Vibrational segregation – simulation, experiment and application to create new classifying machines

I. I. Blekhman L. I. Blekhman L. A. Vaisberg V. B. Vasilkov K. S. Yakimova
iliya.i.blekhman@gmail.com

Abstract

Fundamental physical factors determining particle segregation in granular materials under vibration have been considered. Three main types of vibrations are examined on the base of alien particle movement: horizontal circular, horizontal straight and vertical straight vibrations. Formulas for direction and segregation rate are obtained for each type of vibration on the base of vibrational mechanics approach. These formulas take into account the effect of relatively heavy particles heaving in the medium of smaller particles under vibration. The effect was called "wedge effect". Sometimes this effect is called also "Brazil Nut Effect". The experiments performed show that the wedge effect is more pronounced for the case of vertical straight vibration. The effect of horizontal segregation in the direction of side walls of the vessel is discovered and explained. New type of a vibrating classifier has been suggested and patented on the base of the effect.

1 Introduction

Vibrational segregation of granular materials is essential for a number of mineral processing operations and may be observed in ore transportation and storage. Segregation also explains the effects of abnormal nodule occurrence and boulder expulsion in sand and sand-and-gravel media. The vibrational segregation problem has a long history and currently concerns numerous Russian and foreign researchers [1–14]. Certain foreign papers refer to it as the Brazil Nut Effect. This term stems from the fact that in a container filled with a mixture of different sized nuts (for example, Brazil nuts and hazelnuts) the largest nuts always rise to the top [11–13]; this problem can not be considered sufficiently solved.

The two following commentes seem relevant:

1) Despite occasional assertions in literature, the resulting particle separation (particle quasi-steady state) does not have to correspond to the minimum potential energy [1, 4]. This fact is closely related to the inaccurate nature of the term of "vibrational liquefaction": vibration does not only cause "liquefaction" of the granular medium, but also generates certain additional "slow forces", referred to as vibrational forces [1]. These forces, when algebraically summed up with gravity, significantly affect the segregation process.

2) Increasing amplitudes between the vertical acceleration a and earth's gravitational acceleration g enhance both granular medium layer separation and mixing processes. The latter may prevail over separation at higher values of a, which brings about the requirement for careful selection of acceleration values.

Current extensive research capabilities for segregation processes are due to the development of advanced computer programs based on the methods of particle dynamics and discrete elements [5].

This study examines the segregation theory with regard to three types of granular bed vibrations. Certain basic provisions were confirmed through experiments, the results of which are included herein.

2 Physical factors contributing to the processes of segregation of particles under the influence of vibration

The following five property-based critical factors affecting the volumetric vibrational separation of particles in a granular mixture may be established:

1) *Fluidization (pseudo-liquefaction) of the particle mixture.* This effect is well known [2, 4, 10].

2) *Differences in resistance to particle motion towards the lower boundary of the medium F_-, in the opposite direction F_+ (or F_v)* and in a direction parallel to the surface F_h (Fig. 1).

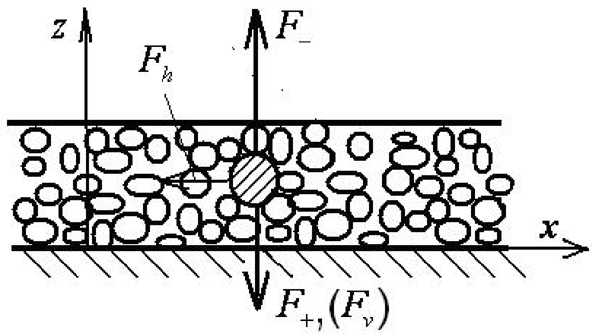

Figure 1: Resistance force particles down relative to the layer of the medium: F_-– down, F_+ (or F_v) – up, F_h – parallel to the plane.

This is due to the fact that, in downward motion, a particle has to “wedge off” underlying particles based on a hard surface; and in upward movement, it has to lift them. It is clear, of course, that this factor is only seen in cases of fluidization and particle oscillations relative to the medium. Another is largest and the power resistance F_h in the direction parallel to the surface.

It would, therefore, be reasonable to study segregation processes by examining the behavior of a single particle. It would be much easier, and it is quite natural to suggest that the detected patterns would persist in the presence of a group of interacting particles of the same type.

