Effect of viscosity on convection and heat transfer in rotating horizontal cylindrical layer of liquid

Alexey A. Vjatkin   Victor G. Kozlov
vjatkin_aa@pspu.ru   kozlov@pspu.ru

Abstract

Thermal convection of viscous fluid in a coaxial horizontal gap rotating around its own axis is investigated experimentally. The dependence of the averaged convection excitation threshold on the fluid viscosity and the temperature difference between the layer boundaries is studied. The temperature of inner boundary is higher than that of the outer one. It is found that in viscous liquids crisis of heat transfer is associated with the appearance of vortices extended along the azimuth (three-dimensional structures) of the layer, and the longitudinal two-dimensional rolls appear on the background of them. The rolls are excited in a threshold way by action thermovibrational mechanism. In the experiments with low-viscosity fluids the opposite sequence of convective processes development is observed.

With the advent and development of convective flows their slow azimuthal drift relative to the cavity is registered. The dependence of the drift velocity on the Prandtl number, the speed of cavity rotation and the temperature difference between the layer boundaries is studied. It is shown that the drift is associated with the azimuthal motion of the fluid, which is also of a vibrational nature and is generated in the boundary Stokes layers by the wave traveling. Is found that with an increase of viscosity the wavelength of the longitudinal rolls and the drift velocity of the vortex system significantly reduce.

1 The Problem

The problem of the natural convection in the circular channel bounded by two horizontal cylinders was the subject of intense research in the middle of the last century, because of the wide technological applications (different cooling systems - from nuclear reactors and high-voltage transmission lines to electronic components, thermal insulation of the aircraft fuselage, etc.). Free convection in a cylindrical layer under the action of different factors attracts the attention of researchers. A large number of works are devoted to the study of influence of rotation, for example [1].

Let us consider the convection of non-isothermal fluid in the cavity rotating in a static (in the laboratory frame) force field. In the cavity frame the field rotates causing fluctuations of the fluid relative to the cavity and, as a consequence, its averaged motion [2]. In the rotating systems thermovibrational convection acquires a number of specific features associated with the Coriolis force. In addition to the action on the averaged convective flows this force affects the oscillating motion of the fluid, which is the generator of the averaged mass forces [3].

In this paper, which is a continuation of experiments in a rotating horizontal cylindrical layer with water [4], the results of thermal convection of a viscous fluid research are presented.

2 Experimental technic and procedure

Cylindrical layer is formed by two coaxial cylinders. The inner boundary is formed by the aluminum cylinder *1* (fig. 1), playing the role of a heat exchanger: water from the thermostat is pumped inside it that sets and keeps the temperature at the layer boundary. The outer layer boundary is formed by a Plexiglas tube *2*. The cavity (water jacket) between the tubes *2* and *3* serves to cool the outer layer boundary by circulating the cold thermostatic water.

The average radius of the working layer $R = 33.5$ mm, its thickness $h = 7.0$ mm. Detailed description of the experimental setup can be found in [4],[5].

The temperature of the inner boundary T_1 and the temperature of liquid in the jacket T_3 are measured by thermo resistors made of copper wire with diameter 0.05 mm. The temperature of the outer boundary of the fluid layer T_2 is measured by the integral sensor, formed by several loops of copper wire with diameter 0.02 mm, extending along the length of the layer and glued to the border with a transparent self-adhesive film with thickness 0.1 mm. The loops are strictly parallel to each other, so that the width of the sensitive part of the sensor does not exceed 2 mm. Two integral sensors arranged in parallel at a distance of 5 mm from each other are used to study the characteristics of the drift of flows relative to the cavity.

