Sophisticated Core Models Specified in Problems on Elastic Waves Propagation in Designs Laminated Elements

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Abstract

According to this article sophisticated (non-classical) core models can be used for describing the dynamic processes in layered structural elements. Arguments are given by example of a two-layer rod executing longitudinal vibrations.

1 Introduction

Along with engineering (classic) models there are so-called sophisticated or non-classical models in rods' dynamics [1]. These models comprehend additional factors that have impact on the dynamic process, or cannot be based on some hypotheses which are accepted in engineering theories and which restrict the area of its applicability. Bernoulli classical theory, which is accepted for the description of longitudinal vibration of a rod, is summarized by Rayleigh - Love's model (the kinetic energy of transverse movements of rod particles accounting), Bishop's model (in accordance with the statement mentioned above including potential energy of the shear deformations), Mindlin - Herrmann's model (freedom from the hypothesis of strain rod uniaxiality) [2.3]. As a rule the sophisticated models are applicable for describing high-frequency wave processes, when the wavelength becomes comparable to the diameter of the rod cross section and engineering models cannot be essentially applicable. However, for the frequency range mentioned above the multimode of wave process should be taken into account and based on it models of solid-state multimode waveguides - resilient layer (Lamb's problem) and thick-walled cylinder (Pohgammera - Kri) are generally preferred than sophisticated core models [5.6].

According to this article sophisticated core models can be used for describing the dynamic processes in layered structural elements. Arguments are given by example of a two-layer rod executing longitudinal vibrations. The issues are considered under conditions of elastic, visco-elastic and nonlinear elastic productions.

2 Compound rod

The compound rod is considered as a combination of two rods contacting with each other. The contact interaction force is supposed to be linear-elastic. The motion of the rods is described by a system of equations when at initial time a kinematic or power is in effect on the left rods end, the right rods end is unforced:

$$\begin{cases} E_1S_1\frac{\partial^2 u_1}{\partial x^2} = \rho_1S_1\frac{\partial^2 u_1}{\partial t^2} + R\left(u_1 - u_2\right), \\ E_2S_2\frac{\partial^2 u_2}{\partial x^2} = \rho_2S_2\frac{\partial^2 u_2}{\partial t^2} + R\left(u_2 - u_1\right), \end{cases} \tag{1}$$

where u_i – longitudinal movements of rods, E_i,S_{i,ρ_i}- rods parameters (Young's modulus, the areas of cross sections and density) (i=1,2), R - force of elastic interaction of rods.

The system mentioned above can be reduced to a single equation in relation to the median line displacement of one of the rods:

$$\left(1+\frac{\rho_1 S_1}{\rho_2 S_2}\right)\frac{\partial^2 u}{\partial t^2}-\left(C_2^2+C_1^2\frac{\rho_1 S_1}{\rho_2 S_2}\right)\frac{\partial^2 u}{\partial x^2}+\frac{\rho_1 S_1}{R}\left(\frac{\partial^4 u}{\partial t^4}-(C_2^2+C_1^2)\frac{\partial^4 u}{\partial t^2\partial x^2}+C_2^2C_1^2\frac{\partial^4 u}{\partial x^4}\right)=0. \tag{2}$$

Here $u=u_1(x,t)$, $C_1=\sqrt{\frac{E_1}{\rho_1}}$, $C_2=\sqrt{\frac{E_2}{\rho_2}}$ – longitudinal waves' velocities in rods.