3) *Oscillations* of particles with density different from the density of the medium relative to the medium. When granular medium contains a particle with its density differing from that of the surrounding particles, the particle in question will oscillate about the medium at sufficiently intense vibrations of the medium. Since a relatively large particle of the same material as the medium has a higher density than the medium (bulk density ρ_0 may be less than the density of particles in the medium ρ by up to 30%), larger particles will also oscillate about the medium.

This is due to the fact that the inertial force amplitude acting on the particle under consideration in relative motion and under medium vibrations under law $A \sin \omega t$, is determined by

$$R = |\rho - \rho_0|\, V A\omega^2$$

where V is particle volume.

Particle oscillations about the medium (in case of straight vertical vibration) occur under the condition of

$$R = |\rho - \rho_0| VA\omega^2 > Inf(F_+, F_-).$$

Here F_- and F_+ represent resistance to downwards and upwards particle motion relatively to the medium, respectively. As a rule, $F_- > F_+$; and it may be assumed that F_+ and F_- are proportional to the squared particle size, and the difference of $F_- - F_+$ increases towards the top and bottom of the vessel.

Note that vertical axis z is directed upwards, as opposed to the data in book [1]; and, therefore, the forces of F_+ and F_- are reversed.

4) *The wedge effect*: when a particle shifts upwards in the medium, its left and right adjacent medium layers slide down in a tapered pattern under the particle (Fig. 2; see also [10]). For this, vertical vibration of the medium is assumed, with the particle in question being larger than those of the medium. A similar effect occurs in horizontal vibration of the medium. The only difference is that smaller particles under the particle in question collapse alternately on the right and on the left.

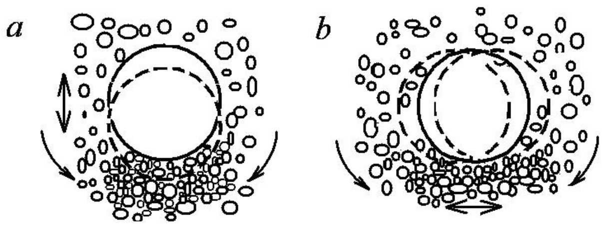

Figure 2: Wedge Effect: a – under vertical medium vibration; b – under horizontal medium vibration

This effect causes the particles with density exceeding apparent density of the granular media to rise to the top.

5) *Fine particles filling the gaps between large particles, smaller particles being pulled to the walls of the vessel.* In case of vibrational liquefaction, fine particles tend to fill the gaps between larger particles. As is known, the particle packing is considerably less dense closer to the walls of the vessel than at a certain distance from the walls. Consequently, in a vibrating vessel, fine particles in a medium of large particles move both towards the bottom of the vessel and towards its walls (Fig. 3).

The motion of fine particles towards the bottom of the vessel (including by gravity) is used in vibration screening, while the effect of their motion towards its vertical walls have not yet found any industrial application. However, it has already been confirmed by experiments (see section 6) and used for the creation of a new, patented type of a vibrating classifier [7].

3 Segregation under vessel circular vibrations, pseudo-resonance effect, wedge effect

I.I. Blekhman, V.V. Gortinsky and G.E. Ptushkina generated formula [1] for the velocity of particle motion towards a less dense medium under circular oscillations in a horizontal plane with amplitude A and frequency ω (Fig. 4, a).

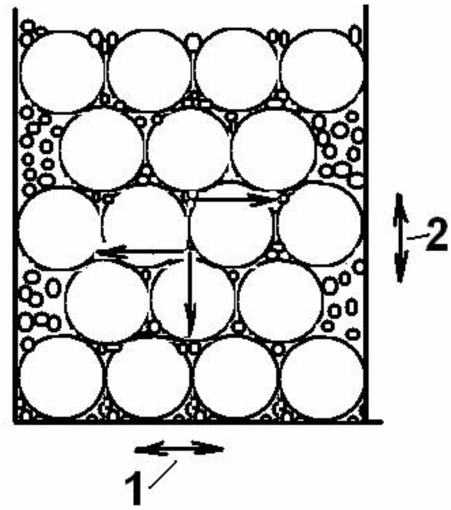

Figure 3: Movement of fine particles in a medium of larger particles towards the walls of the vessel. The effect takes place under horizontal (1), vertical (2) and complex vessel oscillations.

