# The Creep Fracture Problem of Brittle and Quasibrittle Materials

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**Abstract**

In the world scientific literature numerous investigations on development of fracture criteria of brittle and quasibrittle materials are carried out. At the same time the creep fracture or static fatigue problem for these materials is investigated not enough. To solve this problem the conception of damage and fracture of Kachanov-Rabotnov is applied. It was taken into account, that imperfectness and microinhomogeneity of brittle and quasibrittle materials have casual character and at formulation of creep fracture criterion the probability methods are applied. It is considered that the specimen made of the brittle and quasibrittle materials consists of elements with the different initial states of damage. Distinctions of the damaged state of elements are determined by such factors, as porosity, phase composition, presence of micropores and microcracks of different size and other. So the damage distribution in small microvolumes are considered as the random. The kinetic equation for development of damage parameters with the random initial size distribution is proposed. The time to fracture, i.e. achievement of critical size of one of damaged elements, can be also considered as random. Thus at formulation of the creep fracture criterion the statistical models, based on the hypothesis of weak link and on the Weibull's distribution are applied.

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On the creep fracture curves, plotted using experimental results on high-temperature creep, it is possible to distinguish the part of viscous and brittle fractures corresponding to two limit states. The first case, closed to a viscous flaw, is take place at influence of relative high level of stress and high temperatures. The second, brittle case is take place at long influence of small stresses and the high temperatures, known as the effect of thermal brittleness, discovered in 1936 [1]. It was established that during process of long operation of the power equipment at high temperatures in the loaded or not loaded state many types of steels and alloys loses their plasticity and becomes brittle. Generally for the most of steels it is possible to note a tendency of decreasing of residual strain to one percent with the increasing the time to fracture. Until the middle of the twentieth century prevailed the opinion that the effect of thermal brittleness is connected with technology factors. However, when this effect was discovered on pure metals, for example, pure copper [2] arose the need of explanation and description of this effect. It was established that thermal brittleness is caused by physical processes of intensive accumulation of porosity on borders of grains according the mechanism of diffusion of vacancies and grain boundary slipping.

The problem of brittle fracture in the high-temperature creep conditions is most actual in engineering practice, therefore it is a subject of intensive researches by experts in the field of mechanics of materials and materials science. In particular, in mechanics of materials the equations, in which processes of deformation and fracture are described by means of various damage parameters, are developed for the description of brittle fracture. Founders of such approach are L.M. Kachanov and Yu.N. Rabotnov. The criterion of purely brittle fracture was formulated by Kachanov in 1958 [3]. In the formulation of this criterion Kachanov proceeded from the assumption that deformation and fracture are independent processes. In Rabotnov's theory, stated in works published in 1959, 1963 and 1966 [4-6], the interrelated processes of deformation and damage are considered. For this purpose it is introduced the system of two interrelated equations for the rate of creep $\dot{\varepsilon}$ and damage parameter $\dot{\omega}$ in a problem of creep and fracture of a specimen under the influence of stress $\sigma$:

$$\dot{\varepsilon} = b\sigma^m (1-\omega)^{-q}, \tag{1}$$

$$\dot{\omega} = c\sigma^n (1-\omega)^{-r}, \tag{2}$$

where $b$, $c$, $q$, $r$, $m$, $n$ are constants, $\sigma = \sigma_0 \dfrac{F_0}{F}$, $\sigma_0 = \dfrac{F}{F_0}$, $l_0$, $F_0$ are initial, $l$, $F$ are current length and cross section area of the specimen.

In case of purely brittle fracture it is possible to consider $F \approx F_0$, $\sigma = \sigma_0 = Const$, then, accepting initial conditions $t = 0$, $\omega = 0$, the solution of the equation (2) will be found as

$$\omega = 1 - \left[(1-\omega_0)^{r+1} - (r+1)\, c\sigma_0^n t\right]^{\frac{1}{r+1}}. \tag{3}$$

Accepting a fracture condition $\omega = 1$, from the solution (3) of the equation (2) we will receive brittle creep fracture criterion

$$t_f^h = \frac{1}{c(1+r)\sigma_0^n}. \tag{4}$$

where $t_f^h$ is time of brittle fracture.

According to the damage conception, the criterion (4) describes a brittle part of creep fracture curve with reference to destruction of metallic materials in the conditions of high-temperature creep.

In the paper the conception of brittle fracture is used to formulate the probabilistic criterion of creep fracture of brittle and quasibrittle materials. Thus, we proceed from the following positions. It is considered that a specimen made from examined brittle materials consists from elements with various initial damage conditions $\omega_0$ and a limit of damage condition at the time of fracture $\omega_*$ ($\omega_0 \leq \omega \leq \omega_*$). Distinctions of the damaged conditions of elements are defined by such factors, as porosity, phase structure, existence of micropores and microcracks of the different size, etc. It is believed that temporary evolution of a defective state is defined by the kinetic equation of the type (2). At that, time to fracture, i.e. the achievement of a critical value of one damage parameter, is also a random variable. Thus, it is possible to use the statistical models [7, 8] based on a hypothesis of a weak link. As reliability function the Weibull's distribution [9] is considered

$$R(\omega) = \frac{e^{-\lambda\omega^\alpha} - e^{-\lambda\omega_*^\alpha}}{e^{-\lambda\omega_0^\alpha} - e^{-\lambda\omega_*^\alpha}}, \tag{5}$$

where $\lambda$, $\alpha$ are constants.