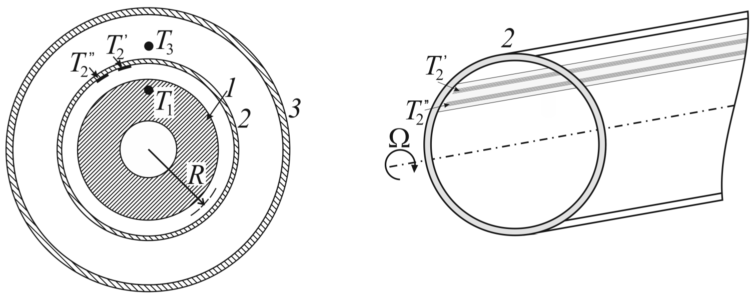

Figure 1: The cuvette scheme

Working liquids are aqueous-glycerol solutions with the concentration $C = 25$ and 50 %. The experimental procedure is following. The temperature of inner layer boundary T_1 and a fluid in the jacket T_3 is set by thermostats, cuvette is rotated at a relatively high speed. During the experiment the rotational speed decreases stepwise. The temperature measurements at each step are made after the time necessary to get the steady convection regime. The sensors data is used to calculate the temperature difference at layer boundaries $\Theta = T_2 - T_1$ and the heat flux through the layer $\Delta T = T_3 - T_2$. The values of T_1 and T_3 remain constant over the entire range of rotational speed (in a definite series of experiments).

Thermo-physical properties of the fluid are determined according the average value of the temperature in the layer. For visualization of convective structures the working fluid is added with aluminum powder with a small amount of a surfactant (respectively 0.03 and 0.2 % on the mass of liquid).

3 Experimental Results

When the inner layer boundary is hotter the centrifugal force plays a stabilizing role. The temperature of outer layer boundary is practically independent of rotational speed n (fig.

2, area I). This condition corresponds to the mechanical equilibrium of liquid. Heat flow at this is close to the molecular one.

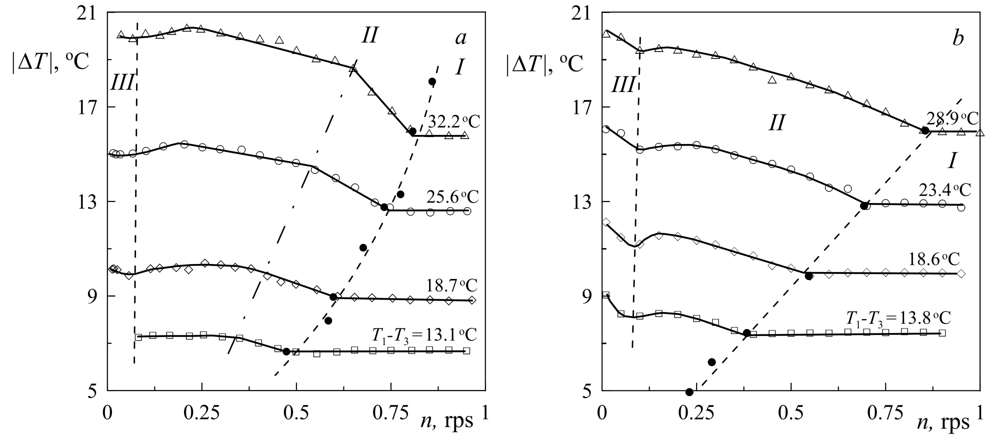

Figure 2: The dependence of temperature ΔT on the velocity of rotation; a – water, b – aqueous glycerol solution with a concentration $C = 25$ %

With decreasing n the averaged convection develops in a threshold manner. The threshold of convection onset is determined by the critical growth of temperature ΔT on the thermal resistance. By lowering the temperature difference between the layer boundaries the threshold shifts to lower values of n (border between areas I and II).

Depending on the viscosity of the liquid at the threshold there are various currents. In experiments with water heat transfer crisis (fig. 2, a, border between areas I and II) is associated with the appearance of two-dimensional convective rolls extending along the axis of rotation [4]. With further decreasing the speed of rotation on the background of longitudinal rolls the convective transverse vortices periodically disposed along the axis are developed. At the appearance of the three-dimensional vortex structures heat transfer through the layer varies slightly: at fig. 2 (a) boundary of the transition to the transverse structure is marked by dash-dotted curve in the region *II*. In a certain range of cavity speed change longwave longitudinal structure and shortwave transverse coexist.

