Generalized Time, “Horizontal” and “Vertical” Scaling at Modelling of Deformation, Structural Transitions and Destruction of Rheologyically Complex Media

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Abstract

Variants of the phenomenological description of the mechanical stressed-deformed and structural condition, damageability and durability of linear and nonlinear tenacity-elasticity-plasticity media are considered on the concept of generalized time, various complexity, different “horizontal” and “vertical” scales.

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1 Introduction

The problem of time [1-3, etc.] is one of the basic and thorny questions of knowledge. There is a wide contrast spectrum of scientific and other representations of time in various fields of knowledge and activity: in physics, mechanics, chemistry, biology, medicine, psychology, philosophy, sociology of religion, etc. All stored up scientific and other representations of time, ways of its definition and measurement are useful and can be rather valuable and productive for many aspects of studying, creation, mastering and creation of objects of activity of Man.

For modelling of determining mechanical properties of various media a new “endochronic” approach was developed in the last decades which is based on the application of generalized (own, internal, etc.) time [4]. Introduction of generalized time is based on physical-chemical-mechanical-time conformity of the time of process and the size of influence, shown in various real conditions and experiments, such, as temperature-, humidity-, radiation irradiation-, structure-, stresses-, deformation-time and other conformities. The greatest difficulty of description represent processes of nonmonotonic deformation, structural changes and accumulation of damages, modelling of phenomena of splitting damageability.

In the present work ways of physical and mathematical modelling of linear and nonlinear processes of deformation, physical (structural, phase) transitions, durability and the destructions which are based on “horizontal” and “vertical” scaling of time are considered; with the use of “simple” and “complex” functions, and also “a functional kind” of scales that enables to describe and predict “instant” and very long mechanical processes adequately and effectively, as well as structural transitions in various media. It allows to generalize uniformly and rather simply the known theories of nonlinear tenacity-elasticity-plasticity, structural transitions, durability and destructions, based on other principles, clearly and consistently classify various theories on designs of scales. Scales are physical parameters of structure reflecting its changes at influences.

2 The equations of tenacity-elasticity-plasticity

Generalized time ξ of a mechanical condition of media in case of influence on it of any mechanical, physical and chemical fields with parameters m_i (mechanical), T_k (physical and chemical); $i = 1, 2, \ldots$, $k = 1, 2, \ldots$; can be expressed through laboratory t by "horizontal" scaling (scale multiplication) of the last:

$$\xi = g(t,\ m_1,\ m_2, \ldots,\ T_1,\ T_2, \ldots)t. \tag{1}$$

A "complex" scale g, dependent on time t is presented here. In a "simple" case, it can have a form of a product of scales of separate fields: $g(m_1,\ m_2,\ \ldots,\ T_1,\ T_2, \ldots) = g^{m_1}(m_1) \cdot g^{m_2}(m_2) \cdots g^{T_1}(T_1) \cdot g^{T_2}(T_2) \cdot$, and it does not depend on t. At nonmonotonic influence the increment of generalized time looks like $d\xi = \tilde{G}(t,\ m_1,\ m_2,\ \ldots,\ T_1,\ T_2,\ \ldots)dt$, where $\tilde{G}$ - is a functional (the hereditary integrated operator considering "memory" of medium on influence).

Determining ratio of media is constructed at the complex stressed-deformed condition, with the application of generalized time for the deviatored and spherical parts of mechanical tensors in case of the isotropic medium and anisotropic generalized time - for anisotropic [9] medium.

Let's consider the behaviour of media at single-axial stretching-compression under the absence of other influences. At nonmonotonic load communication of deformation with a stress (nonlinear creep-plasticity) looks like a quasy-linear integrated ratio:

$$\varepsilon(t) = \bar{P}^{\xi}\sigma = \int_{0^-}^{t} P(\xi^{\sigma} - \zeta)\dot{\sigma}(\tau)d\tau. \tag{2}$$

"The opposite" to it, the equation of relaxation has the similar form. We shall accept $\xi^{\sigma} = \xi^{\varepsilon} = \xi$, then functions of creep P and relaxation R as well as scales can be calculated one on another - on linear and nonlinear ratio

$$\bar{P}R = \bar{R}P = \int_{0^-}^{t} P(\xi - \zeta)\dot{R}(\zeta)d\zeta = \int_{0^-}^{t} R(\xi - \zeta)\dot{P}(\zeta)d\zeta = 1.\xi = \xi^{\sigma} = \xi^{\varepsilon}. \tag{3}$$

Experiments have shown, that already at the moderate deformations many media possess a "complex" time scale $g^{\sigma}(t, \sigma)$ in a mode load $\sigma(t) = H(t)\sigma^0$ ($H(t)$-united Heaviside function, $\sigma^0 = const$). In case of nonmonotonic process $\sigma(t)$ the argument in the ratio (2) is a functional. For media to which we shall apply a principle of addition of Bolzman-Volterra-Perso [6] an argument of the kind is received [9]

$$\xi^{\sigma} - \zeta = \bar{G}^{\sigma}_{\tau,t} = \int_{\tau^+}^{t^-} G^{\sigma}\left[t - \rho, \sigma(\rho)\right]d\rho = \int_{\tau^+}^{t^-} \left\{ g^{\sigma}\left[t - \rho, \sigma(\rho)\right] - \frac{\partial g^{\sigma}\left[t - \rho, \sigma(\rho)\right]}{\partial \rho}\rho \right\} d\rho. \tag{4}$$

