Pattern formation inside a rapidly rotating horizontal cylinder partially filled with liquid and granular medium

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Abstract

The dynamics of liquid and granular medium in a rapidly rotating horizontal cylinder is experimentally studied. In a rapidly rotating cylinder liquid and granular medium coat the cylindrical wall under centrifugal force. Oscillating (in the rotating frame) gravitational force generates inertial waves with various axial and azimuthal wavenumbers [1, 2]. Inertial waves generate intensive oscillations in liquid and granular medium and fluidize the latter. Oscillatory liquid flow near the flat sand bed provoke the onset of regular patterns in the form of ripples perpendicular to the axis of oscillation. In nature, sand ripples are formed on sea beds under back and forth motion of liquid. Although the formation of these patterns has been studied for a long time, the dominant wavelength selection mechanism is not clear yet. The surface waves also generate averaged liquid flows in the direction of its propagation [3] and bring ripples on the sand bed to the continuous drift [4]. In nature, drift of patterns is caused by wind in a desert or by water in a river.

1 Experimental set-up

The experimental set-up used in the investigation of pattern formation in the rotating cavity is shown in Fig. 1. The experiments are conducted with the transparent Plexiglas circular cylinder *1* of length $L = 7.8$ cm and inner diameter $D = 12.6$ cm mounted on the rigid stand *2*. The cylinder is rotated by the stepper motor FL86STH80-4208A *3*. The angular velocity of rotation Ω varies in the range from 0 to 50 s^{-1} with use of the generator of electric signals Gw Instek GAG-810. The experiments are conducted in natural or stroboscopic illumination through the end wall of the cell. The frequency of stroboscopic illumination equals to the rotation rate and is managed by use of DAC ZETLab. At the center of the rear wall of the cylinder there is a hole to fill it with liquid and granular medium. The experiments are conducted with water-glycerol mixtures; viscosity varies in the range from 1 to 30 cSt. The quantity of liquid is characterized by the relative filling $q = V/V_0$ (V is fluid volume and V_0 is a cavity volume) which varies in the range from 0.1 to 0.4. The experiments are performed with spherical glass particles of diameter of 0.15 - 0.25 mm; granular medium mass m equals to 125 or 250 g.

Images of the cell are recorded via digital camera Nikon D40 positioned along the axis of rotation *4*, facing a transparent side wall. In order to provide high quality images of the liquid and sand the cross-section of the cell is illuminated by a LED flexible tape placed around the cylindrical wall. These images are then analyzed in order to explore the dynamics of inertial waves on the free surface of liquid and spatiotemporal evolution of patterns on the liquid вЂ“ sand interface. In certain cases it is required to measure the velocity of wave propagation and the free surface of liquid is illuminated by a stroboscope lamp.

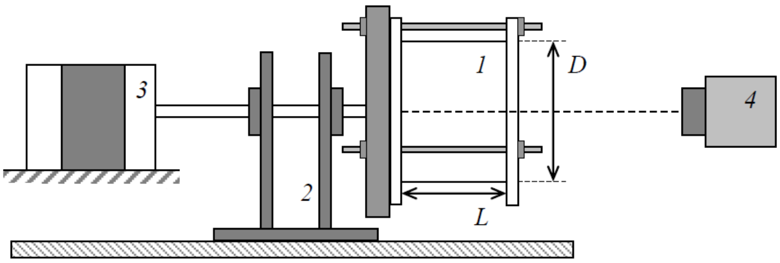


Figure 1: Experimental set-up

2 Experimental data

At the beginning of the experiment the cell rotates sufficiently fast and liquid and granular medium rest in the rotating frame. The decrease of Ω results in the excitation of liquid oscillations induced by gravitational force. When the angular velocity Ω is equaled to the natural frequency of liquid oscillations then the inertial waves are excited [1, 2]. In the studied range of $q > 0.20$ the gravitational force induces two-dimensional traveling waves with azimuthal wavenumber l equaled to 1 or 2 [5].