In a layer of viscous fluid at low rotational speed there appears weak flow in the form of azimuthally elongated rolls located primarily near the cavity ends (fig. 3, a). The crisis of heat transfer (fig. 2, b, border between areas I and II) is associated with the appearance of convective rolls extending along the azimuth of the layer (fig. 3, b) in the entire volume of the coaxial gap. The transverse dimension of structures is comparable to the thickness of the layer. At the same time on the background of these three-dimensional flows two-dimensional structures вЂ“ rolls extending along the axis of rotation вЂ“ are registered. The wavelength of longitudinal rolls is about $10h$ at the threshold and significantly decreases (fig. 3, c) with intensification of convection (with decrease of the rotation speed).

With decreasing speed n of cavity rotation the temperature difference ΔT increases (fig. 2, area II), indicating an increase in the intensity of averaged convection. Further reduction of n leads to a quasi-stationary gravitational convection, which prevails in the area *III*. Averaged convection is replaced by the convective structures which are quasistationary in the laboratory frame. The transition to this regime is accompanied by a reduction of heat transfer (border of areas II and III).

With the advent of convective flows (fig. 2, border between areas I and II) in the layer containing sensor regular low-frequency oscillations of temperature T_2 are detected

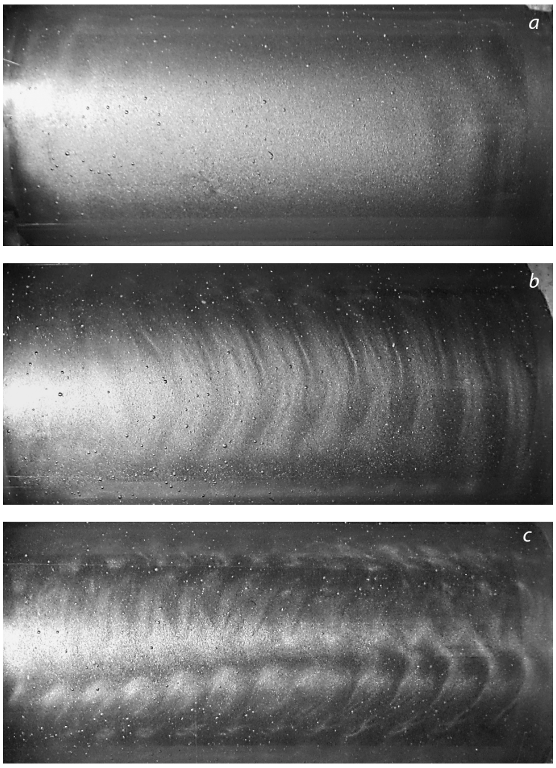

Figure 3: Photos of convective structures; $C = 25\%$, $T_3 - T_1 = 25.5$ K; $n = 0.9$ (a), 0.8 (b) and 0.7 rps (c)

(fig. 4, a). Observations show that the temperature fluctuations are caused by the slow drift of the vortices relative to the cavity (rolls move in a direction opposite to the cavity rotation).

The system of convective vortices drifting relative to the cavity causes temperature variations recorded by sensors. To study the dynamics of wavelength of two-dimensional longitudinal currents and drift rate a pair of integral sensors mounted on the outer edge of the working layer by a predetermined distance from each other are used. Thus, the sensors detect temperature variations, time-shifted and substantially matching in the shape (fig. 4, a). This conditionвЂ™s fulfill refers to the fact that the structure move from one sensor to another without serious distortion. The time offset and the distance between the centers of the sensors allow us to calculate the drift velocity. For the experimental results shown at fig. 4 (a) time shift is $\Delta t = 58.6$ (*1*), 72.1 (*2*), 49.6 (*3*) and 45.1 c (*4*). Drift velocities equal $U = 0.12$, 0.1, 0.14, 0.16 mm/s respectively.

The amplitude of the temperature fluctuations in the threshold increases, and practically does not change in a certain range of values n (fig. 4, a). The magnitude increases with the temperature difference of the layer boundaries in a series of experiments with one fluid. With the increase of viscosity the amplitude decreases and simultaneously violations of the strict waveforms associated with irregular structural rearrangements become frequent. This occurs with the appearance and disappearance of pairs of rolls.

