

Strength analysis of axisymmetric shells moving in deformed media at high speed

Nikolay V. Banichuk Svetlana Yu. Ivanova Evgeniy V. Makeev
 banichuk@ipmnet.ru

Abstract

The study concerns the problem movement of rigid axisymmetric shell (penetrator or striker) in deformed media. The model of thin-walled shell of revolution is formulated and the two-term quadratic expression is used for estimation of the resistance force as a function of striker velocity in high-speed penetration processes. General analytical representations are found for shell acceleration and arising membrane stresses. Dynamical strength analysis is performed and presented in particular cases of axisymmetric shells of different shapes.

1 Introduction

The problems of penetration of rigid projectiles (striker) into deformable media at supersonic entry velocities and the problems of design of layered shield structures are of significant practical and theoretical importance. Wide survey on the mechanics of penetration of projectiles into targets was presented by Backman and Goldsmith [1] and Goldsmith [2] (taking into account complicated conditions). Bivin et al. [3, 4] devoted their study to determination of the dynamic characteristics of deformed media by the method of penetration. Cavitation and the influence of head shape in attack of thick targets by non-deformable projectiles was investigated by Hill [5]. Experimental aspects of the problems of penetration and perforation are described by Bivin in the monograph [6]. Laboratory scale penetration experiments into geological targets up to impact velocities of 2.1 km/sec were studied by Forrestal et al. [7]. Some shape optimization problems have been solved for rigid striker penetrating into deformable media and published by Ben-Dor et al. [8], Banichuk et al. [9, 10, 11] and Ostapenko et al. [12, 13].

2 Basic relations of the shell model and expressions for acting forces

Let us consider a shell which has the shape of a surface of revolution, the axis of which coincides with the x -axis (Fig. 1). The position of the meridian plane is specified by the angle θ which is measured from a certain fixed meridian plane, the position of the parallel circle is defined by the angle φ between the normal to the surface and the axis of rotation, $r = r(x)$ is the radius of the parallel circle, which determines the distance from a point on the neutral surface of the shell to its axis of rotation and $0 \leq x \leq l$, where l the specified length of the shell. The meridian plane and the plane which is perpendicular to the meridian are the planes of principal curvatures of the shell surface at the point being considered. The corresponding principal radii of the curvature are r_φ and r_θ . Very well known relations

between meridional curvature radius r_φ , circumferential curvature radius r_θ and radius r are presented, for example, in [15, 16]. For simplicity the thickness is supposed to be constant ($h(\varphi) = h(\varphi(x)) = h = \text{const}$). The radius r and the curvature radii r_φ , r_θ distributions and the value h are assumed to satisfy the well known conditions $h \ll r_m = \min\{\min_\varphi r_\varphi(\varphi), \min_\varphi r_\theta(\varphi)\} = \min\{\min_x r_\varphi(\varphi(x)), \min_x r_\theta(\varphi(x))\}$ of the theory of thin elastic shells [15, 16]. The maxima and minima with respect to x are defined at the interval $[0, l]$ and the external operation \min in front of the braces denotes the search for the smaller of the two quantities.

The shell is loaded by axisymmetric forces which can be considered as the loads applied to the neutral surfaces. The intensities of the external loads, which act on the directions normal and tangential to the meridian, are denoted by q_n and q_φ (see Fig. 1). In the considered case the elementary resistance force of external media is determined as ($\vec{q}_\varphi = 0$, Fig. 1, 2)