A similar equation can be derived in Mindlin - Herrmann's model describing the rod longitudinal vibrations [2-4]:

$$4\left(\frac{\lambda+\mu}{\lambda}\right)\frac{\partial^2 u}{\partial t^2}-4\left(C_l^2\frac{\lambda+\mu}{\lambda}-\frac{\kappa_2^2\lambda}{\rho}\right)\frac{\partial^2 u}{\partial x^2}+\frac{H^2\rho}{2\kappa_2^2\lambda}\left(\frac{\partial^4 u}{\partial t^4}-\left(C_l^2+\kappa_1^2C_\tau^2\right)\frac{\partial^4 u}{\partial t^2\partial x^2}+C_l^2\kappa_1^2C_\tau^2\frac{\partial^4 u}{\partial x^4}\right)=0. \tag{3}$$

Here u (x, t) – longitudinal and cross movements of particles of a rod, H – rod thickness, ρ - material density, $C_l=\sqrt{\frac{\lambda+\mu}{\rho}}$,$C_\tau=\sqrt{\frac{\mu}{\rho}}$ - longitudinal and shift waves' velocities, λ,μ - constants of Lame, κ_1,κ_2 - the correcting coefficients, allowing to increase the frequency range of model applicability.

Thus, longitudinal vibrations of a compound rod can be described by Mindlin - Herrmann's equation of longitudinal vibrations of some hypothetical rod, parameters of which are evaluated through parameters of initial rods as follows:

$$\begin{cases} 4\frac{\lambda+\mu}{\lambda}=1+\frac{\rho_1 S_1}{\rho_2 S_2};\ 4\left(!_l^2\frac{\lambda+\mu}{\lambda}-\frac{\kappa_2^2\lambda}{\rho}\right)=C_2^2+C_1^2\frac{\rho_1 S_1}{\rho_2 S_2}; \\ \frac{H^2\rho}{2\kappa_2^2\lambda}=\frac{\rho_1 S_1}{R};\ \frac{H^2\rho}{2\kappa_2^2\lambda}\left(C_l^2+\kappa_1^2C_\tau^2\right)=\frac{\rho_1 S_1}{R}\left(C_2^2+C_1^2\right); \\ \frac{H^2\rho}{2\kappa_2^2\lambda}\kappa_1^2C_\tau^2C_l^2=C_1^2C_2^2\frac{\rho_1 S_1}{R}. \end{cases} \tag{4}$$

Reduction to Mindlin-Herrmann's model is possible if the parameters of the compound rod satisfy the following condition: $\rho_1 S_1>3\rho_2 S_2$, or $\frac{h_1}{h_2}>3\frac{\rho_1}{\rho_2}$, where $h_{1,2}$- thickness of rods. For simultaneity of the system it is also necessary to suggest that the longitudinal and shear waves' velocities are equal to$C_l=C_1$, $\kappa_1 C_\tau=C_2$. In this case, the thickness of the equivalent rod $H=\sqrt{\frac{(C_1^2-C_2^2)R}{2\rho_1 S_1}}$ will increase with increasing the force of rods elastic interaction and will decrease with decreasing the linear density of the first rod. The correcting coefficients in Mindlin - Herrmann's model are related with the parameters of the initial rods by such dependences as$\kappa_1^2=2\frac{C_2^2}{C_1^2}\frac{\rho_1 S_1-\rho_2 S_2}{\rho_1 S_1-3\rho_2 S_2}$, $\kappa_2^2=\frac{C_1^2-C_2^2}{8C_1^2}\frac{\rho_1 S_1-\rho_2 S_2}{\rho_2 S_2}$, allowing deriving of the one expression for the shear waves' velocity to the following form: $C_\tau=C_1\sqrt{2\frac{\rho_1 S_1-3\rho_2 S_2}{\rho_1 S_1-\rho_2 S_2}}$.

