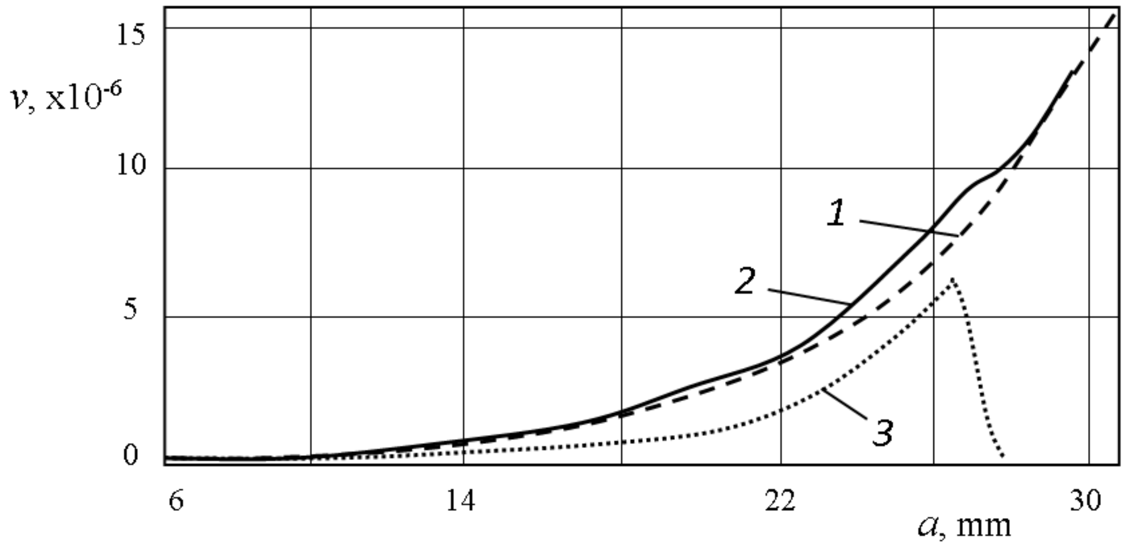

Figure 2: Crack growth rate vs crack length, Eq. (7):
1 - $B = 7.6 \cdot 10^{-15}, a_0 = 0.3\ mm$; **2** - $B = 2.8 \cdot 10^{-13}, a_0 = 2.0\ mm$; **3** - Paris law (6)

zero-to-tension mode. Material is the structural steel 09G2; parameters of the criterion (1) are: $C = 0.340, \alpha = 0.653$; cyclic stabilized stress-strain curve — as described in [8]. Parameters of (6) are: $C_c = 10^{-12}, m = 3.0$ [9].

Cyclic stress-strain analysis was carried out by applying finite-element software developed for solution of cyclic elastic-plastic problems; stress intensity factor values at assumed crack extensions were estimated using the extrapolation to the "effective" crack tip procedure described in [7]. Parameter B was obtained at varied initial crack size, a_0, and the crack length, a.

Analysis showed that parameter B occurred strongly affected by both, initial crack size, and assumed crack length. For the crack growth analysis the two pairs of B and a_0 were selected: $a_0 = 0.3\ mm, B = 7.6 \cdot 10^{-15}$ and $a_0 = 2.0\ mm, B = 2.8 \cdot 10^{-13}$. For comparison, crack growth was analyzed by using Paris equation (6); stress intensity factor values were calculated using the handbook [10] data.

Results of the crack propagation evaluation are given in Fig.2, where $\nu = da/dN$, is the crack growth rate, *m/cycle*. In should be noted that when the crack grows up to about 0.7 of the plate width, $a \approx 27\ mm$, plasticity develops through the whole ligament, singularity of the stress field vanishes and stress intensities do not exist anymore. The further crack extensions are completely defined by the plastic damage.

It may be seen from Fig.2 that crack propagation predictions are insignificantly affected by selection of initial crack size and parameter B value (curves 1 and 2). The discrepancy between growth rates assessed by applying Eq. (7) and Paris law (6) may be explained by the role of the parameter C of the criterion (1) which may be different in the combined growth rate formula (7).

In general, the relationship (7) provides qualitatively realistic description of the fatigue crack propagation regularity and allows covering the whole range of the process, up to conclusive phase preceding complete failure of the component.

4 Conclusion

An approach based on combined model of fatigue is suggested which incorporates the two mechanisms of fatigue damage: damage accumulation due to macro and microplasticity and failure of microstructural barriers associated with the crack extensions caused by the stress field. The combined model of fatigue may be promising in procedures of assessment the in-service reliability of structures. Although the model is based on application of the well-known in Fatigue mechanics approaches, the parameters characterizing materials properties should be refined based on test data, and its efficiency further needs in experimental justification.

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