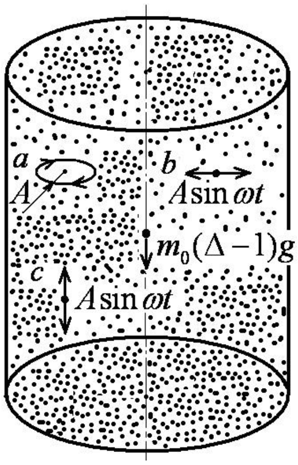

Figure 4: A foreign particle in a medium under: a – circular oscillations in a horizontal plane, b – straight horizontal harmonic oscillations, c – straight vertical harmonic oscillations.

In our case, this formula has the following form:

$$V_z = \frac{m_0(\Delta - 1)g}{F_v} A\omega \sqrt{\left[\frac{m_0(\Delta - 1)}{m_1\sqrt{1-\delta^2}}\right]^2 - \left(\frac{F_h}{m_1 A\omega^2}\right)^2}, \tag{1}$$

where F_v and F_h are dry friction resistance forces at vertical and horizontal particle motion, respectively; m_1 is the particle mass, including the added medium mass; m_0 is the medium mass per particle volume; $\Delta = \rho/\rho_0$ is the particle density ratio to the average (bulk) density of the medium; g is the free fall acceleration,

$$\delta = m_0(\Delta - 1)g/F_v. \tag{2}$$

Note that the values of force F_v may differ for upward and downward particle movement. As a rule, it may be assumed that $\delta \ll 1$. Then, equation (1) takes the form of

$$V_z = -A\omega \frac{m_0(\Delta - 1)g}{F_v} \sqrt{\left[\frac{m_0}{m_1}(\Delta - 1)\right]^2 - \left(\frac{F_h}{m_1 A\omega^2}\right)^2}. \tag{3}$$

Formula (3) implies the inequality of

$$m_0 \left|\Delta - 1\right| A\omega^2 > F_h, \tag{4}$$

which requires the amplitude of the inertial force at relative particle motion in the medium to exceed resistance force F_h.

According to formula (1), a denser particle, as compared to the medium ($\Delta > 1$), moves downwards, while a less dense particle ($\Delta < 1$) goes upwards ($V > 0$).

However, this formula does not take into account the wedge effect mentioned above. In order to include this effect, the following expression may be suggested for additional particle displacement during period $T = 2\pi/\omega$:

$$S_{\bigcirc} = k_{\bigcirc} A \left|\Delta - 1\right| \frac{m_0 \left|\Delta - 1\right| A\omega^2 - F_h}{F_h}. \tag{5}$$

Similarly to formula (3), the latter formula requires fulfillment of condition (4), and includes a certain dimensionless coefficient $k_{\bigcirc}$. Here and below, parameter indices correspond to the vibration trajectories: $\bigcirc$ stands for cyclic vibration, — stands for straight horizontal vibration; and | means straight vertical vibration. The value of $S_{\bigcirc}$ is always positive, as it should be, since the effect causes the particle to move up.

As a result, with account of formulae (4) and (5), the particle velocity equation (with the wedge effect) is as follows:

$$V_{\bigcirc} = \left[A\omega - \frac{m_0(\Delta - 1)g}{F_v} \sqrt{\left[\frac{m_0}{m_1}(\Delta - 1)\right] - \left(\frac{F_h}{m_1 A\omega^2}\right)^2} + \right.$$
$$\left. + \frac{k_{\bigcirc}}{2\pi} \left|\Delta - 1\right| \frac{m_0 \left|\Delta - 1\right| A\omega^2 - F_h}{F_h}\right]. \tag{6}$$

At $V_{\bigcirc} > 0$, the particle moves up; and at $V_{\bigcirc} < 0$, the particle moves down.

The segregation effect is, therefore, preconditioned by inequality (4), which causes either downward or upward displacement of the particle. Considering that force F_h is proportional to the squared particle size D , and mass m_0 is proportional to its cube, segregation condition (4) may be expressed as the following correlation between the values of D and ω:

$$q_{\bigcirc} D \left|\Delta - 1\right| A\omega^2 > 1, \tag{7}$$

where $q_{\bigcirc} = m_0/F_h D$ is a certain constant coefficient.