In the particular case, if the one of the rods density is considered as small quantity ($\rho_2\to 0$), the equation system (1) will be derived to the equation of rod longitudinal vibrations of Bishop's model:

$$\rho S\frac{\partial^2 u}{\partial t^2}-ES\frac{\partial^2 u}{\partial x^2}-\rho\nu^2 I_0\frac{\partial^4 u}{\partial t^2\partial x^2}+\mu\nu^2 I_0\frac{\partial^4 u}{\partial x^4}=0. \tag{5}$$

Here ν - Poisson's coefficient, I_0 - the polar moment of inertia, and parameters of an equivalent rod are connected with parameters of initial rods by ratios:

$$\begin{cases} \rho S = \rho_1 S_1; \;\; ES = E_1 S_1 + E_2 S_2 \\ \rho \nu^2 I_0 = \frac{\rho_1 S_1 E_2 S_2}{R}; \;\; \mu \nu^2 I_0 = \frac{E_1 S_1 E_2 S_2}{R}. \end{cases} \tag{6}$$

In such a case, the parameters of the compound rod should meet a such condition as: $\frac{E_2}{E_1} > \frac{S_1}{S_2}$, and the polar radius of inertia and Poisson's coefficient of the equivalent rod are determined by the following relations: $r_p = \frac{2E_1S_1}{E_2S_2 - E_1S_1}\sqrt{\frac{E_2S_2}{R}}$, $\nu = \frac{E_2S_2 - E_1S_1}{2E_1S_1}$. The longitudinal and shear waves' velocities in Bishop's model rod are evaluated through the longitudinal wave velocity in the initial rod: $C_0 = \sqrt{C_1^2 + \frac{E_2S_2}{\rho_1 S_1}}$, $C_\tau = C_1$.

The wave energy in homogeneous dispersing systems is known to be transferred with a group velocity [6]. In this article investigate whether this pattern is kept for laminated elements.

The system (1) can be received from variation principle Hamilton's - Ostrogradsky by the following equations:

$$\begin{cases} \frac{\partial}{\partial t}\frac{\partial L}{\partial\left(\frac{\partial u_1}{\partial t}\right)} + \frac{\partial}{\partial x}\frac{\partial L}{\partial\left(\frac{\partial u_1}{\partial x}\right)} - \frac{\partial L}{\partial u_1} = 0, \\ \frac{\partial}{\partial t}\frac{\partial L}{\partial\left(\frac{\partial u_2}{\partial t}\right)} + \frac{\partial}{\partial x}\frac{\partial L}{\partial\left(\frac{\partial u_2}{\partial x}\right)} - \frac{\partial L}{\partial u_2} = 0. \end{cases} \tag{7}$$

Here Lagrangian L is set as:

$$L = \frac{\rho_1 S_1}{2}\left(\frac{\partial u_1}{\partial t}\right)^2 - \frac{E_1 S_1}{2}\left(\frac{\partial u_1}{\partial x}\right)^2 + \frac{\rho_2 S_2}{2}\left(\frac{\partial u_2}{\partial t}\right)^2 - \frac{E_2 S_2}{2}\left(\frac{\partial u_2}{\partial x}\right)^2 - \frac{R}{2}(u_1 - u_2)^2. \tag{8}$$

The equation of transfer energy (Umov-Poynting's equation), which are related to (7), will be set down as:

$$\frac{\partial W}{\partial t} + \frac{\partial S}{\partial x} = 0. \tag{9}$$

Here [3]

$$\mathrm{W} = \left(\frac{\partial L}{\partial\left(\frac{\partial u_1}{\partial t}\right)}\left(\frac{\partial u_1}{\partial t}\right) + \frac{\partial L}{\partial\left(\frac{\partial u_2}{\partial t}\right)}\left(\frac{\partial u_2}{\partial t}\right) - L\right) \tag{10}$$

– energy density;

$$\mathrm{S} = \left(\frac{\partial L}{\partial\left(\frac{\partial u_1}{\partial x}\right)}\left(\frac{\partial u_1}{\partial t}\right) + \frac{\partial L}{\partial\left(\frac{\partial u_2}{\partial x}\right)}\left(\frac{\partial u_2}{\partial t}\right)\right) \tag{11}$$

– density of a stream energy.