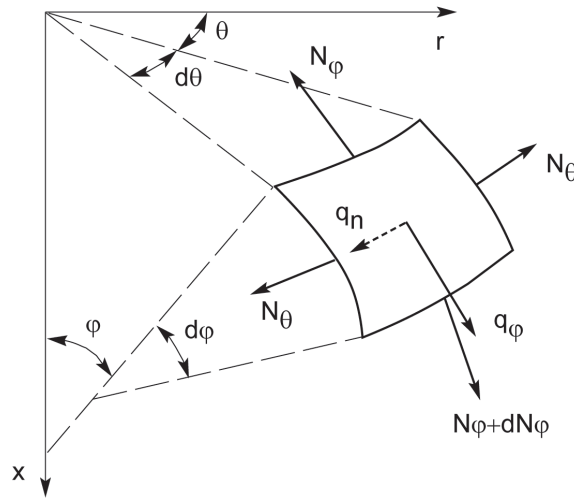


Figure 1: Element of the axisymmetric shell

$$\begin{aligned} \vec{q}_n &= \sigma(v_n) \vec{n} dS, \quad v_n = v(\vec{e} \cdot \vec{n}) = v \cos \varphi, \\ \sigma(v_n) &= a_0 + a_2 v_n^2 = a_0 + a_2 v^2 \cos^2 \varphi, \\ dS &= d\Gamma d\theta, \quad d\Gamma = r \sqrt{1 + r_s^2}, \quad r_s = dr/ds, \end{aligned} \quad (1)$$

where a_0 , a_2 are given positive material constants of the external media [14], v and v_n are, respectively, velocity of the shell oriented in negative direction of the x -axis and the component of the velocity normal to the middle surface.

In what follow the position of considered moving shell (penetrator) is characterized by the coordinate x ($x_0 \leq x \leq L$) of the nose of penetrator and the position of the parallel circle is described by the coordinate s ($0 \leq s \leq l$), where l is the length of the shell measured along the axis of symmetry.

The mass M_φ of the portion of the shell above parallel circle AB (Fig. 2) and the forces D_φ , D_N applied to this portion and acted in x -direction are evaluated with the help

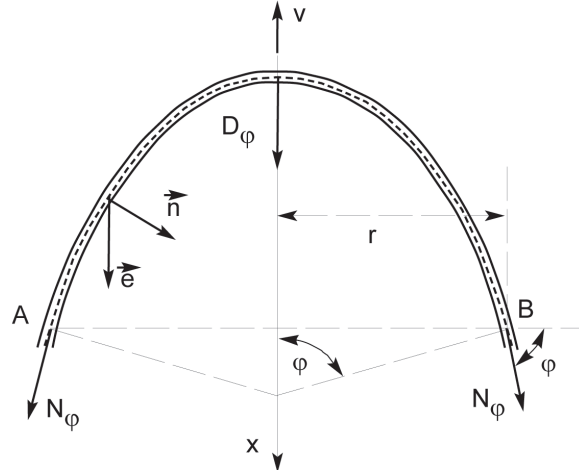


Figure 2: Portion of the shell above parallel circle AB

of the following expressions

$$\begin{aligned}
 M_\varphi(s) &= 2\pi\rho h \int_0^{\varphi(s)} d\Gamma = \rho h S(s), \quad S(s) = 2\pi \int_0^s r \sqrt{1+r_s^2} ds, \\
 D_\varphi(x, s) &= 2\pi \int_0^{\varphi(s)} \sigma(v_n) (\vec{e} \cdot \vec{n}) d\Gamma = \Psi_0(s) + v^2(x) \Psi_2(s), \\
 D_N(x, s) &= 2\pi r(s) N_\varphi(x, s) \sin \varphi(s), \\
 \Psi_0(s) &\equiv \pi a_0 (r^2(s) - r^2(0)), \quad \Psi_2(s) \equiv 2\pi a_2 \int_0^s \frac{rr_s^3}{1+r_s^2} ds.
 \end{aligned} \tag{2}$$

Note that the bottom of the shell is characterized by the condition $\varphi = \varphi_f(s = l)$ and for these given values φ_f and l we have

$$M = M_\varphi(l) = \rho h S(l), \tag{3}$$

$$D = D_\varphi(x, l) = \Psi_0(l) + v^2(x) \Psi_2(l), \tag{4}$$

$$D_N = D_N(x, l) = 2\pi r(l) N_\varphi(x, l) \sin \varphi(l) = 0, \tag{5}$$

where M and D are the total mass of the shell and the total load on the shell. Here and in what follows N_φ , N_θ (see Fig. 1) are the magnitudes of the normal membrane forces per unit length, which can be determined from the system of dynamical equations. Note also the condition $N_\varphi(x, l) = 0$ corresponding to the bottom of the shell.