Taking into account (3), the relation (5) can be expressed through time $t$

$$R(t) = \frac{e^{-\lambda\omega^{\alpha}(t)} - e^{-\lambda\omega_*^{\alpha}}}{e^{-\lambda\omega_0^{\alpha}} - e^{-\lambda l_*^{\alpha}}}. \tag{6}$$

Setting reliability level $R(t) = R_*$, from (6) we will receive the following creep fracture criterion

$$t_f = \frac{1}{c(r+1)\sigma_0^n}\left\{(1-\omega_0)^{r+1} - \left[1 - \left(\frac{\ln(1/A)}{\lambda}\right)^{1/\alpha}\right]^{r+1}\right\}. \tag{7}$$

where $A = e^{-\lambda\omega_*^{\alpha}} + R_*\left(e^{-\lambda\omega_0^{\alpha}} - e^{-\lambda\omega_*^{\alpha}}\right)$.

The main difference of criterion (7) from criterion of brittle fracture of Kachanov-Rabotnov (4) is that the formula (7) contains values of initial and limit damage, and also statistical characteristics of defective condition of a specimen. Thus, in the context of suggested criterion there are opportunities for the description of experimental creep fracture curves taking into account natural scatter of time of fracture. Really, by means of family of creep fracture curves corresponding to equal probability of fracture, it is possible to describe a scatter band with the indication of the upper and lower limits of a material work capacity.

The relation for creep deformation can be received from the solution of equation (1). For this purpose let's introduce the current value of damage in this equation according to (3). In this case the solution of the equation (1) under the initial conditions $t = 0$, $\varepsilon = 0$ will be received in the form

$$\varepsilon = \frac{b\sigma_0^{m-n}}{c(r+1-q)}\left\{(1-\omega_0)^{r+1-q} - \left[(1-\omega_0)^{r+1} - (r+1)c\sigma_0^n t\right]^{\frac{r+1-q}{r+1}}\right\}. \tag{8}$$

Taking $\omega_0 = 0$ in (8), we will have

$$\varepsilon = \frac{b\sigma_0^{m-n}}{c(r+1-q)}\left\{1 - \left[1-(r+1)c\sigma_0^n t\right]^{\frac{r+1-q}{r+1}}\right\}. \tag{9}$$

On Fig. 1 the creep curves using the relation (8) for three values of stresses 200 $MPa$ (curve 1), 150 $MPa$ (curve 2), 100 $MPa$ (curve 3) are plotted. These curves are in agreement with the corresponding curves received according to the theory of Rabotnov [4-6].

In calculations according to formula (8) it was accepted the following values of coefficients: $\omega_0 = 0,1$, $m = 4$, $n = 2$, $q = 2$, $r = 2$, $b = 10^{-13}\,[MPa]^{-4}\,[h]^{-1}$, $c = 10^{-7}\,[MPa]^{-2}\,[h]^{-1}$.

## References


[1] Borzdyka A.M. Methods of hot mechanical tests of metals. M.: Metallurgizdat. 1955. 352p. (in Russian).

[2] Chadek Y. Creep of metallic materials. M.: Mir. 1987. 304p. (in Russian).

[3] Kachanov L.M. About the time to facture in the conditions of creep // Izv. Academy of Sciences of the USSR. OTN. 1958. N 8. P. 26-31. (in Russian).

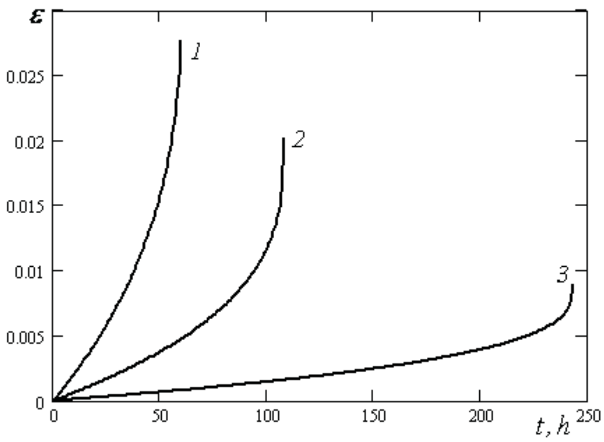


Figure 1: Creep curves according to relation (8) for three values of stresses: 200 $MPa$ (curve 1), 150 $MPa$ (curve 2), 100 $MPa$ (curve 3).

[4] Rabotnov Yu.N. About the mechanism of long destruction // Fracture problems of durability of materials and designs. M.: Publishing house of Academy of Sciences of the USSR. 1959. P. 5-7. (in Russian).

[5] Rabotnov Yu.N. About the fracture under the creep conditions // PMTF. 1963. N 2. P. 113-123. (in Russian).

[6] Rabotnov Yu.N. Creep of construction elements. M.: Nauka. 1966. 752p. (in Russian).

[7] Bolotin V.V. Statistical methods in construction mechanics. M.: Stroyizdat. 1965. 279p. (in Russian).

[8] Arutyunyan R.A. Problem of deformation aging and prolonged fracture in material sciences. St.-Petersburg: St.-Petersburg University Press. 2004. 253p. (in Russian).

[9] Weibull W. A statistical distribution function of wide applicability // J. Appl. Mech. 1951. V. 18. N 3. P. 293-297.

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