For Lagrangian (8) obvious type of expressions (10), (11) is the following:

$$W = \frac{\rho_1 S_1}{2}\left(\frac{\partial u_1}{\partial t}\right)^2 + \frac{E_1 S_1}{2}\left(\frac{\partial u_1}{\partial x}\right)^2 + \frac{\rho_2 S_2}{2}\left(\frac{\partial u_2}{\partial t}\right)^2 + \frac{E_2 S_2}{2}\left(\frac{\partial u_2}{\partial x}\right)^2 + \frac{R}{2}(u_1 - u_2)^2, \tag{12}$$

$$S = -E_1 S_1\left(\frac{\partial u_1}{\partial x}\right)\left(\frac{\partial u_1}{\partial t}\right) - E_2 S_2\left(\frac{\partial u_2}{\partial x}\right)\left(\frac{\partial u_2}{\partial t}\right). \tag{13}$$

The wave energy transfer velocity should be entered as the ratio of:

$$v_{M=} = \frac{< S >}{< W >}, \tag{14}$$

where the numerator is presented as an average value of stream energy density, the denominator is presented as an average value of energy density.

Movements $u_1(x,t), u_2(x,t)$ are considered as changing under the law of a running harmonious wave:

$$u_1(x,t) = Ae^{i\theta} + A^* e^{-i\theta}, u_2(x,t) = e^{i\theta} +^* e^{-i\theta}, \tag{15}$$

where A, B are the complex amplitudes, A* and B* are the conjugate complex values, $\theta = \omega t - kx$– a wave phase, ω– a circular frequency, k– a wave number.

Averaging in (14) is carried out on the period of a phase change of a harmonious wave ($< S >= \frac{1}{2\pi}\int_0^{2\pi}(S)d\theta$,$< W >= \frac{1}{2\pi}\int_0^{2\pi}(W)d\theta$).

Energy transfer velocity, calculated by a formula (14), is described by the following expression:

$$v_{ve} = \frac{2E_1S_1\omega \mathrm{k}R^2 + 2E_2S_2\omega \mathrm{k}(-\rho_1S_1\omega^2 + E_1S_1\mathrm{k}^2 - R)^2}{R^2(-\rho_1S_1\omega^2 + 3E_1S_1\mathrm{k}^2 - R) + (\rho_2S_2\omega^2 + E_2S_2\mathrm{k}^2 + R)(-\rho_1S_1\omega^2 + E_1S_1\mathrm{k}^2 - R)^2}, \tag{16}$$

where the link between A and B complex amplitudes is taken into account:

$$B = -\frac{(-\rho_1S_1\omega^2 + E_1S_1\mathrm{k}^2 - R)}{R}. \tag{17}$$

The frequency and the wave number are connected with the dispersion law:

$$\omega = \pm\frac{\sqrt{2}}{2}\sqrt{\frac{k^2\frac{\rho_1S_1}{R}\left(C_1^2+C_2^2\right)+(1+\frac{\rho_1S_1}{\rho_2S_2})\pm\sqrt{k^4\frac{\rho_1^2S_1^2}{R^2}\left(C_1^2-C_2^2\right)^2+(1+\frac{\rho_1S_1}{\rho_2S_2})^2+2k^2\frac{\rho_1S_1}{R}\left(C_1^2-C_2^2\right)(1-\frac{\rho_1S_1}{\rho_2S_2}})R}{\rho_1S_1}}. \tag{18}$$

This ratio is turned out from (1) by substitution of the derivation in the form of (15).