With a decrease in the cavity rotational speed the velocity of structures drift increases monotonically (fig. 5, a) вЂ“ the period of temperature fluctuations recorded by the integrated sensors on the outer layer wall decreases (fig. 4, a). Increasing the viscosity reduces the rate of drift. The increase in temperature recorded by sensor occurs (for example, fig. 4, a, moment i at the diagram *1*) when in the vicinity of the sensor the pair

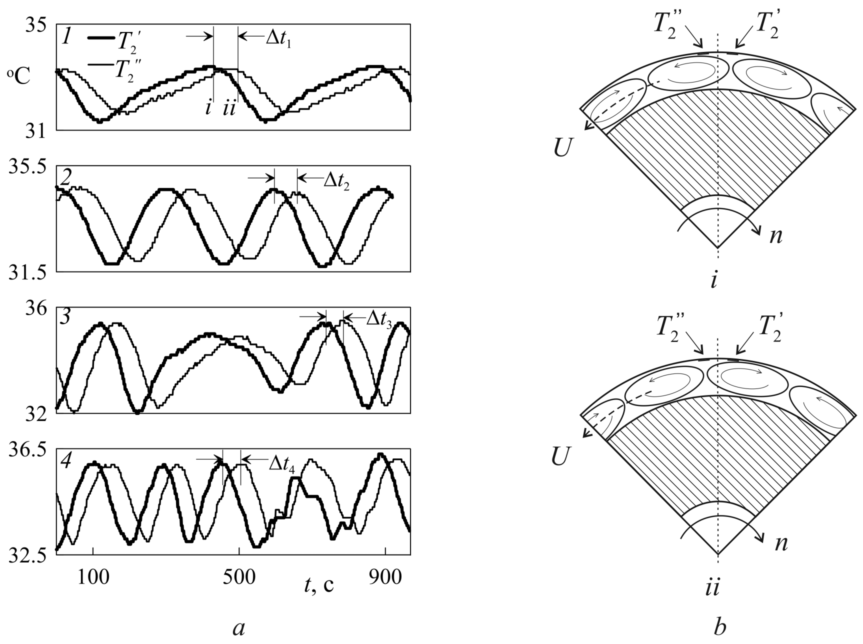

Figure 4: The dependence of temperature on the external boundary of layer on time (a) and a scheme of currents (b); C = 25 %, $T_3 - T_1 = 28.9$ K, $n = 0.85$ (*1*), 0.75 (*2*), 0.65 (*3*), 0.55 rps (*4*)

rolls carries the heat from the hotter inner boundary to the cold outer one (fig. 4, b, time moment i). Thus, one complete oscillation of temperature is interpreted as the passing of a pair of longitudinal rolls over the sensor. After some time Δt the neighboring sensor registers the same section of the structures system (fig. 4, time moment ii).

According to the drift velocity, shown in fig. 5 (a) the wavelength of the two-dimensional structure on each step of the experiment is determined. A general regularity is reducing the wavelength with viscosity (fig. 5, b). Another regularity is the change of the wavelength with increasing supercriticality. Near the threshold and at the developed convection the wavelength values differ several times. The drift velocity at high n (weak supercriticality) significantly diverge from the general pattern (fig. 5, a).

It should be noted that the period of oscillation of temperature registered by the integral sensor located strictly along the cylinder generatrix does not accurately reflect the passage time of a pair of rolls in the case of their bending. This occurs when a pair of rolls appears or disappears in the layer. Recording the temperature for long period of time allows to improve the accuracy of measurement. For example at fig. 4 (a, diagram *3*) one of the oscillations has a period not typical for a given speed of rotation. Large confidence intervals at fig. 5 (b) indicate the difference in structures form.

4 Discussion

In this formulation the force of gravity has a specific effect on a layer of non-uniformly heated fluid. In the cavity rotating around the horizontal axis the gravity causes fluid oscillations in the cavity frame. Fluid oscillations give rise to an averaged convective motion (thermovibrational mechanism), characterized by the vibrational parameter $R_v = (g\beta\Theta h)^2/2\nu\chi\Omega^2$ [2]. The action of centrifugal force of inertia and the Coriolis force on the fluid in a rotating system is characterized by a centrifugal Rayleigh number $Ra =$