The normal membrane stresses are evaluated as

$$\sigma_\varphi = \sigma_\varphi(x, s) = \frac{N_\varphi(x, s)}{h}, \quad \sigma_\theta = \sigma_\theta(x, s) = \frac{N_\theta(x, s)}{h}. \tag{6}$$

The shear membrane stresses vanish, i.e. $\sigma_{\varphi\theta} \equiv 0$, because the shape of the shell and loads are supposed to be axisymmetric.

Safety condition for considered moving shell can be written as a condition imposed on the arising membrane stresses

$$\max \{ |\sigma_\varphi|, |\sigma_\theta| \} \leq \sigma_* \tag{7}$$

or as a constraint on admissible values of the thickness

$$h \geq \frac{1}{\sigma_*} \max \{ |N_\varphi|, |N_\theta| \}, \tag{8}$$

where $\sigma_* > 0$ is a given constant.

3 Dynamical equations and dynamical stress state analysis

To find kinematical values w , v and membrane forces N_φ , N_θ we will use the equations

$$Mw = -D, \quad w = \frac{dv}{dt} = \frac{1}{2} \frac{dv^2}{dx}, \quad (9)$$

describing the dynamics of the total shell, the equation

$$M_\varphi w = -D_\varphi - D_N, \quad (10)$$

describing the movement of the shell portion above parallel circle AB and the equation

$$\rho h w_n = \sigma(v_n) - \frac{N_\varphi}{r_\varphi} - \frac{N_\theta}{r_\theta}, \quad (11)$$

describing the movement of the element shown in Fig. 1 in the direction normal to the element.

At first we use equations (3), (4), (9) and transform the equation of dynamics (9) to the form

$$\frac{dv^2}{dx} = -\frac{2}{M} \left(\Psi_0(l) + v^2 \Psi_2(l) \right). \quad (12)$$

Performing integration of the differential equation (12) and taking into account the initial condition $v(x_0) = v_0$ we obtain

$$v^2(x) = \left(\frac{\Psi_0(l)}{\Psi_2(l)} + v_0^2 \right) \exp \left[-\frac{2\Psi_2(l)}{M} (x - x_0) \right] - \frac{\Psi_0(l)}{\Psi_2(l)}, \quad x_0 \leq x \leq L. \quad (13)$$

Consequently we will have the following expression for shell acceleration

$$\begin{aligned} w(x) &= -\frac{D(x)}{M} = -\frac{1}{M} \left[\Psi_0(l) + v^2(x) \Psi_2(l) \right] = \\ &= -\frac{1}{M} \left\{ \left[\Psi_0(l) + v_0^2 \Psi_2(l) \right] \exp \left(-\frac{2\Psi_2(l)}{M} (x - x_0) \right) \right\}. \end{aligned} \quad (14)$$

To find the membrane force $N_\varphi(x, s)$ at some $x \in [x_0, L]$ and $s \in [0, l]$ we will use the equation of the shell portion above the parallel circle (10) and representations (2), (14). We obtain

$$\begin{aligned} N_\varphi(x, s) &= -\frac{1}{2\pi r \sin \varphi} (D_\varphi - w M_\varphi) = \\ &= -\frac{\sqrt{1+r_s^2(s)}}{2\pi r(s)} \left[\Psi_0(s) + v^2(x) \Psi_2(s) - \rho h w(x) S(s) \right]. \end{aligned} \quad (15)$$

Determination of the membrane force $N_\theta(x, s)$ is based on the dynamical equation (11) and the expressions (1) for σ_n . Performing necessary elementary operations we will have

$$N_\theta(x, s) = r_\theta(s) \left\{ a_0 + a_2 v^2(x) \frac{r_s^2}{1+r_s^2} - \frac{N_\varphi(x, s)}{r_\varphi(s)} - \rho h w \frac{r_s}{\sqrt{1+r_s^2}} \right\}, \quad (16)$$

where $v(x)$, $w(x)$, $N_\varphi(x, s)$ are given, respectively, by the expressions (13), (14) and (15) at $x \in [x_0, L]$ and $s \in [0, l]$.