The group velocity v_{gv} is determined by differentiating (18) on the wave number. It is equal to the following expression:

$$v_{gv} = \frac{\sqrt{2}}{4}\frac{\left(k\frac{\rho_1S_1}{R}\left(C_1^2+C_2^2\right) + \frac{2k^3\frac{\rho_1^2S_1^2}{R^2}\left(C_1^2-C_2^2\right)^2+\frac{2k\rho_1S_1\left(C_1^2-C_2^2\right)(1-\frac{\rho_1S_1}{\rho_2S_2})}{R}}{\sqrt{k^4\frac{\rho_1^2S_1^2}{R^2}\left(C_1^2-C_2^2\right)^2+(1+\frac{\rho_1S_1}{\rho_2S_2})^2+2k^2\frac{\rho_1S_1}{R}\left(C_1^2-C_2^2\right)(1-\frac{\rho_1S_1}{\rho_2S_2})}}\right)R}{\sqrt{\frac{\{k^2\frac{\rho_1S_1}{R}\left(C_1^2+C_2^2\right)+(1+\frac{\rho_1S_1}{\rho_2S_2})+\sqrt{k^4\frac{\rho_1^2S_1^2}{R^2}\left(C_1^2-C_2^2\right)^2+(1+\frac{\rho_1S_1}{\rho_2S_2})^2+2k^2\frac{\rho_1S_1}{R}\left(C_1^2-C_2^2\right)(1-\frac{\rho_1S_1}{\rho_2S_2})}\}R}{\rho_1S_1}}\rho_1S_1}. \tag{19}$$

If the frequency entering into (16), will be replaced with the wave number by an evaluation of a formula (18), expression $\mathrm{v_{M=}} = \mathrm{v_{3@}}$.will be approved.

Now therefore, it demonstrates that the wave energy velocity and the group velocity are equal; and the elastic wave energy is transferred over layered structural elements with the group velocity as well.

1. Visco-elastic rod

Then a compound rod is considered; the contact interaction force is assumed to be linear visco-elastic. The motion of the rods is also described by a system of equations:

$$\begin{cases} E_1 S_1 \frac{\partial^2 u_1}{\partial x^2} = \rho_1 S_1 \frac{\partial^2 u_1}{\partial t^2} + R\left(u_1 - u_2\right) + R_1 \left(\frac{\partial u_1}{\partial t} - \frac{\partial u_2}{\partial t}\right), \\ E_2 S_2 \frac{\partial^2 u_2}{\partial x^2} = \rho_2 S_2 \frac{\partial^2 u_2}{\partial t^2} + R\left(u_2 - u_1\right) + R_1 \left(\frac{\partial u_2}{\partial t} - \frac{\partial u_1}{\partial t}\right). \end{cases} \tag{20}$$

The system can be reduced to a single equation in relation to the median line displacement of one of the rods:

$$\begin{aligned} &\left(1 + \frac{\rho_1 S_1}{\rho_2 S_2}\right) \frac{\partial^2 u}{\partial t^2} - \left(C_2^2 + C_1^2 \frac{\rho_1 S_1}{\rho_2 S_2}\right) \frac{\partial^2 u}{\partial x^2} + \frac{\rho_1 S_1}{R} \left(\frac{\partial^4 u}{\partial t^4} - (C_2^2 + C_1^2) \frac{\partial^4 u}{\partial t^2 \partial x^2} + \right. \\ &\left. + C_2^2 C_1^2 \frac{\partial^4 u}{\partial x^4}\right) + \frac{R_1}{R} \left(\left(1 + \frac{\rho_1 S_1}{\rho_2 S_2}\right) \frac{\partial^3 u}{\partial t^3} - \left(C_2^2 + C_1^2 \frac{\rho_1 S_1}{\rho_2 S_2}\right) \frac{\partial^3 u}{\partial t \partial x^2}\right) = 0. \end{aligned} \tag{21}$$

Here $\frac{R_1}{R}$ – dissipation coefficient, $u = u_1(x,t), C_1 = \sqrt{\frac{E_1}{\rho_1}}, C_2 = \sqrt{\frac{E_2}{\rho_2}}$– longitudinal waves' velocities in rods.