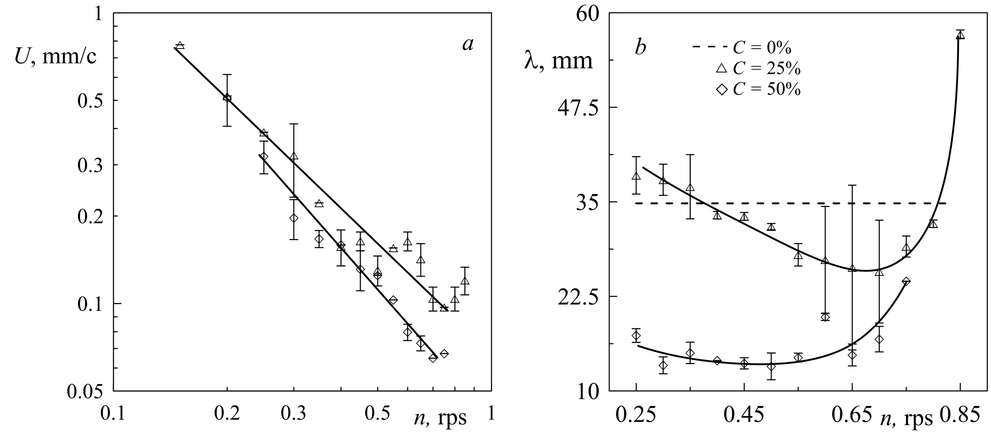

Figure 5: The dependence of the rate of rolls azimuthal drift (a) and wavelength (b) on the velocity of rotation

$\Omega^2 R\beta\Theta h^3/\nu\chi$ and dimensionless speed of rotation $\omega = \Omega^2 h/\nu$. Thus, the averaged thermal convection in a cavity with non-isothermal boundaries rotating around a horizontal axis is defined by the action of two different mechanisms, centrifugal [1] and vibrational [2]. Threshold curves in case of low viscous liquid are studied in detail in [4],[5].

Consider the dependence of the Nusselt number Nu defined as the ratio of the heat flow through the layer to the heat flow in the absence of convection at the same value of Θ on the dimensionless control parameter R_v. In the area of negative Ra and moderate values of R_v (at rapid cavity rotation) the liquid is in mechanical equilibrium state. With a decrease in the rotational speed the stabilizing effect of the centrifugal mechanism reduces while the role of thermovibrational one increases, which leads to excitation of averaged thermal convection. At the excitation threshold of convection the critical change in the value Nu is recorded (fig. 6).