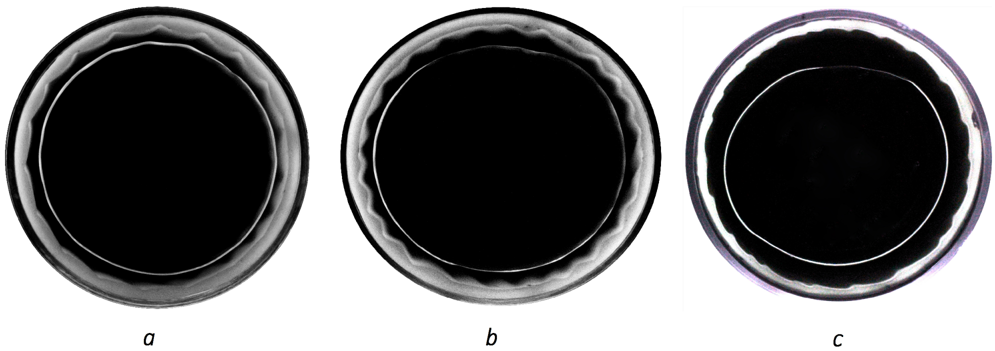
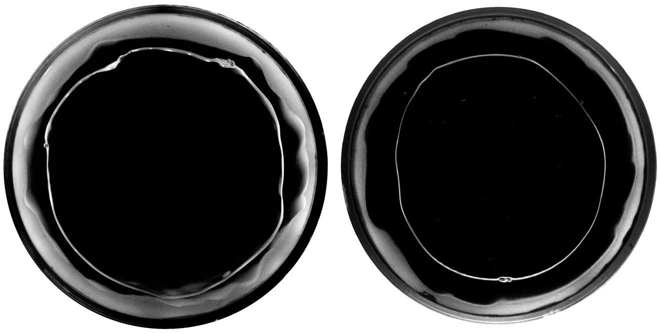


Figure 2: Regular patterns on the sand - liquid interface: $\Omega = 35.6\ s^{-1}$, $\nu = 1.4$ cSt, $q = 0.215$ (a), $\Omega = 33.0\ s^{-1}$, $\nu = 3.2$ cSt, $q = 0.257$ (b) and $\Omega = 31.1\ s^{-1}$, $\nu = 20.4$ cSt, $q = 0.360$ (c); $m = 250$ g

Intensive inertial waves produce liquid oscillations and provoke onset of regular patterns. In the threshold of pattern formation azimuth waves generate a quasistationary relief in the form of regular squashed ripples (Fig. 2a). In the supercritical domain regular patterns take the form of series of symmetric hills extended along the axis of rotation (Fig. 2b). In the range of large values of relative filling $q > 0.3$ the interface is unstable to excitation of regular dunes (Fig. 2c). A noteworthy fact, that within tens of minutes regular hills and regular dunes transform to the series of irregular patterns (Fig. 3).

It is found that decrease of rotation rate is followed by increase of spatial period of regular patterns λ. Fig. 4 demonstrates averaged value of λ measured by images taken within a few minutes after appearance of regular patterns in dependence on rotation rate Ω.

In the experiments with low viscous liquids the regular patterns are observed in a wide range of rotation velocity. The increase of viscosity brings to the narrowing of a domain of the relief existence. In the range of relative filling $q = 0.2$ - 0.3 experimental data agree with each other for the fixed viscosity in the plane Ω, λ . Experimental data obtained in the range $q > 0.3$ do not agree with each other in the plane of chosen parameters.



a *b*

Figure 3: Time evolution of ripples on the liquid вЂ“ sand interface: $\Omega = 33.0\ s^{-1}$, $\nu = 3.2$ cSt, $q = 0.257$ and $t = 60$ min (a); $\Omega = 31.1\ s^{-1}$, $\nu = 20.4$ cSt, $q = 0.360$ and $t = 120$ min (b); $m = 250$ g

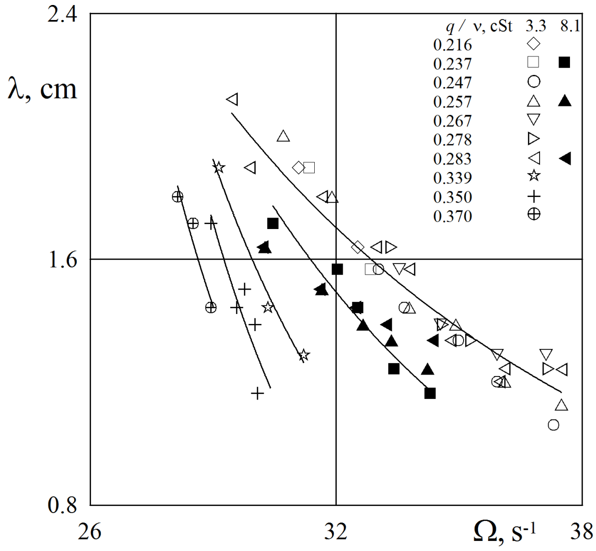


Figure 4: The spatial period of regular ripples versus angular velocity; mass of sand $m = 250$ g

3 Discussion

Both liquid and sand inside a horizontal rotating cylinder are affected by oscillating (in the rotating frame) gravitational force. It results in excitation of liquid oscillation and under certain conditions the inertial wave excites. Intensive inertial waves produce tangential oscillations on the liquid - sand interface and induce fluidization of the upper layer of the granular medium. Inertial waves propagated along the liquid - fluidized sand interface induce oscillatory motion in both media and provoke Kelvin-Helmholtz instability [6] which is followed by onset of series of regular ripples extended along the axis of rotation.