It is noted that an analogous equation can be derived in Mindlin-Herrmann's system:

$$\begin{aligned} &4\left(\frac{\lambda+\mu}{\lambda}\right) \frac{\partial^2 u}{\partial t^2} - 4\left(!_l^2 \frac{\lambda+\mu}{\lambda} - \frac{\kappa_2^2 \lambda}{\rho}\right) \frac{\partial^2 u}{\partial x^2} + \frac{H^2 \rho}{2\kappa_2^2 \lambda} \left(\frac{\partial^4 u}{\partial t^4} - \left(C_l^2 + \kappa_1^2 C_\tau^2\right) \frac{\partial^4 u}{\partial t^2 \partial x^2} + \right. \\ &\left. + C_l^2 \kappa_1^2 C_\tau^2 \frac{\partial^4 u}{\partial x^4}\right) + \chi \left(4\left(\frac{\lambda+\mu}{\lambda}\right) \frac{\partial^3 u}{\partial t^3} - 4\left(!_l^2 \frac{\lambda+\mu}{\lambda} - \frac{\kappa_2^2 \lambda}{\rho}\right) \frac{\partial^3 u}{\partial t \partial x^2}\right) = 0. \end{aligned} \tag{22}$$

A similar equation can be derived in Mindlin-Herrmann's model describing rod longitudinal vibrations:

$$\begin{cases} 4\frac{\lambda+\mu}{\lambda} = 1 + \frac{\rho_1 S_1}{\rho_2 S_2}; 4\left(!_l^2 \frac{\lambda+\mu}{\lambda} - \frac{\kappa_2^2 \lambda}{\rho}\right) = C_2^2 + C_1^2 \frac{\rho_1 S_1}{\rho_2 S_2};\ \frac{H^2 \rho}{2\kappa_2^2 \lambda} = \frac{\rho_1 S_1}{R}; \\ \frac{H^2 \rho}{2\kappa_2^2 \lambda}\left(C_l^2 + \kappa_1^2 C_\tau^2\right) = \frac{\rho_1 S_1}{R}(C_2^2 + C_1^2);\ \frac{H^2 \rho}{2\kappa_2^2 \lambda} C_l^2 \kappa_1^2 C_\tau^2 = \frac{\rho_1 S_1}{R} C_2^2 C_1^2; \\ \chi = \frac{R_1}{R};\ \chi^4 \left(\frac{\lambda+\mu}{\lambda}\right) = \frac{R_1}{R}\left(1 + \frac{\rho_1 S_1}{\rho_2 S_2}\right);\ \chi^4 \left(!_l^2 \frac{\lambda+\mu}{\lambda} - \frac{\kappa_2^2 \lambda}{\rho}\right) = \frac{R_1}{R}\left(C_2^2 + C_1^2 \frac{\rho_1 S_1}{\rho_2 S_2}\right). \end{cases} \tag{23}$$

Now therefore, the longitudinal vibrations of a compound rod, both for the elastic and visco-elastic contact interaction, are described by Mindlin-Herrmann's equation of hypothetical rod longitudinal vibrations.

2. Nonlinear-elastic compound rod

If geometrical and physical nonlinearities are considered for each rod, dynamics of the system will be described by the following equations:

$$\begin{cases} E_1 S_1 \left(1 + \alpha_1 \frac{\partial u_1}{\partial x}\right) \frac{\partial^2 u_1}{\partial x^2} = \rho_1 S_1 \frac{\partial^2 u_1}{\partial t^2} + R(u_1 - u_2) \\ E_2 S_2 \left(1 + \alpha_2 \frac{\partial u_2}{\partial x}\right) \frac{\partial^2 u_2}{\partial x^2} = \rho_2 S_2 \frac{\partial^2 u_2}{\partial t^2} + R(u_2 - u_1) \end{cases} \tag{24}$$

The motion of the rods is described by an equation system: with elastic and viscous interaction forces. The system (24) can be reduced to one equation. Actually, such parameters should be entered as dimensionless variables

$$U = \frac{u}{u_0};\ y = \frac{x}{X};\ \tau = \frac{t}{T}; \gamma = 1 + \frac{\rho_1 S_1}{\rho_2 S_2},$$

and designations

$$D = C_2^2 + C_1^2 \frac{\rho_1 S_1}{\rho_2 S_2};\ \% = \Lambda;\ T^2 = \frac{\Lambda^2 \gamma}{D},$$

where u_0- conveyance, Λ- wave's length, satisfying to a ratio $u_0/\Lambda = 10^{-4}$, T- wave period.