Note the above assumption that the oscillation amplitude of particles in the medium coincides with the oscillation amplitude of the vessel. This assumption is true for a medium-sized vessel (in terms of transverse dimensions), with significantly high influence of its sidewalls. When a medium layer is located in a large vessel or on a flat surface, it is set into motion solely by dry friction against the surface. The amplitude of particle oscillations in the medium, in this case, will never exceed the oscillation amplitude of the vessel. This is referred to as a pseudo-resonance effect, implying a peak-based dependence of the particle immersion rate on oscillation frequency ω. This effect was previously explained by resonance phenomena, whereas in fact it is due to the combined action of inertia and dry friction forces [1].

4 Segregation under straight horizontal harmonic vessel oscillations

In this case (see Fig.4, b), particle motion in the medium is described by differential equations

$$\begin{array}{l} m_1\dot{u} = m_0(\Delta - 1)A\omega^2\cos\omega t - F_h\frac{u}{\sqrt{u^2+w^2}}, \\ m_1\dot{w} = -m_0(\Delta - 1)g - F_w\frac{w}{\sqrt{u^2+w^2}}, \end{array}, \tag{8}$$

resulting from the equations applying to circular oscillations [2] at $v \equiv 0$. In these formulae u , v and w are particle velocities relative to the medium in cases of horizontal and vertical movement, respectively.

Upon introduction of dimensionless quantities

$$\begin{array}{l} u_1 = \frac{u}{A\omega}, \quad w_1 = \frac{w}{A\omega}, \quad \tau = \omega t, \quad w_* = \frac{A\omega^2}{g}, \\ f_h = \frac{F_h}{m_1 A\omega^2}, \quad f_w = \frac{F_w}{m_1 A\omega^2}, \quad q = \frac{m_0}{m_1}(\Delta - 1), \end{array}, \tag{9}$$

equation (8) may be rewritten in the form of

$$\begin{array}{l} u'_1 = q\cos\tau - f_h\frac{u_1}{\sqrt{u_1^2+w_1^2}}, \\ w'_1 = -\frac{q}{w_*} - f_w\frac{w_1}{\sqrt{u_1^2+w_1^2}}, \end{array}, \tag{10}$$

where the dash indicates differentiation by τ .

In contrast to circular oscillations, equations (10) do not allow exact solutions. Analysis of the equations shows that in case of circular oscillations, the particle moves steadily along a helical line, while in this case it moves steadily along a saw-tooth trajectory. Approximate solutions for equations (10) were obtained through direct separation of motions [1].

The resulting equation for determining dimensionless average particle velocity $W_1 = \langle w_1 \rangle = W/A\omega$ is as follows[1]:

$$|\delta| = \frac{2}{\pi\sqrt{1+\frac{q^2}{W_1^2}}} K\left(\frac{q^2}{q^2+W_1^2}\right), \tag{11}$$

where $K(x)$ is the complete elliptic integral of the first kind and, according to (2) and (9),

$$\delta = \frac{q}{f_w w_*} = \frac{m_0 g(\Delta - 1)}{F_w}, \qquad q = \frac{m_0}{m_1}(\Delta - 1).$$

Equation (11) determines dimensionless particle downward velocity W_1/q as a function of downward displacement parameter δ (Fig. 5).

Note that a constant average particle downward velocity is only ensured when $\delta = m_0 g(\Delta - 1)/F_w < 1$, as shown in Fig. 5. At $\delta > 1$, accelerated particle movement is observed, which is of no practical interest. It should also be noted that ratio (11) is true under condition (4).

This ratio does not account for the wedge effect. When the effect is included, the particle downward (or upward) velocity formula will be similar to formula (6):

$$V_- = A\omega\left[-|W_1| + \frac{k_-}{2\pi}|\Delta - 1|\frac{m_0|\Delta - 1|A\omega^2 - F_h}{F_h}\right], \tag{12}$$

where W_1 is determined from ratio (11) or using the curves in Fig. 5.

[1] Formula (11) and curves in Fig. 5 were developed by V.S. Sorokin.