The experimental data prove that evolution of sand ripples is influenced by liquid dynamics. According to the observations, ripples rise in the presence of inertial waves only. The observations are supplemented by the measurements of dimensionless acceleration $\Gamma = g\ /\ \Omega^2 a$ (g is the acceleration due to gravity, a is the radius of air column) in the thresholds of inertial waves' excitation and rise of ripples in dependence on q (Fig. 5). The nonmonotonic dependence of Γ on q in the threshold of wave excitation is determined by

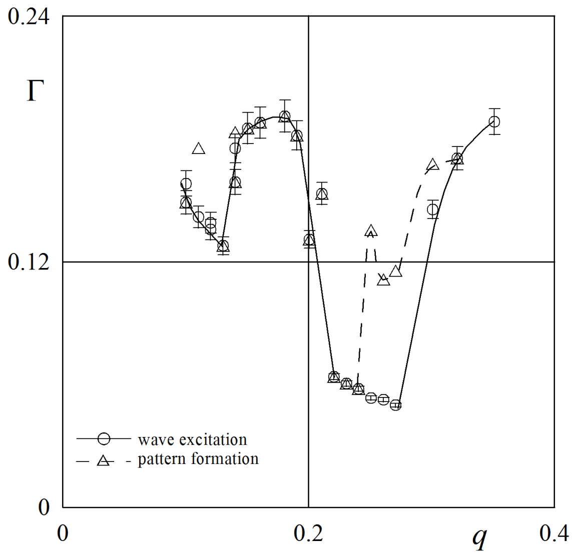


Figure 5: The thresholds of appearance of inertial waves and regular patterns; ν = 1 cSt, m = 125 g

the resonant intensification of various modes of liquid oscillations and is studied in detail in [2]. One can find that inside the resonant sack regular patterns rise in the form of symmetric hills, outside - in the form of spatially periodic dunes.

The analysis of experimental data in the exclusive domain q = 0.2 - 0.3 shows that λ is determined by dimensionless parameter $\Gamma\omega_R^{1/2}$, where $\omega_R = \Omega R^2/\nu$ is the dimensionless frequency based on radius of a cylinder R. As the unit for the wavelength we introduce the thickness of viscous boundary layer $\delta = \sqrt{2\nu/\Omega}$.

The experimental data obtained in liquids of different viscosities are in good agreement with each other in the range of moderate relative filling q = 0.2 - 0.3 (Fig. 6). If the regular dunes are generated then the experimental data do not fit to the general law $\lambda/\delta \sim (\Gamma\omega_R^{1/2})^{4/3}$.

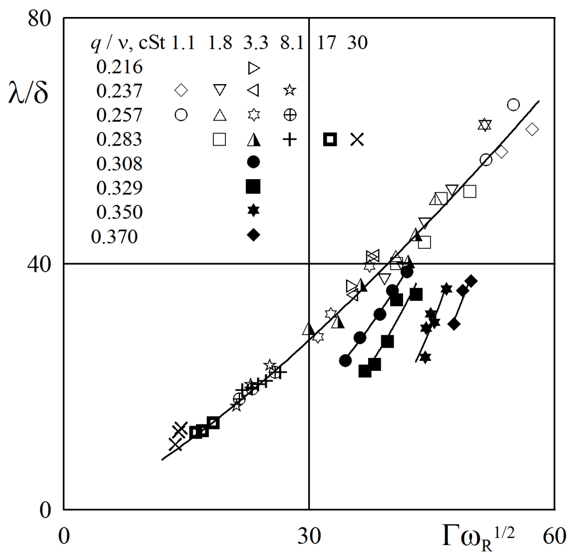


Figure 6: Dimensionless spatial period of regular ripples versus dimensionless parameter $\Gamma\omega_R^{1/2}$; solid line corresponds to $\lambda/\delta \sim (\Gamma\omega_R^{1/2})^{4/3}$, m = 250 g

4 Conclusion

The dynamics of liquid and granular medium in a rapidly rotating horizontal cylinder is experimentally studied. It is found that axisymmetrical sand bed is unstable to the onset of regular patterns in the form of ripples extended along the axis of rotation. In dependence on relative filling of liquid q regular patterns take a form of symmetric hills or dunes extended along the rotation axis. The pattern formation is associated with excitation of 2D inertial waves caused by gravitational force. In the range of relative filling $q = 0.2$ - 0.3 the wavelength of regular ripples is determined by dimensionless parameter $\Gamma\omega_R^{1/2}$.

Acknowledgements

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