Taking into account that values for which relation degree $u_0\Lambda/$ above 3 are negligible the system takes on the following form:

$$\frac{\partial^2 U}{\partial \tau^2} - \frac{\partial^2 U}{\partial C^2} + \frac{\rho_1 S_1 D}{R\gamma^2 \Lambda^2}\frac{\partial^4 U}{\partial \tau^4} - \frac{\rho_1 S_1 (C_2^2 + C_1^2)}{R\gamma \Lambda^2}\frac{\partial^4 U}{\partial C^2 \partial \tau^2} + \frac{\rho_1 S_1 C_2^2 C_1^2}{R\Lambda^2 D}\frac{\partial^4 U}{\partial C^4} - \\ - \frac{\left(C_2^2 \alpha_2 + C_1^2 \alpha_1 \frac{\rho_1 S_1}{\rho_2 S_2}\right)}{D}\frac{u_0}{\Lambda}\frac{\partial U}{\partial C}\frac{\partial^2 U}{\partial C^2} = 0. \tag{25}$$

Here: $C_1 = \sqrt{\frac{E_1}{\rho_1}}, C_2 = \sqrt{\frac{E_2}{\rho_2}}$- longitudinal waves' velocities in rods.

For the decision of the equation (25) we will search in a class of stationary waves, that is in the form of function U=U(y–vτ), depending y–vτ=ξ, where v=const - speed of a stationary wave.

The equation in private derivatives (25) will be reduced in this case to the equation non-harmonic oscillator concerning longitudinal deformation$\frac{dU}{d\xi} = w$:

$$\frac{d^2 w}{d\xi^2} + 0w + bw^2 = 0, \tag{26}$$

where

$$a = \frac{v^2 - 1}{B};\ b = -\frac{1}{2}\frac{C_2^2\alpha_2 + C_1^2\alpha_1\frac{\rho_1 S_1}{\rho_2 S_2}}{BD}\frac{u_0}{\Lambda}, B = \frac{\rho_1 S_1 D}{R\gamma^2\Lambda^2}v^4 - \frac{\rho_1 S_1 (C_2^2 + C_1^2)}{R\gamma\Lambda^2}v^2 + \frac{\rho_1 S_1 C_2^2 C_1^2}{RD\Lambda^2}.$$

It should be noticed that roots of equation B=0 are the following: $v_1^2 = \frac{C_2^2\gamma}{D}; v_2^2 = \frac{C_1^2\gamma}{D}$. Specifically, it can satisfy to a condition when$\frac{C_2^2\gamma}{D} = 5 - 4\frac{C_1^2\gamma}{D}$ (for definiteness it is considered that $C_1 > C_2$). In this case $0 < \frac{C_2^2\gamma}{D} < 1; 1 < \frac{C_1^2\gamma}{D} < \frac{5}{4}$, then $0 < v_1^2 < 1; 1 < v_2^2 < \frac{5}{4}$.

Additionally, signs on roots are defined as: between roots (-):$\frac{C_2^2\gamma}{D} < v^2 < \frac{C_1^2\gamma}{D}$; out of roots (+):$v^2 > \frac{C_1^2\gamma}{D}, v^2 < \frac{C_2^2\gamma}{D}$.

According to the analysis (26) private decisions of the equation (25) are nonlinear lonely stationary waves (solitons).