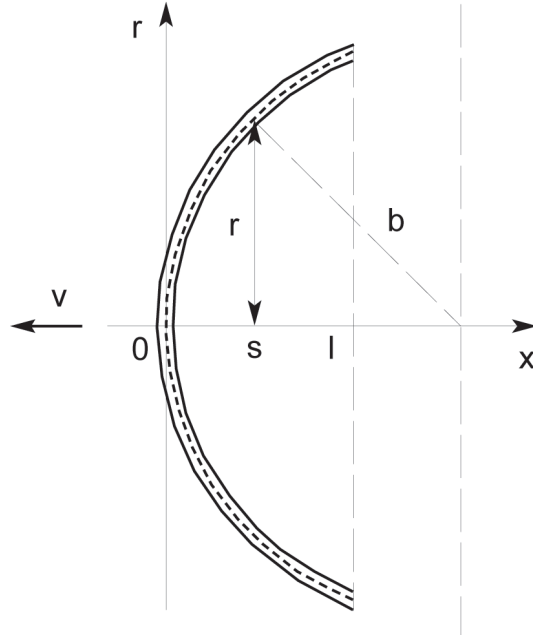


Figure 3: Spherical segment

4 Spherical and conical shells

In what follows we consider particular cases of axisymmetric shells. Movement analysis and determination of membrane stresses (forces) will be performed with the help of relations (1)-(6) and expressions (13)-(16).

Let us consider the movement of the striker having the shape of the segment of thin-walled spherical shell (Fig. 3) of radius b . In this case

$$\begin{aligned} r &= r(s) = \sqrt{s(2b-s)}, \quad 0 \leq s \leq l \leq b, \\ r_\varphi &= r_\theta = b, \quad rr_s = b-s, \quad r\sqrt{1+r_s^2} = b. \end{aligned} \quad (17)$$

Taking into account geometrical relations (17) we obtain

$$\begin{aligned} S(s) &= 2\pi bs, \quad M = 2\pi\rho hbl, \quad \Psi_0(s) = \pi a_0 s(2b-s), \\ \Psi_2(s) &= \frac{\pi a_2}{2b^2} (b^4 - (b-s)^4). \end{aligned} \quad (18)$$

Using the initial condition $v(0) = v_0(x_0 = 0)$ and expressions (13)-(18) we find the movement velocity

$$v^2(x) = \left(v_{sph}^2 + v_0^2 \right) \exp\left(-\frac{x}{x_{sph}} \right) - v_{sph}^2, \quad (19)$$

where

$$x_{sph} = \frac{2\rho hlb^3}{a_2(b^4 - (b-l)^4)}, \quad v_{sph}^2 = \frac{2a_0b^2}{a_2(2b^2 - 2bl + l^2)}. \quad (20)$$

As it follows from expression (19) and the condition $v = 0$ at $x = H$ the depth of penetration (DOP) of spherical penetrator is evaluated as

$$H = H_{sph} = x_{sph} \ln\left(1 + \frac{v_0^2}{v_{sph}^2} \right). \quad (21)$$