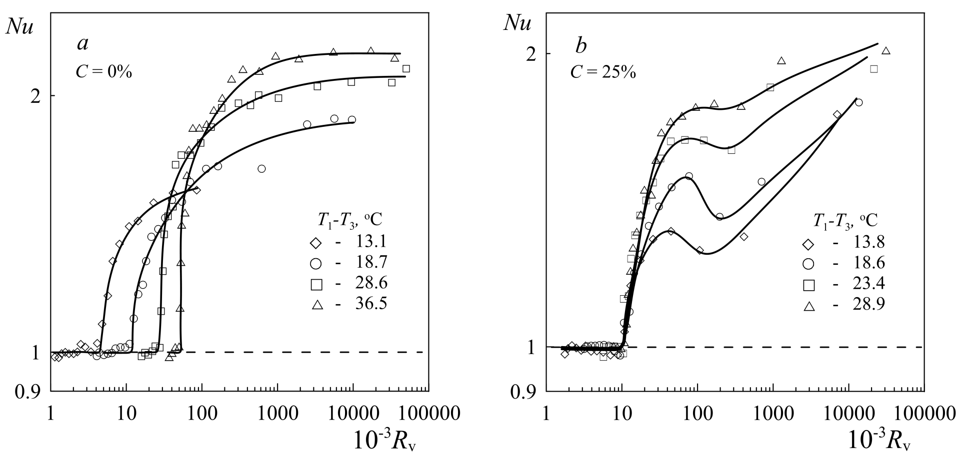

Figure 6: Dependence of the Nusselt number on the vibrational parameter

In supercritical domain the heat transfer increases sharply. In experiments with water at low supercritical mismatch of heat transfer curves corresponding to different values of $|\ Ra\ |$ is observed (fig. 6, a). With increase of the temperature difference of the layer boundaries the stabilizing effect of the centrifugal force of inertia rises that increases the threshold vibrational parameter. Mismatch of the curves at substantially supercritical area is associated with the effect of the dimensionless velocity of rotation. The experiments were performed with a low-viscous liquid in a relatively thick layer that defines a sufficiently high value of the dimensionless speed of rotation at the threshold $\omega = 200\text{-}300$. However, as the supercritical rise (with decreasing speed) ω reduces significantly. Value ω characterizes the ratio of the cavity size to the thickness of Stokes layer existing near solid boundaries due to fluctuations of fluid and plays an important role in the vibrational convection. It is known that at low frequency the effect of thermovibrational mechanism reduces with decreasing ω. Only in the limiting case of thin boundary layers, $\omega \gg 1$, the frequency does not play role, and the only parameter that determines the two-dimensional vibrational convection remains R_v.

In the experiments with viscous fluids (fig. 6, *b*) near the threshold the heat transfer curves are in good agreement. Mismatch of the curves with substantial supercritical is also associated with the effect of the dimensionless velocity of rotation.

Consider the period of oscillation in experiments with liquids of different viscosities on the plane of the dimensionless parameters $\widetilde{\tau}, \Gamma$ (fig. 7). Here $\widetilde{\tau} = \tau\Omega/2\pi$, $\Gamma = (g\beta\Theta/\Omega^2 R)$. The dashed lines show the theoretical dependency $\widetilde{\tau} = 16/3L\Gamma^2$. The difference is in the azimuthal numbers L (the number of pairs of rolls on the perimeter of the layer). For overcritical convection of water the azimuthal number $L = 6$ (curve P°), aqueous glycerol solution with a concentration of 25% вЂ“ 8 (curve *b*), 50% вЂ“ 14 (curve *c*). The theory is based on the assumption that the system of vortices is transferred by azimuthal movement of the whole liquid layer. Such movement of the liquid may be of the vibrational nature and is generated in the Stokes layers by progressive azimuth wave.

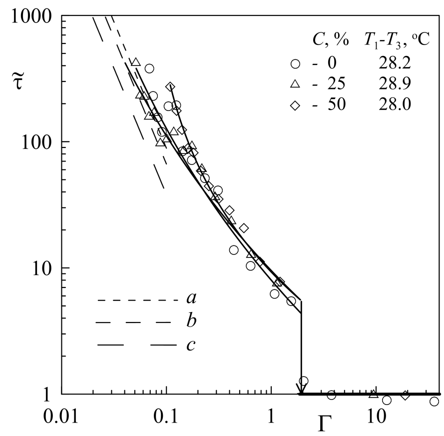

Figure 7: Dimensionless period $\widetilde{\tau}$ vs parameter Γ

At low supercritical (at small values of Γ) the theoretical dependences are in satisfactory agreement with the experimental results in case of low viscous liquid: water (fig. 7, curve *a*) and glycerol solution with concentration 25% (curve *b*). In case of viscous fluid there is a significant discrepancy between theory and experimental data (curve *c*). Increasing viscosity of the solution lowers the dimensionless speed of rotation. It is known that the intensity of flows of vibrational nature decreases with viscosity. This effect manifests itself in a significant decrease in the rate of drift of structures with the increase in viscosity of fluid (fig. 5). What is the cause of misalignment of theory and experimental results at viscous fluids.

5 Conclusion

Thermal convection of a viscous fluid in a coaxial horizontal gap uniformly rotating around its own axis is investigated experimentally. The case of the hot inner boundary is described.

It is found that in viscous liquids the instability associated with the emergence of toroidal vortices in the layer is the most dangerous. The longitudinal rolls appear on the background of three-dimensional flows. In experiments with low-viscous fluids the reversed sequence of the convective processes development is registered. The convection develops under the influence of thermovibrational mechanism.

The azimuthal drift of vortex system is investigated systematically. Is found that with increasing viscosity the speed of the drift motion and the wavelength of the longitudinal rolls decreases.

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Victor G. Kozlov, Alexey A. Vjatkin, The Laboratory of Vibrational Hydromechanics, Perm State Humanitarian-Pedagogical University, Perm, Russia