When $\sigma \le \sigma_l$ (σ_l - border of linear creep) is $g^{\sigma}(t, \sigma) = 1$. At $\sigma(t) = const$ $\xi^{\sigma} - \zeta = g^{\sigma}(t - \tau, \sigma)(t - \tau)$, $\xi^{\sigma}(t) = g^{\sigma}(t, \sigma) \cdot t$. For the description of the accelerated or slowed down response (hardening or dishardening) a scale-functional $\tilde{G}^{\sigma}$ constructed by a hierarchical principle and representing product of complex scale-function $G^{\sigma}(t,\sigma)$ and scale-functional $\tilde{g}^{\sigma}(\dot{\sigma})$, - correcting G^{σ}, is entered:

$$\tilde{G}^{\sigma}(t,\sigma,\dot{\sigma}) = G^{\sigma}(t,\sigma)\cdot\tilde{g}^{\sigma}(\dot{\sigma}), \tilde{g}^{\sigma}(\dot{\sigma}) = 1 + \int_{0}^{\rho} q^{\sigma}(\lambda)\dot{\sigma}(\lambda)d\lambda; q^{\sigma}(0) = 0. \tag{5}$$

The nonlinear variant of scale $\tilde{g}^{\sigma}$ is considered also in [9]. Thus, generalized time and scale-functional possess "memory" since they consider background influences on media.

Endochronic updatings of the nonlinear equations of tenacity-elasticity of Bolzman-Volterra-Perso have been received and checked up experimentally [9], the kinetic theory of durability of Zgurkov with above-barriered and subbarriered (tunnel) transitions [10], the equations of nonlinear creep of Rabotnov [11], technical theories of creep [12], and Moskvitin's ratios [13]. Comparison of modifications has shown the difference between the kind and degree of "complexity" of the received scales of mentioned theories.

The additional opportunity of the account of nonlinear properties is given with the introduction of a "vertical" scale g^{v} [4, 14] by which function of creep P in (2) is multiplied: $g^{v\sigma}(\sigma)\cdot P(\xi^{\sigma})$, or - a function of relaxation: $g^{v\varepsilon}(\varepsilon)\cdot R(\xi^{\varepsilon})$. It enables to correct and make nonlinear the instant module, to modify the function of tenacity-plasticity and to simplify the form of determining functions.

3 Criteria of long durability and structural stability.

For some materials, for example aluminium and polymetilmetakrilat, an endochronic criterion of durability at the stretching [15] of the final form is established experimentally:

$$\xi^{\sigma}(t) \le \xi^{\sigma}_{c} = const, \tag{6}$$

based on "complex" stressed-time analogy (1). The criterion of this kind is effective for the estimation of characteristic zones of transitions on curves of long durability and a limit of fluidity, for example, for the allocation of area of transition from a dynamic mode to static and at transition of media from one to other structural (physical, phase) conditions, for example

$$\xi^{\sigma}(t) \le \xi^{\sigma}_{m_i} = const, m_1 = d, m_2 = p. \tag{7}$$

A uniform endochronic criterion of long durability at the influence of stress and constant temperature is constructed. The criterion is based on the following connection of generalized time with the laboratory time [16]:

$$\xi^{\sigma,T}_{c}(t|_{\sigma_0,T_0},\sigma,T) = g^{\sigma,T}(\sigma,T)\cdot t|_{\sigma_0,T_0}. \tag{8}$$

Here $t|_{\sigma_0,T_0}$ - is the size of long durability at some chosen values of a stress σ_0, , and $g^{\sigma,T}$ - is the scale (measure) of time $t|_{\sigma_0,T_0}$, being a factor of similarity and a function of two sizes - stress σ, and temperature T, - time $t|_{\sigma_0,T_0}$, stantartized on values σ_0 and temperature T_0. At the stress σ, and temperature T the generalized time $\xi^{\sigma,T}_{c}$ coincides with the time t_c. Analytical expressions of time scales for fragile, viscous and mixed of media, the time of destruction which is described by various theories by Gul, Zgurkov, Regel, Slucker, Tomashevsky, Trunin and other researchers are received.

At variable processes of influences on media its long durability can be calculated under the linear or nonlinear law of accumulation of damages, including the use of generalized time:

$$\omega(t)=\int\limits_0^{\zeta}\frac{d\zeta}{\xi_c^{\sigma,T}(\sigma,T)}=\frac{1}{t_{\sigma_0,T_0}}\int\limits_0^{t}\frac{d\rho}{g_c^{\sigma,T}\left[\sigma(\rho),T(\rho)\right]}=\leq 1. \tag{9}$$

For the construction of nonlinear schemes of accumulation of damages in the equation (9) it is necessary to enter a different kernel, or to replace in it the "simple" depending only on σ, T, time scale $g_c^{\sigma,T}(\sigma,T)$ by a "complex", and, even, "functional" scale (with "memory" of nonmonotonic influences)$\tilde{G}_c^{\sigma,T}(\rho,\sigma,\dot{\sigma},T,\dot{T})$.

4 Conclusion

Models and criteria of linear and nonlinear tenacity-elasticity-plasticity, of structural transitions, damageability and destruction of rheology complex media, based on the endochronic concepts allowing to describe effectively mechanical behaviour of media are considered. Application of generalized time with the use of "horizontal" and "vertical" scales of various complexity and hierarchy is analysed. The given approach allows to predict the behaviour of media at small and big times, and to carry out the planning of the express-tests of materials.

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