Acceleration $w(x)$ and membrane stresses $\sigma_\varphi(x, s)$, $\sigma_\theta(x, s)$ be found with the help of (6), (14)-(18) and represented in the following form

$$\begin{aligned} w(x) &= -\frac{\pi l(2b-l)}{M} \left[a_0 + \frac{a_2}{2b^2} (2b^2 - 2bl + l^2) v^2(x) \right], \\ \sigma_\varphi(x, s) &= -\frac{b}{2s(2b-s)h} \left[a_0 s(2b-s) + v^2(x) \frac{a_2}{2b^2} (b^4 - (b-s)^4) - 2\rho h b s w(x) \right], \\ \sigma_\theta(x, s) &= \frac{b}{h} \left[a_0 + a_2 v^2(x) \left(\frac{b-s}{b} \right)^2 - \frac{N_\varphi(x, s)}{b} - \rho h \left(\frac{b-s}{b} \right) w(x) \right]. \end{aligned} \quad (22)$$

Suppose now that the striker has a shape of thin-walled conical shell. In this case

$$\begin{aligned} r &= r(s) = \kappa s, \quad r_\theta(s) = \kappa \sqrt{1 + \kappa^2} s, \quad r_\varphi = \infty, \\ S(s) &= \pi \kappa \sqrt{1 + \kappa^2} s^2, \quad M = \pi \rho h l^2 \kappa \sqrt{1 + \kappa^2}, \\ \Psi_0(s) &= \pi a_0 \kappa^2 s^2, \quad \Psi_2(s) = \pi a_2 \frac{s^2 \kappa^4}{1 + \kappa^2}. \end{aligned} \quad (23)$$

Velocity and acceleration distributions for conical striker movement given by

$$\begin{aligned} v^2(x) &= (v_{con}^2 + v_0^2) \exp\left(-\frac{x}{x_{con}}\right) - v_{con}^2, \quad 0 \leq x \leq H_{con}, \\ w(x) &= -\frac{\rho \kappa^2 l^2}{M} \left(a_0 + \frac{a_2 \kappa^2}{1 + \kappa^2} v^2(x) \right), \end{aligned} \quad (24)$$

where

$$x_{con} = \frac{\rho h (1 + \kappa^2)^{3/2}}{2a_2 \kappa^3}, \quad v_{con}^2 = \left(\frac{1 + \kappa^2}{\kappa^2} \right) \frac{a_0}{a_2}, \quad H_{con} = x_{con} \ln \left(1 + \frac{v_0^2}{v_{con}^2} \right). \quad (25)$$

Here by means of H_{con} we denote the depth of penetration of conical shell. To find $\sigma_\varphi = N_\varphi(x, s)/h$ and $\sigma_\theta = N_\theta(x, s)/h$ it is possible to use formulas (6), (14)-(16) and (23), (24). We have

$$\begin{aligned} \sigma_\varphi(x, s) &= -\frac{s\sqrt{1+\kappa^2}}{2h} \left[a_0 \kappa + v^2(x) a_2 \kappa^3 - \rho h \sqrt{1 + \kappa^2} w(x) \right], \\ \sigma_\theta(x, s) &= \frac{s\kappa\sqrt{1+\kappa^2}}{h} \left[a_0 + a_2 v^2(x) \frac{\kappa^2}{1+\kappa^2} - \rho h \frac{\kappa^2}{\sqrt{1+\kappa^2}} w(x) \right]. \end{aligned} \quad (26)$$

5 Some notes and conclusions

In the paper we presented formulation and analysis of the problem of high speed penetration of rigid axisymmetric shell into dense media. The model of dynamical interaction of thin-walled shell of arbitrary axisymmetric shape and deformable media at high speed has been developed using two-term approximation of the external media resistance forces. Dynamical system of equations was presented and the dynamical stress state analysis was performed. General analytical representations were found for shell velocity acceleration, depth of penetration, arising dynamical membrane stresses. As particular examples the movement of the strikers having the conical shape and the shape of the segment of spherical shell has been considered and arising membrane stresses have been evaluated and presented.

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Nikolay V. Banichuk, Prospekt Vernadskogo, 101, bld.1, Moscow, Russia

Svetlana Yu. Ivanova, Prospekt Vernadskogo, 101, bld.1, Moscow, Russia

Evgeniy V. Makeev, Prospekt Vernadskogo, 101, bld.1, Moscow, Russia