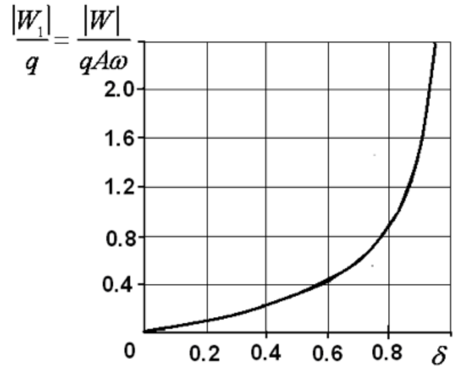

Figure 5: Dependence of the dimensionless particle downward velocity W_1/q under horizontal harmonic oscillations of the medium (without wedge effect) on parameter δ.

5 Segregation in straight vertical oscillations of the vessel

This case (excluding the wedge effect) was previously studied and described in [1]. In particular, an approximate formula was obtained for the average particle velocity

$$V_z = -A\omega \frac{m_0\,|\Delta - 1|}{m_1} \sin \frac{\pi}{F_+ + F_-} \left[\frac{1}{2}(F_- - F_+) - m_0(\Delta - 1)g\right], \tag{13}$$

which is true if

$$m_0\,|\Delta - 1|\,A\omega^2 > Inf\,[F_+,\ F_-]$$

This inequality implies that the inertial force amplitude in particle relative motion should exceed, in absolute magnitude, the smallest of forces F_+ and F_-. This condition that, in this case, replaces inequality (4) is required for particle motion relative to the medium, i.e. is prerequisite to segregation.

Another requirement for the validity of approximate formula (13) is associated with the specifics of its mathematical derivation and has the form of

$$m_0\,|\Delta - 1|\,A\omega^2 \gg Sup\,[F_+,\ F_-]\,, \tag{14}$$

ensuring that the previous inequality is true.

Formula (13) implies that a denser particle, as compared to the medium, moves upwards if

$$\tfrac{1}{2}(F_- - F_+) > m_0(\Delta - 1)g\ ,$$

i.e. the half-difference between forces F_- and F_+ exceeds the particle weight in the medium; in other cases, the particle moves downwards.

It should be noted that the wedge effect, in this case, may be taken into account by selection of a sufficiently high value of particle downward movement resistance force F_-. However, if it is assumed that forces F_+ and F_- are derived from "static" experiments (without any wedge effect), the particle velocity shall be calculated by a formula similar to (6) and (12) and having the form of

$$V_| = V_z + A\omega \frac{k_1}{2\pi}\,|\Delta - 1|\,\frac{m_0\,|\Delta - 1|\,A\omega^2 - F_-}{F_-}.$$

where k_1 is the respective dimensionless factor.

Note that, in a different form, the wedge effect has already been considered in papers [5, 6], covering the mechanism of expulsion of heavy objects to the ground surface under seismic vibrations.

6 Experimental investigations

The detailed content of the experimental results is supposed to state in a subsequent article. Here we present the results of only two sets of experiments.

1) Experiments on the surfacing of steel balls of different diameters in the sand in the case of the horizontal vibration of the vessel.

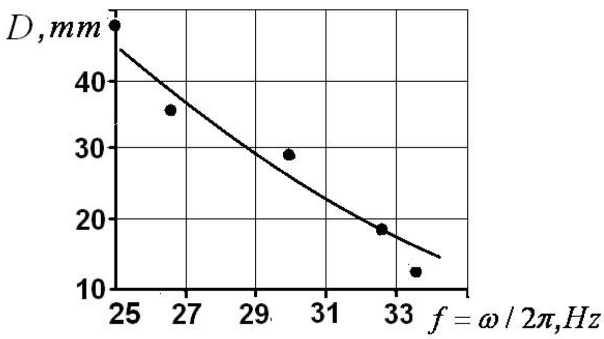

Figure 6: The smallest diameters of steel balls coming up to the sand surface from the depth of 100 mm, depending on the frequency of horizontal vibrations

Experiments were performed at the oscillation amplitude A=2,2 mm and for different values of frequency $f = 2\pi/\omega$. Figure 6 and experimental points show theoretical curve $D = C/f^2$ (C =const) obtained from relation (7) by replacing the inequality sign with the equality sign, in compliance with the experimental conditions. It is quite obvious that correlation (7) remains true in this case; moreover, the same correlation between D and $\omega = 2\pi f$ follows from (4), (7) and (14), if the inequality sign is replaced with the equality sign. Therefore, the above curve well agrees with the experimental data.

2) Experiments on horizontal segregation.