For the first case < 0, $b > 0$ and soliton has positive polarity. Soliton amplitude $A_c$and its width Δ are described by expressions:

$$A_c = \frac{3(v^2 - 1)D}{(C_2^2\alpha_2 + C_1^2\alpha_1\frac{\rho_1 S_1}{\rho_2 S_2})\frac{u_0}{\Lambda}};\quad \Delta = \frac{2}{\sqrt{\frac{v^2-1}{B}}}.$$

dependences of amplitude and width soliton from its speed are presented in Figure 1.

In this case with growth of speed of a lonely stationary wave its amplitude increases and width decreases. Such behavior is specific for classical soliton [7].

For the second case < 0, $b < 0$ and soliton has negative polarity. Its amplitude and width are described by expressions:

$$A_c = \frac{3(1 - v^2)D}{(C_2^2\alpha_2 + C_1^2\alpha_1\frac{\rho_1 S_1}{\rho_2 S_2})\frac{u_0}{\gamma\Lambda}};\quad \Delta = \frac{2}{\sqrt{\frac{1-v^2}{B}}}.$$

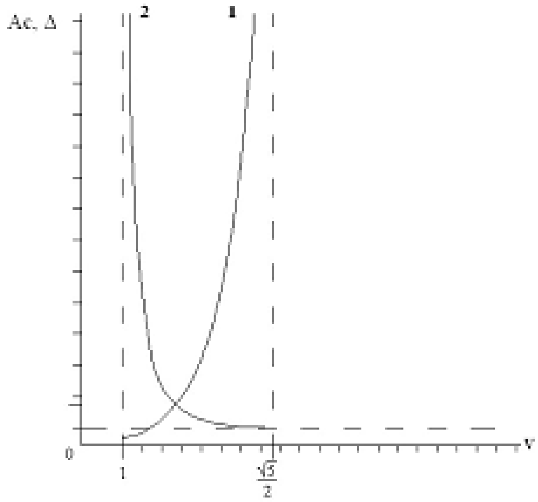

Figure 1: Dependence of amplitude (a curve 1) and width (a curve 2) of positive polarity soliton from its speed.

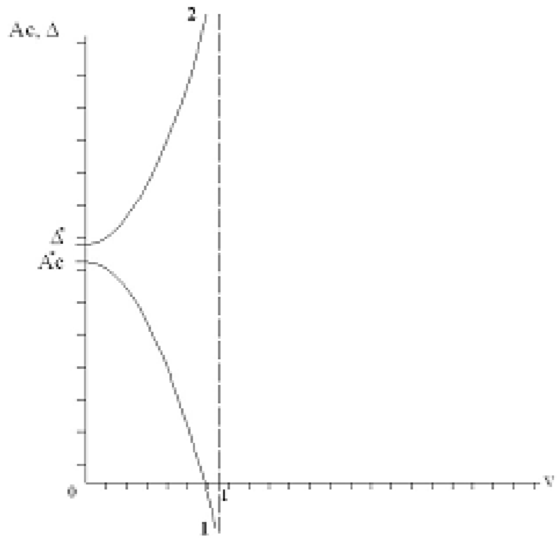

Figure 2: Dependence of amplitude (curve 1) and width (curve 2) of negative polarity soliton from its speed.

Dependences of amplitude and width soliton from its speed are presented in Figure 2.

$$A_c^* = \frac{3D}{(C_2^2\alpha_2 + C_1^2\alpha_1\frac{\rho_1 S_1}{\rho_2 S_2})\frac{u_0}{\gamma\Lambda}}; \quad \Delta^* = \frac{2}{\sqrt{\frac{RD\Lambda^2}{\rho_1 S_1 C_2^2 C_1^2}}}.$$

In this case with growth of a lonely stationary wave's velocity its amplitude and width increases simultaneously. Such behavior is not specific for classical soliton and it is abnormal.

Based on all statements mentioned above it is shown that the deformations localized waves (solitons) with both negative and positive polarity can be formed in a compound nonlinear-elastic rod.

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