Experimental installation was fixed on the table of the vibration stand interfacing bottom flange 5 (Fig. 7) with the working surface of the table using bolted connections so that its axis coincides with the axis of the table stand. As the bulk material in a vessel filled mixture of pea seeds - 200 cm^3 and nuts (hazelnuts) - 800 cm^3. Density of $\rho_o = 0,48$ g/cm^3 nuts, typical size 14 - 17 mm and a density of 0.90 g/cm^3 pea seeds, the characteristic size of 5.5 - 6.5 mm. The experiments were performed at an amplitude of A=2.2 mm, a frequency until 35 Hz for horizontal and vertical linear oscillations. The experiment was to evaluate the total amount of pea seeds that came out of the holes of the vessel for a certain period of vibration of varying intensity. Grains were collected in a transparent plastic bag, which was placed inside the vessel with particulate material (Fig. 7).

Experimental results for the vertical and horizontal vibrations are presented in Figures 8 and 9. Each mode of vibration exposure duration was 3 min. A longer duration of exposure to vibration did not make sense, since grain peas, as preliminary studies, then almost did not get out of the holes of the vessel.

Figure 7: Photo of the experimental setup for the study of the classification of bulk materials.

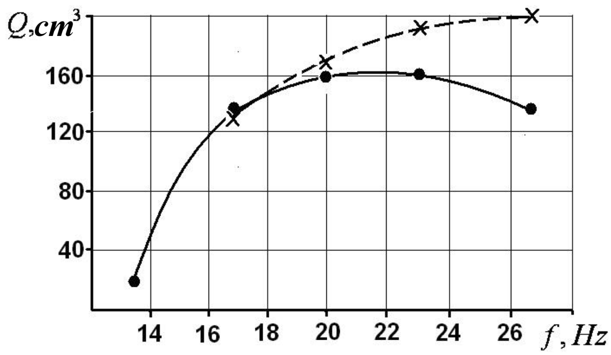

Figure 8: Volume pea seeds, emerging from the container side openings, depending on the frequency of vibration (–•– horizontal vibration; –×–vertical vibration).

For horizontal vibration, as seen from Fig. 8, the maximum output of pea seeds Q= 160 cm^3 accounts for the frequency near 22 Hz. In this case the vessel is one fifth of the original volume of filled seeds of peas. Increase or decrease of the frequency leads to a decrease in yield pea seeds from the vessel.

When vertical vibration with frequency increase from the output 13.3 Hz pea seeds from the vessel and at a frequency f = 7.26 Hz vessel pea grains not remains. The results of experiments to determine the dependence of the pea seeds on the duration of exposure to vibration with frequency f = 26.7 Hz are shown in Fig. 9. As can be seen, even at the two-minute exposure vessel remains practically without pea seeds.

7 Conclusion

This article uses the model of a single particle that is alien to other particles of the medium to provide a physical explanation and a mathematical description of segregation effects occurring under three characteristic types of vibration. Considered the influence factor called wedge effect. Detected and attributed the effect of segregation in a horizontal direction toward the sidewall of the vessel. A new type of vibration classifier based on this effect. It

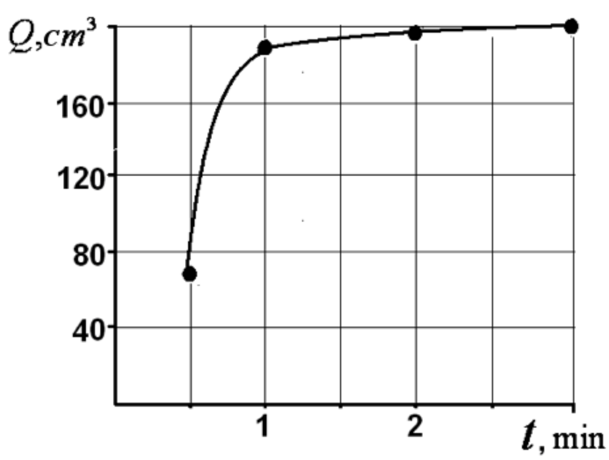

Figure 9: Volume Q pea seeds that came from the vessel during exposure t to vertical vibration.

is also shown that the theoretical results well agree with the experimental data.

Acknowledgements

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I.I. Blekhman, V.O., Bolshoy pr. 61, St.Petersburg, 199178, Russia